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NONLINEAR RANDOM VIBRATION OF BASE ISOLATED BUILDINGS UNDER EVOLUTIONARY SEISMIC INPUT

JORGE E. HURTADO
National University of Colombia, Apartado 127, Manizales, Colombia

ALEX H. BARBAT
Faculty of Civil Engineering, Technical University of Catalonia
c/ Gran Capitán s/n, 08034 Barcelona, Spain

and ALVARO A. M. F. da CUNHA
Porto University, 4099 Porto Codex, Portugal

ABSTRACT

This paper presents the results of a study on the stochastic response of buildings supported by hysteretic elastomeric base-isolators subject to bidirectional seismic input. The main purpose of the research is to analyse the effects on the response of (a) the two-fold nonstationary character of the earthquake motion, both in amplitude and frequency, and (b) the effects of biaxial loading and torsional excentricities. The structures are idealized as linear spatial frames supported by hysteretic rubber bearings. The amplitude and frequency nonstationary ground motion is modeled as biaxial, modulated white noises filtered by time-varying Clough-Penzien systems using the Wen-Yeh instantaneous power spectrum formulation. The study is performed by means of the method of stochastic equivalent linearization. The paper shows the high increase of the displacement response of the isolation system when the ground motion exhibits low frequency surface waves and illustrates the applicability of the method of stochastic linearization for the analysis of base isolated buildings.

KEYWORDS

Base isolated structures; stochastic linearization; surface waves; random vibrations.

INTRODUCTION

In the last two decades the possibility of using base isolation devices in buildings located in earthquake areas has attracted the attention of both researchers and designers, due to the possibility of reducing the earthquake damage expected to occur in fixed - base structures designed by the current methods. This possibility resides in that the base isolation produces(1) an increase of the period of the structure to values associated with low acceleration responses in the dominant mode shape, in which the structure behaves essentially as a rigid body, and (2) a reduction of the period of the second mode, in which the structure shows a deformed pattern.

This overall behavior restricts the use of base isolated devices to those cases where the dominant period of the ground motion is not to high (that is, no longer than, say, 1.5 s), as can be the case of soft soil profiles. Since this extreme conditions are not too common, the use of base isolation for earthquake resistant design seems to have a broad field of potential application.

However, a major handicap for the application of this simplified, spectrum-oriented conception of the base isolation philosophy arises when considering the effect of surface waves of high energy in the response of the the resulting high period structures. In some cases, this low frequency waves contain a large amount energy and can then have a serious effect on the overall response of the structure, in spite of that they do no dominate the response spectra.

METHOD OF ANALYSIS

The equation of motion of a non linear three dimensional building structure with a mass matrix $\mathbf{M}$ and a viscous damping matrix $\mathbf{C}$ is given by

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{F}(\mathbf{u}, \dot{\mathbf{u}}) = \mathbf{p}(\mathbf{t}) \tag{1}$$

where $\mathbf{F}(\cdot,\cdot)$ is the matrix of restoring forces, $\mathbf{u}$ the vector of displacements (translational and rotational) of the system and $\mathbf{p}(\mathbf{t})$ the vector of external excitations. For random vibration purposes is convenient to consider the ground motion in terms of the response of a filter to white noise excitation, so that the equations governing the dynamics of the filter can be appended to the system given by (1). In the classical Kanai - Tajimi formulation the ground acceleration is given by

$$\ddot{v}_g = 2\xi_g\omega_g\dot{x}_g + \omega_g^2 x_g \tag{2}$$

where ω_g and ξ_g are the angular frequency and damping ratio, respectively, of a time invariant filter and x_g is its displacement response to a white noise $n(t)$:

$$\ddot{x}_g + 2\xi_g\omega_g\dot{x}_g + \omega_g^2 x_g = n(t) \tag{3}$$

As it is well known, this leads to modeling the ground motion as a stationary process with power spectral density given by

$$G_{KT}(\omega) = G_0 \frac{1 + 4\xi_g^2\omega_g^2\omega^2}{(\omega_g^2 - \omega^2)^2 + 4\xi_g^2\omega_g^2\omega^2} \tag{4}$$

in which G_0 is the power spectral density of the white noise. The non stationarity character of the earthquake amplitude is usually introduced by multiplicating the compound response $\ddot{v}_g$ by a modulating function $I(t)$. This leads to a model of ground excitation in which the frequency variation of the earthquake along time is disregarded. For the case of base isolated buildings, however, the consideration of that variation is of great importance, due to, first, the concentration of non linearities of the entire building in one level only, and second, the low frequency of surface waves which can be in tone with the natural frequency of the structure. In order to model this kind of non stationarity of the ground motion, use can be made of the model proposed by Yeh and Wen (1989)

$$\ddot{v}_g = 2\xi_g\omega_g \frac{\dot{x}_g}{\dot{\phi}(t)} + \omega_g^2 x_g \tag{5}$$

where $\phi(t)$ is a smooth, strictly increasing function intended to reflect the time variation of frequencies, and which can be estimated on the basis of the zero crossings history of an earthquake record. The function $\phi(t)$ can be viewed as a curvilinear coordinate of time that stretches and squeezes the original earthquake record in the high and low frequency regions respectively. The so-called instantaneous spectrum is then given by

$$G_{KT}(t,\omega) = \frac{1}{\dot{\phi}(t)} G_{KT}\left(\frac{\omega}{\dot{\phi}(t)}\right) \tag{6}$$

The main advantage of this over other formulations of evolutionary earthquake spectra is that it allows the use of linear filter equations as in the stationary or amplitude non stationary cases mentioned above.

For the analysis of the base isolated system it can be assumed that the superstructure responses linearly while the elastomeric bearings display a non linear behavior. For both deterministic and stochastic non linear analyses of base isolated buildings the use of the Bouc-Wen-Baber constitutive model for the bearings seems to be highly adequate (Yang *et al.*, 1992). In the biaxial extension of the model given in Yeh and Wen (1989), the restoring forces in each direction for an isotropic oscillator are given by

$$f_x = \alpha k x + (1-\alpha) k z_x$$

$$f_y = \alpha k y + (1-\alpha) k z_y \tag{7}$$

where k is a constant of proportionality having units of stiffness, x, y are the displacement in the x and y direction respectively, and z_x, z_y are nonlinear functions with displacement units which are governed by the following differential equations:

$$\dot{z}_x = a\dot{x} - \beta|\dot{x} z_x| z_x - \gamma \dot{x} z_x^2 - \beta|\dot{y} z_y| z_x - \gamma \dot{y} z_x z_y$$

$$\dot{z}_y = a\dot{y} - \beta|\dot{y} z_y| z_y - \gamma \dot{y} z_y^2 - \beta|\dot{x} z_x| z_y - \gamma \dot{x} z_y z_x \tag{8}$$

where a, β, and γ are parameters that control the shape of the hysteretic loop. The equivalent linearization method can be applied in this case by substituting this equations by the linear ones

$$\dot{z}_x = C_1 \dot{x} + C_2 \dot{y} + C_3 z_x + C_4 z_y$$

$$\dot{z}_y = D_1 \dot{x} + D_2 \dot{y} + D_3 z_x + D_4 z_y \tag{9}$$

where the unknown coefficients must be estimated by the minimization of the expected value of the squared difference between the non linear and linear equations. Closed forms expressions of the coefficients for the case of an assumed gaussian response are given in Yeh and Wen (1989). If the linear superstructure is modeled according to the commonlly invoked hypothesis of infinite rigidity of the slabs in their own plane, there will be a total number of $3n + 2m + 2$ equations to be solved in the deterministic case, where n is the number of stories and m the number of isolating bearings. The second moment statistics of the zero mean state vector $\mathbf{y}^{\mathbf{T}} = [\mathbf{u} \quad \dot{\mathbf{u}} \quad \mathbf{z_x} \quad \mathbf{z_y} \quad x_g \quad y_g]$ is obtained by solving the differential equation

$$\dot{\mathbf{S}} = \mathbf{A}\mathbf{S} + \mathbf{S}\mathbf{A}^{\mathbf{T}} + \mathbf{B} \tag{10}$$

in which $\mathbf{S}$ is the covariance matrix of the response $\mathbf{S} = \mathrm{E}\{\, \mathbf{y}\ \mathbf{y}^{\mathbf{T}} \}$, $\mathbf{A}$ is the state matrix which includes the linearization matrices and $\mathbf{B}$ is the excitation matrix. The order of these matrices is $6n + 2m + 2$ (A detailed description of the method of stochastic linearization can be found in Roberts and Spanos (1990)). It is observed that this number is not too much greater than the number of equations required in the deterministic case, and which is more important, the results of the calculation are close to the statistics that otherwise need to be obtained by a large number of deterministic analyses.

The solution of the above system of differential equations let to know the second moment statistics of the response in the global coordinates. If b_x, b_y, b_θ express the displacements in the global coordinates of the center of mass of the isolating slab, the variance of the displacements of the j-bearing with distances (x_j, y_j) with respect to that point can be calculated by

$$\sigma^2_{x_j} = \sigma^2_{b_x} + y^2_j \sigma^2_{b_\theta} - 2y_j \sigma_{b_x b_\theta}$$

$$\sigma^2_{y_j} = \sigma^2_{b_y} + x^2_j \sigma^2_{b_\theta} + 2x_j \sigma_{b_y b_\theta} \tag{11}$$

The variance of the relative story drifts of the i-story in the x-direction $d_i = x_i - x_{i-1}$ can be obtained through

$$\sigma^2_{d_i} = \sigma^2_{x_i} + \sigma^2_{x_{i-1}} - 2\sigma_{x_i x_{i-1}} \tag{12}$$

and similarly in the orthogonal direction.

Finally, it must be mentioned that the maximum biaxial displacement of the bearings, which are the most important design variable, can be estimated on the basis of the covariance matrix **S** through a procedure explained in Yeh and Wen (1989), which involves the rotation of coordinate axes, the estimation of the crossing rate of the vector stochastic process $x(t), y(t)$ across an elliptical boundary and the integration with respect to time. It is observed that the use of an approximate equation for the calculation of the mean crossing rates given in the quoted reference is almost exact for cases when the root mean square displacements in the two orthogonal directions are similar ($\sigma_x \approx \sigma_y$). Since this is just the expected case of base isolating bearings, due to their orthotropic nature and the similarity of the natural frequencies of the building in the two orthogonal directions, the use of this procedure, which has been succesfully tested by simulations, conducts to a realistical assessment of the mean maximum displacement of the building.

NUMERICAL ANALYSES

In order to show the effect of the frequency changes of the input ground motion in the response of base isolated buildings as well as the applicability of the stochastic linearization method to the design of this kind of structures, a series of analyses was conduted using as structural model a square building of 10 m of length with two slabs, one supported on four elastomeric bearings and the other on a three dimensional one story frame. The natural period of the entire structure in any of the global axis was 2.12 s. P-Δ effects on the bearing deformation have been included by means of a geometric stiffness matrix whose coefficients have been calculated by an equation reported by Kelly (1993). The input motion was taken the same in each direction, and characterized by a unit variance, Kanai -Tajimi power spectrum with $\omega_g = 18.85$ rad/s and $\xi_g = 0.65$ and an intensity function $I(t)$ given by

$$I^2(t) = A \frac{t^B}{D + t^E} e^{-Ct} \tag{13}$$

with $A = 0.098$ m^2/s^4, $B = 32.5, C = 0.32, D = 0.001, E = 29.0$

Use was made of three the frequency modulating functions calculated from the following records: El Centro (S00E Imperial Valley, 1940); SMART (SMART- 1 No. 45, E00N, 1986), and Ventura (N78W, 12724 Ventura Boulevard, L. A. 1971). The first record is in contrast with the third in that the later shows a clear evolution of dominant frequencies from high to low values, while in the first low frequency pulses occur randomly along the time axis. The second record has an intermediate appearance. All results that will be mentioned in the following correspond to root mean square displacements.

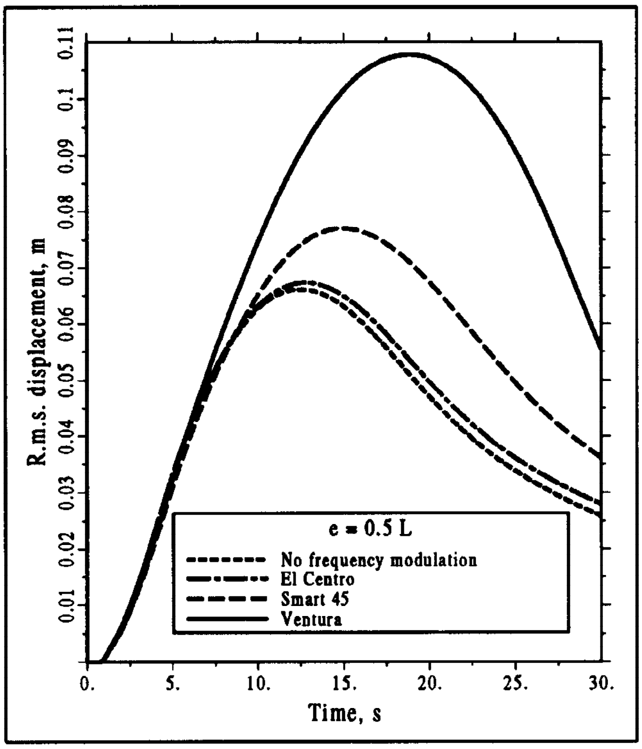

Fig. 1 Effect of frequency evolution in the displacement of the isolation system

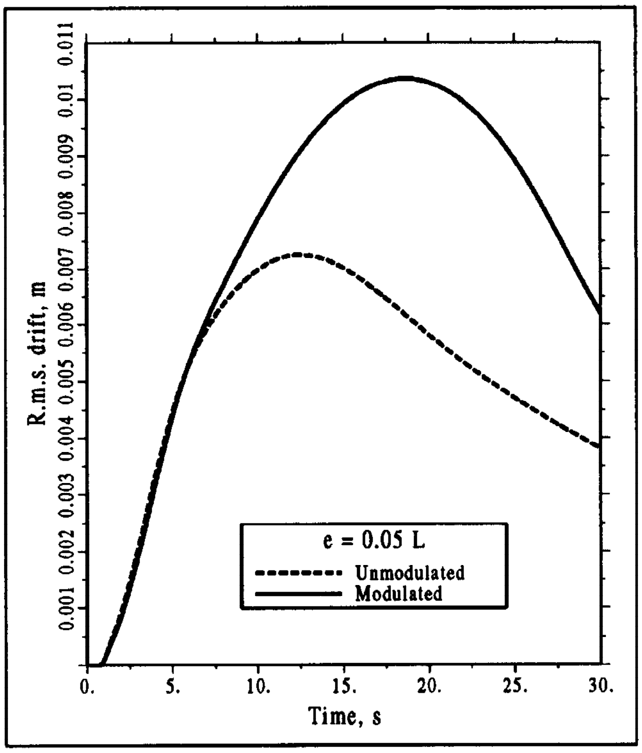

Fig. 2 Effect of frequency evolution in the interstory drift of the superstructure

Figure 1 shows the effect of the frequency modulation on the displacement of the center of mass of the base slab, when the building has a torsional excentricity of 5% the dimension of the square plant L. The use of El Centro function gives almost the same result as that obtained disregarding the frequency evolution, while the case of the Ventura earthquake function produces a very high increase (63 %) with respect to that case. Similar results were obtained for the symmetric (no torsion) and other unsymmetric cases. In figure 2 the same effect is observed for the case of the interstory drift of the building using the Ventura modulating function. Figure 3 compares the results

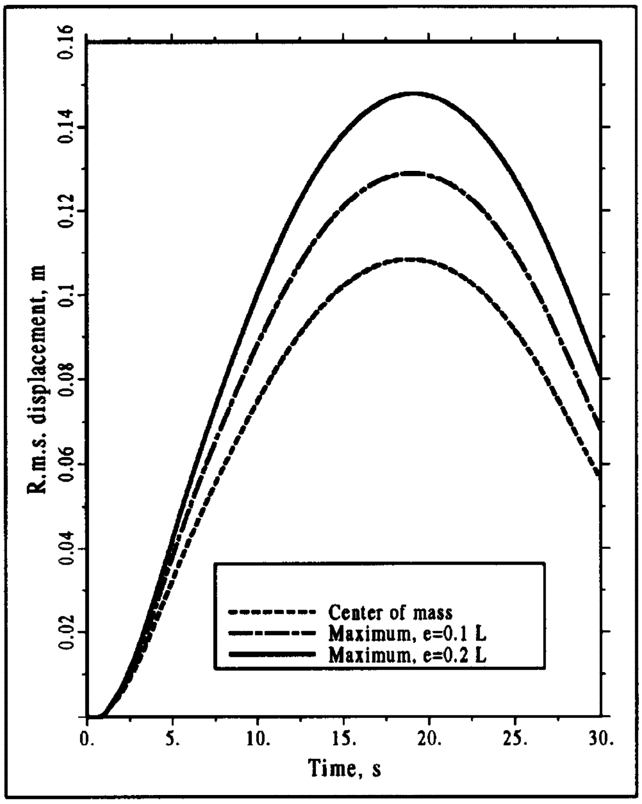

Fig. 3 Effect of torsional excentricity on bearing displacement

corresponding to maximum bearing displacements when the excentricities are 10 and 20 % of the slab length respectively, using the Ventura earthquake function. It can be observed that when passing from the first to the second excentricity there is an increase of about 15 % in the maximum r. m. s. response. Finally, using the procedure mentioned above for calculating the expected maximum biaxial responses, the results corresponding to an excentricity of 5 % are the following: 0.168 m (no frequency modulation); 0.172 m (El Centro); 0.200 m (SMART); and 0.284 m (Ventura). The difference between the non modulated case and that using the Ventura modulating function is 69 %, somewhat higher than the existing between the corresponding r.m.s. responses.

CONCLUSIONS

The main conclusions arising from the present work are:
1. The non stationary nature of earthquake actions with respect to frequency content can produce a large increase in the displacements of both the isolation system and the superstructure of base isolated buildings, especially when the excitation includes a sequence of low frequency cycles which are associated with surface waves, and not necessarily to soft soil conditions.
2. The method of stochastic equivalent linearization is an excellent tool for analysing the response of this kind of structural systems, due to its ability to predict both the root mean square and maximum expected responses without resort to simulation techniques.

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