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Optimization of Steel Structural System under Earthquake Excitation

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Abstract

In most optimization design methods, continuous optimization techniques have been often used. However, in general, structural design parameters consist of discrete elements. And, optimization design by computers makes it possible not only to improve the quality and efficiency of designs but also to discover novel design process beyond human capabilities. Hence, an algorithm which is relevant to such problems is desirable.
This paper deals with a new stochastic combinational optimization algorithm which uses the importance sampling procedure based on a Monte Carlo method. In this proposed procedure, discrete design parameters are randomly sampled from region of solution space, and allowable region is updated and condensed from the objective function of minimum weight and minimum displacements of nodes. An example of a 16 bar truss structure under earthquake excitation is considered, and numerical analysis is carried out for an optimum steel structural design where specified ready-made steel products are dealt with as elements of structure.

Key Words

Optimum design, Combinational optimization procedure, Stochastical process, Steel and truss structure

1.Introduction

In conventional optimization process of structural design under seismic loading, designers have to search for optimal design values of element parameters so as to satisfy constraints strictly and quantitatively. Since these design parameters such as cross sections of ready-made steel products are discrete, the objective function of design process is also discrete. And, to solve a combinational optimization problem, means to find the best or at least semi-optimal solution among a finite or countable numbers of alternative solutions.
In most optimization design problems, optimization techniques such as Newton method which deal with only continuous variables have been often used. However, in general, structural design parameters consist of discrete elements. And, optimization design by computers makes it possible not only to improve the quality and efficiency of designs but also to discover novel design process beyond human capabilities. In addition, these combinational optimization problems are always subject to danger of falling in local minima. Hence, an algorithm which is relevant to such combinational optimization problems is desirable.
In order to circumvent these situations, a new combinational optimization procedure which uses an importance sampling method based on a Monte Carlo method [Rubinstein (1981)] is proposed. This

procedure is to randomnize the cross sectional area of a member of truss structure, and then updating the governing conditional probability density function, the expectation of the minimum weight and minimum displacements of nodes, is obtained recursively [Sutoh et al (1995)]. In other words, proposed procedure define state variables as probabilistic and turns out to be a Markovian approach such as an importance sampling procedure in sense that the sampling region for location of optimal solution is recursively condensed by the objective function of minimum weight and minimum displacements of nodes.
Regarding the importance sampling procedure, a series of research works by Bucher (1988) and Pradlwarter et al (1994) must be mentioned. These studies are mainly concerned with evaluation of structural reliability for complicated structures. Recently, a new artificial life approach which is named genetic algorithm has been applied on combinational optimization problems. Genetic algorithm is an optimization procedure representing simplified natural mechanism in which fitness function is used instead of objective function. Basic functions of genetic algorithm are selection, crossover, reproduction and mutation. Since these functions are of random nature except for reproduction, genetic algorithm may be called a probabilistic search procedure. This principle was applied to engineering problems as genetic algorithm [Goldberg (1989)]. An earliest model of genetic algorithm, however, was developed by Holland (1975) based on the research on group automaton of Fogel et al (1967).
Finally, an example of a 16 bar truss structure under earthquake excitation is considered, and numerical analysis is carried out for an optimum steel structural design where specified ready-made steel products are dealt with as elements of structure. And it was found that procedure is a stable and robust determination of design parameters, and can be easily applied even for optimum design problems under seismic loading, since the procedure handles with the algorithm of an importance sampling procedure, in parallel with structural analysis.

2. Formulation of Combinational Optimization

General combinational optimization problem can be formulated in the following way. Find a vector of discrete design variables $y \in \Omega$ so as to minimize an objective function $Z(y) = [g_1(y), g_2(y), \cdots, g_n(y)]$.

$$z(y) \rightarrow \min \tag{1.a}$$
$$y \in D, D \subset \Omega \tag{1.b}$$

where D is the allowable solution space, Ω is the solution space, **y** is a vector of discrete variables and $g_i(y), i = 1, \cdots n$ are elements of system. In this combinational optimization problem, greater computer time may be required to find the solution, otherwise the solution may fall into local minima.

2.1 Proposed Combinational Optimization Algorithm

The concept of the proposed combinational optimization algorithm has a strong analogy with the importance sampling procedure based on a Monte Carlo method.
The methodology of the proposed algorithm is expressed as follows.

Step 1

For the first step, i sets of vectors are sampled in the allowable solution space D_1 and the objective functions $z(y)$ are calculated.

1) i sets of random sampling in the allowable solution space : D_1

(Allowable solution space : D_1 Step (1) - i sets of sample vectors)

$$\begin{aligned} y^1_{(1)} &= \{y^1_1, y^1_2, \cdots, y^1_{m2}\} \\ &\vdots \\ y^i_{(1)} &= \{y^i_1, y^i_2, \cdots, y^i_{m2}\} \end{aligned} \tag{2}$$

(Objective functions from i sets of sample vectors)

$$\begin{aligned} Z(y^1_{(1)}) &= [g_1(y^1_{(1)}), g_2(y^1_{(1)}), \cdots, g_n(y^1_{(1)})] \\ &\vdots \end{aligned} \tag{3}$$

$$Z(y^{i}_{(1)}) = [g_1(y^{i}_{(1)}), g_2(y^{i}_{(1)}), \cdots, g_n(y^{i}_{(1)})]$$

where m_2 is number of discrete parameters.

2) Calculate the objective functions and update the next allowable solution space (D_2) to be condensed from the smaller objective function value compared with the mean value.

Step 2

Then, for the second step, i_1 sets of variable vectors are sampled in allowable solution space D_2 and the objective functions $z(y)$ are calculated, and so on.

3) i_1 sets of random sampling in allowable solution space : D_2

(Allowable solution space : D_2 Step (2) - i_1 sets of sample vectors)

$$\begin{aligned} y^{1}_{(2)} &= \{y^1_1, y^1_2, \cdots, y^1_{m2}\} \\ &\vdots \\ y^{i1}_{(2)} &= \{y^{i1}_1, y^{i1}_2, \cdots, y^{i1}_{m2}\} \end{aligned} \tag{4.a}$$

where the number of sample size is $i > i_1$.

(Objective function from i_1 set sample vectors)

$$\begin{aligned} Z(y^{1}_{(2)}) &= [g_1(y^{1}_{(2)}), g_2(y^{1}_{(2)}), \cdots, g_n(y^{1}_{(2)})] \\ &\vdots \\ Z(y^{i1}_{(2)}) &= [g_1(y^{i1}_{(2)}), g_2(y^{i1}_{(2)}), \cdots, g_n(y^{i1}_{(2)})] \end{aligned} \tag{5.a}$$

Then, combinational optimization problems are always exposed to danger of falling in local minima. In order to circumvent this situation, k_1 sets of variable vectors are sampled in the former solution space D_1 and the objective functions are calculated.

(Reproduction from solution space: D_1 Step (R_1) - k_1 sets of sample vectors)

$$\begin{aligned} y^{1}_{(R1)} &= \{y^1_1, y^1_2, \cdots, y^1_{m2}\} \\ &\vdots \\ y^{k1}_{(R1)} &= \{y^{k1}_1, y^{k1}_2, \cdots, y^{k1}_{m2}\} \end{aligned} \tag{4.b}$$

k_1:number of reproduction, $k_1 = \alpha \cdot i_1$ (α:reproduction rate)

$$\begin{aligned} Z(y^{1}_{(R1)}) &= [g_1(y^{1}_{(R1)}), g_2(y^{1}_{(R1)}), \cdots, g_n(y^{1}_{(R1)})] \\ &\vdots \\ Z(y^{k1}_{(R1)}) &= [g_1(y^{k1}_{(R1)}), g_2(y^{k1}_{(R1)}), \cdots, g_n(y^{k1}_{(R1)})] \end{aligned} \tag{5.b}$$

4) Similarly, calculate the objective functions and update the next allowable solution space (D_3) to be condensed from the smaller objective function value compared with the mean value.

Step j

5) i_{j-1} sets of random sampling in allowable solution space : D_j

(Allowable solution space : D_j Step (j)- i_{j-1} sets of sample vectors)

$$\begin{aligned} y^{1}_{(j)} &= \{y^1_1, y^1_2, \cdots, y^1_{m2}\} \\ &\vdots \\ y^{ij-1}_{(j)} &= \{y^{ij-1}_1, y^{ij-1}_2, \cdots, y^{ij-1}_{m2}\} \end{aligned} \tag{6.a}$$

$i > i_1 > \cdots > i_{j-1}$: number of sample size

(Objective function from i_{j-1} sets of sample vectors)

$$\begin{aligned} Z(y^{1}_{(j)}) &= [g_1(y^{1}_{(j)}), g_2(y^{1}_{(j)}), \cdots, g_n(y^{1}_{(j)})] \\ &\vdots \end{aligned} \tag{7.a}$$

$$Z(y_{(j)}^{ij-1}) = [g_1(y_{(j)}^{ij-1}), g_2(y_{(j)}^{ij-1}), \cdots, g_n(y_{(j)}^{ij-1})]$$

(Reproduction from solution space : D_i Step (R_{j-1}) - k_{j-1} sets of sample vectors)

$$\begin{aligned} y_{(Rj-1)}^{1} &= \{y_1^1, y_2^1, \cdots, y_{m2}^1\} \\ &\vdots \\ y_{(Rj-1)}^{kj-1} &= \{y_1^{kj-1}, y_2^{kj-1}, \cdots, y_{m2}^{kj-1}\} \end{aligned} \tag{6.b}$$

k_{j-1}:number of reproduction, $k_{j-1} = \alpha \cdot i_{j-1}$ (α:reproduction rate)

$$\begin{aligned} Z(y_{(Rj-1)}^{1}) &= [g_1(y_{(Rj-1)}^{1}), g_2(y_{(Rj-1)}^{1}), \cdots, g_n(y_{(Rj-1)}^{1})] \\ &\vdots \\ Z(y_{(Rj-1)}^{kj-1}) &= [g_1(y_{(Rj-1)}^{kj-1}), g_2(y_{(Rj-1)}^{kj-1}), \cdots, g_n(y_{(Rj-1)}^{kj-1})] \end{aligned} \tag{7.b}$$

In this way, discrete design parameters are randomly sampled from region of solution space, and with allowable region is updated recursively to be condensed from the objective function and optimum solutions of combinational problems is also obtained.

The flow chart of the proposed procedure is illustrated in Fig. 1.

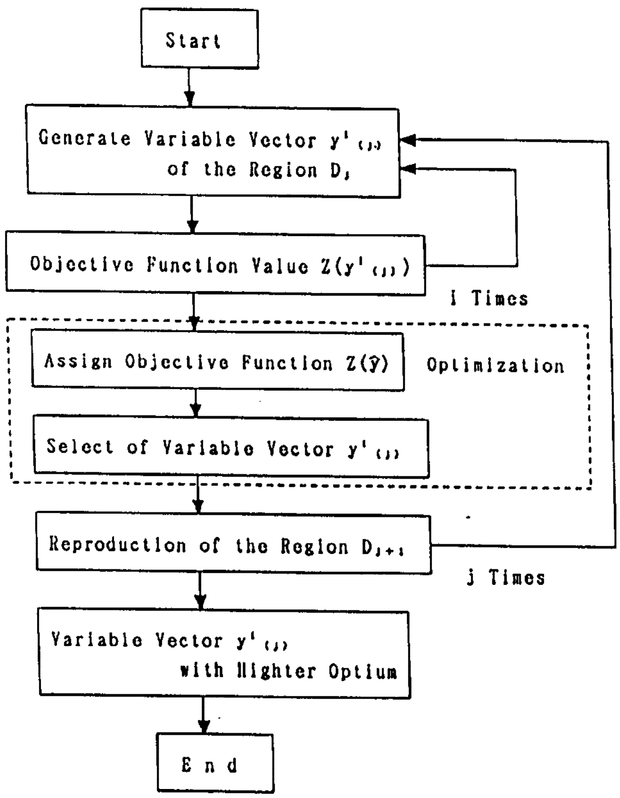

Fig. 1. Algorithm of proposed procedure

2.2 Markov Analysis

The proposed combinetional optimization procedure can be interpreted mathematically by using the theory of discrete Markov chains. This procedure is a sequence of trials, where the probability of the outcome of given sampling vectors depends only on the outcome of the previous sampling vectors.

The conditional probability of the kth outcome sample vectors y is expressed by following equation.

$$f(y_{(k)}^{i} | y_{(1)}^{i}, y_{(2)}^{i}, \cdots, y_{(k-1)}^{i}) = f(y_{(k)}^{i} | y_{(k-1)}^{i}) \tag{8}$$

And similarly, the conditional probability of the kth allowable solution space D_j is expressed by following equation.

$$f(D_k | D_1, D_2, \cdots, D_{k-1}) = f(D_k | D_{k-1}) \tag{9}$$

In this procedure, k^{th} sample vectors are stochastic variables dependent on the outcome of $k\text{-}1^{th}$ sampling vectors, then the transition probability matrix P at the k^{th} sample for each pair i , j of outcomes is expressed by following equation.

$$\pi_k = \pi_{k-1} \cdot P \tag{10}$$

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1N} \\ P_{21} & P_{22} & \cdots & P_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ P_{N1} & P_{N2} & \cdots & P_{NN} \end{bmatrix}$$

where $\pi_k(1 \cdot N)$ is the state vector distribution, $\pi_0 = [y_1, y_2, \cdots, y_N]$ and N is the number of possible states.
In this combinational optimization problem, the transition probability matrix is an unique stationary matrix, and the components p_{ij} are satisfied with the following conditions.

$$p_{ij} \geq 0, \ \sum_j p_{ij} = 1 \tag{11}$$

In the case of the proposed algorithm, the outcome of a sampling vectors depend only on the outcome of the previous sampling vectors. However, to predict the outcome of sampling vectors, optimization problems we must know the unique stationary transition probability matrix P, which are determined in the following manner using the previous sampling vectors.
Now, we consider the unique stationary transition probability matrix is determined by using the state vector distribution $y_0, y_1, \cdots, y_T$ at sampled number of 0 to T.

$$P_{y0,y1} \cdot P_{y1,y2} \cdots P_{yT-1,yT} = \prod_{i=1}^{S} \prod_{j=1}^{S} P_{ij}^{n_{ij}} \tag{12}$$

n_{ij}:Number of transition i to j

y_0:Given, $S = N$

Considering the number of previous sample sets n_{ij} to be integrated in the determination of the unique matrix elements, determine the best estimator based on the logarithm of maximum likelihood which can be expressed as the following equation.

$$\log L = \sum_{i=1}^{S} \sum_{j=1}^{S} n_{ij} \log P_{ij} \tag{13}$$

These elements P_{ij} are satisfied with the following conditions.

$$P_{ij} \geq 0, \ i, j = 1, 2, \cdots, S \tag{14.a}$$

$$\sum_{j=1}^{S} P_{ij} = 1, \ i = 1, 2, \cdots, S \tag{14.b}$$

Thus, these elements P_{ij} are to be evaluated by the following equation.

$$F = \log L + \sum_{i=1}^{S} \lambda_i (\sum_{j=1}^{S} P_{ij} - 1) = \sum_{i=1}^{S} [\sum_{j=1}^{S} n_{ij} \log P_{ij} + \lambda_i (\sum_{j=1}^{S} P_{ij} - 1)] \tag{15}$$

λ_i :Lagrange multiplier, $i = 1, 2, \cdots, S$

In order to evaluate the best estimator P_{ij}, we must solve the equation (15) by maximizing F in the following way.

$$\frac{\partial F}{\partial P_{ij}} = \frac{P_{ij}}{n_{ij}} + \lambda_i \tag{16.a}$$

$$P_{ij} = -\lambda_i n_{ij} \tag{16.b}$$

$$1 = \sum_{j=1}^{S} P_{ij} = -\lambda_i \sum_{j=1}^{S} n_{ij} = -\lambda_i n_i \tag{16.c}$$

From equations (16.a) to (16.c), the unique stationary transition probability matrix P_{ij} is determined by an finite number of previous sampling vectors.

$$P_{ij} = n_{ij} / n_i \tag{17}$$

$$\because n_i = \sum_{j=1}^{S} n_{ij}$$

In this proposed procedure, the unknown unique stationary transition probability matrix P_{ij} is evaluated recursively during the each sampling process, and the probability of the equation (17) determins the allowable solution space D_j in the algorithm.

3. Numerical Example

A numerical example is demonstrated with a 16 bar truss structure in Fig. 2 under earthquake excitation. The model is subjected to the ground motion in Fig. 3.

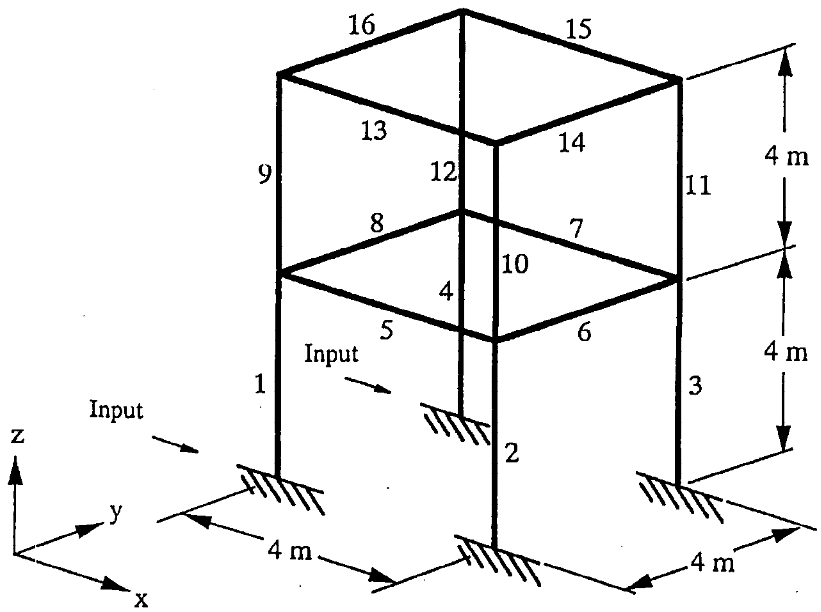

Fig. 2. 16 bar truss structure

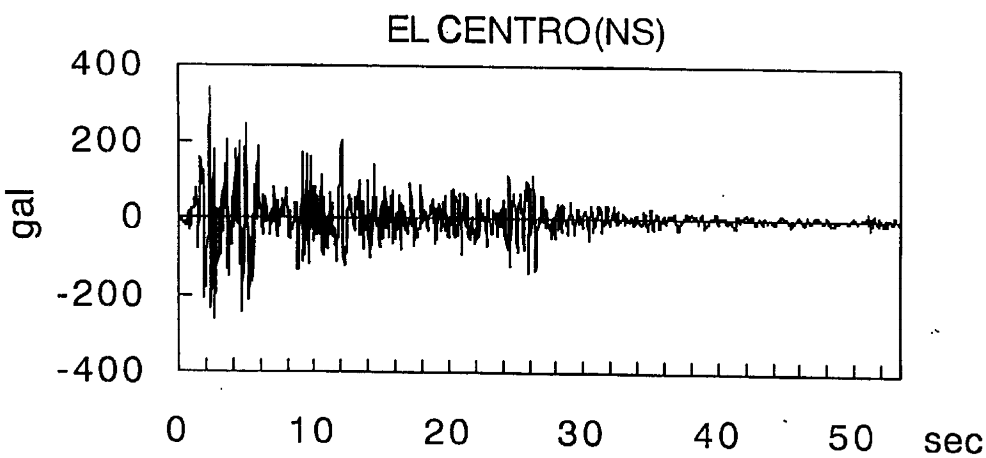

Fig. 3. Input acceleration

In this numerical example, these design parameters such as cross section of ready-made steel products shown in Table 1 are discrete and the objective function of design process is the total weight of the truss structure which is to be minimize under the constrant of 5.0cm maximum displacements of nodes during earthquake excitation. In this algorithm, each discrete design parameters are randomly sampled from the region of solution space, and the allowable region is updated and condensed from the objective function. The, sample size is shown in Table 2.

Table 1. Ready-made steel products

No.	A(cm^2)
1	1.238
2	1.799
3	3.096
4	3.971
5	4.562
6	6.769
7	9.513
8	11.200
9	14.450
10	17.170
11	20.410
12	27.620
13	35.260
14	53.610
15	75.410
16	99.730
17	103.300
18	120.100
19	157.100
20	177.300
21	197.600

Table 2. Sample size

	Sample Size
Case 1	100
Case 2	50
Case 3	20

The results are shown in Fig. 4. In the case of the 50 and 100 sample vectors, the proposed procedure gives good combinational optimization design from the minimum weight and minimum displacements of nodes in the 16 bar truss structure under a seismic loading.

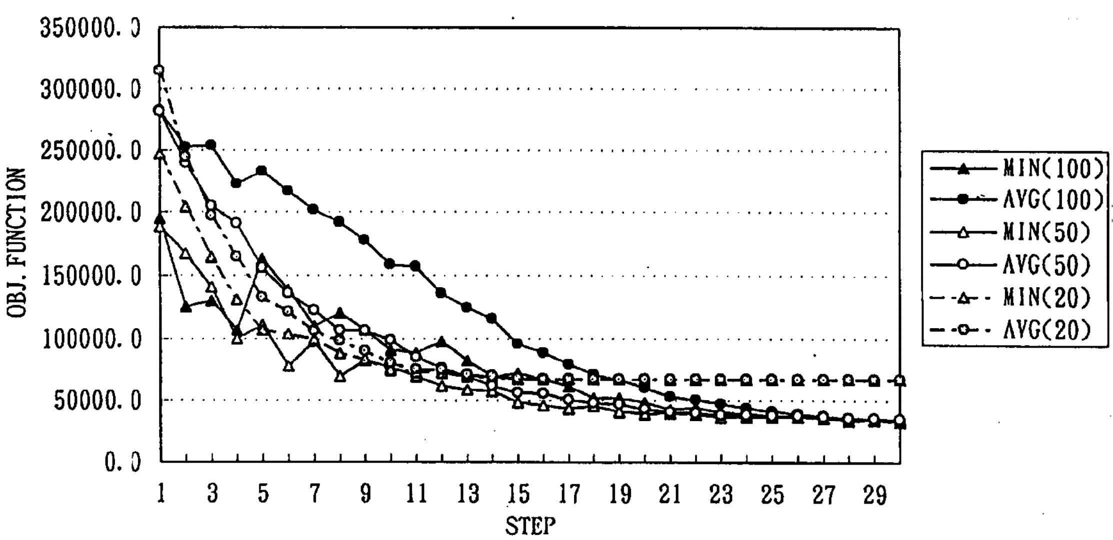

Fig. 4. Convergence Process of the objective function

4. Concluding Remarks

A stochastic combinational optimization procedure is proposed and through numerical example of an optimum steel structural design, the following conclusions can be drawn.

(1) A stochastic combinational optimization algorithm is proposed, which uses an importance sampling procedure based on a Monte Carlo method. In this proposed procedure, discrete design parameters are randomly sampled from region of solution space, and the allowable region is updated and condensed from the objective function.

(2) In the proposed combinational optimization procedure, the unknown unique stationary transition probability matrix of a Markov chain is evaluated recursively during each sampling process.

(3) It was found that the procedure is a stable and robust determination of design parameters, and can be easily applied even for optimum design problems under seismic loading, since the procedure handles with the algorithm of an importance sampling procedure, in parallel with structural analysis.

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