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REDUCED-ORDER SLIDING MODE CONTROL WITH COMPENSATORS FOR SEISMIC RESPONSE CONTROL

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ABSTRACT

Recently, it has been demonstrated that control techniques based on sliding mode control (SMC) are robust and their performances are quite remarkable for applications to active/hybrid control of seismic-excited linear, nonlinear or hysteretic civil engineering structures. In this paper, sliding mode control methods are further extended by introducing compensator and different static output feedback controllers. The incorporation of compensators provides a convenient way of making trade-offs between control efforts and specific response quantities of the structures through the use of linear quadratic optimal control theory. In addition to full state feedback controllers, a systematic design of static output feedback controllers using only a few sensors are presented in this paper to facilitate practical implementations. Since civil engineering structures generally consist of many degrees of freedom, a controller design using a full-order system may be difficult. Based on the dominant modes and the retention of selected degrees of freedom of the original structure, a method to construct a reduced-order structural model is presented. Then, the controller is designed based on the reduced-order system. It is shown that the performance of the controller using the reduced-order system is quite close to that of the controller based on the full-order system.

KEYWORDS

Active structural control, seismic response control, sliding mode control, compensators, reduced order, static output feedback

INTRODUCTION

Recently, various advanced control theories have been investigated for implementations of control systems to seismic-excited structures, such as H_2 control [e.g., Dyke, et al 1994, Suhardjo, et al 1994], H_∞ control [e.g., Schmitendorf, et al 1994, Jabbari, et al 1995], sliding mode control [e.g., Yang, et al 1994a], etc. In particular, the theory of sliding mode control (SMC) or variable structure system (VSS) was developed for robust control of uncertain nonlinear systems. Applications of sliding mode control methods to the following seismic-excited structures have been studied: (i) linear and nonlinear or hysteretic buildings [Yang, et al 1995a, b], and (ii) parametric control, such as the use of active variable dampers (AVD) on bridges [Yang, et al 1995c] and active variable stiffness (AVS) systems [Yang, et al 1994b]. Continuous sliding mode controllers that do not have possible chattering effect were presented [e.g., Yang, et al 1995a, b]. In addition to full state feedback controllers, static output feedback controllers using only a limited number of sensors installed at strategic locations were also presented in the studies above. Shaking table experimental verifications of the SMC methods to linear and sliding-isolated building models have been conducted [Yang, et al 1996a, b]. Based on the numerical simulations and experimental results, it was demonstrated that the sliding mode control methods are robust and their performances are quite remarkable.

In the previous studies, however, the modulation of the control effort versus response quantities was made either by adjusting the sliding surface or by specifying the maximum control level (saturated controller). This paper presents the design of sliding mode controllers by introducing a fixed-order compensator using the linear quadratic optimal control theory [Yallapragada, et al 1992]. The main advantage of using linear quadratic optimal control theory with a fixed-order compensator in sliding mode control designs is that a modulation of response quantities and control effort can be made in a systematic manner for both the full-state feedback and static output feedback control strategies. Compensators are designed through the LQR formulation which allows for the modulation of the control forces and specific response quantities in a similar way as LQR. Since civil engineering structures generally consist of many degrees of freedom, the design of a controller using a full-order system can be difficult. Based on the dominant modes and the retention of selected degrees of freedom of the original structure, a method to construct a reduced-order structural model is presented. Then, the controller is designed based on the reduced-order system. The advantage of the approach based on the reduced-order system over the modal control is that the observers are not needed. This is because the state variables of the reduced-order system are identical to selected state variables of the original structure. It is shown that the performance of the controller using the reduced-order system is very close to that of the controller based on the full-order system. Numerical simulation results are presented to demonstrate the applicability and other desirable features of continuous sliding mode control (CSMC) with a compensator, both for full-order and reduced-order systems.

MATHEMATICAL FORMULATION

The vector equation of motion for a linear elastic structure subjected to a one-dimensional earthquake ground acceleration $\ddot{x}_0(t)$ can be expressed as

$$\overline{M}\ddot{X}(t)+\overline{C}\dot{X}(t)+\overline{K}X(t)=\overline{H}U(t)+\eta\ddot{x}_0(t) \qquad (1)$$

in which $X(t)=[x_1,x_2,\ldots,x_n]'$ = an n vector with x_i being the relative displacement of the i th floor with respect to the ground; a prime indicates the transpose of either a vector or a matrix; $\overline{M}$, $\overline{C}$ and $\overline{K}$ are (nxn) mass, damping and stiffness matrixes, respectively; $U(t)=[u_1,u_2,\ldots,u_r]'$ = a r-control vector; $\overline{H}$ = a (nxr) matrix denoting the location of r controllers; and η= an earthquake excitation influence vector. In the state space, Eq. (1) can be written as

$$\dot{Z}(t)=AZ(t)+BU(t)+E(t) \qquad (2)$$

in which $Z(t)=[X'(t),\dot{X}'(t)]'$ = a 2n state vector; A = a (2nx2n) system matrix; B = a (2nxr) location matrix; and E(t)= a 2n excitation vector, given by,

$$A=\begin{bmatrix}0 & I\\ -\overline{M}^{-1}\overline{K} & -\overline{M}^{-1}\overline{C}\end{bmatrix};\ B=\begin{bmatrix}0\\ \overline{M}^{-1}\overline{H}\end{bmatrix};\ E(t)=\begin{bmatrix}0\\ \overline{M}^{-1}\eta\ddot{x}_0(t)\end{bmatrix} \qquad (3)$$

The measured system output y(t) of Eq. (2) is given by

$$y(t) = CZ(t) \qquad (4)$$

where C is a (mx2n) observation matrix and m is the number of sensors installed on the structure.

Design of Compensator and Sliding Surface

The minimum order of the compensator is (r+1), where r is the number of controllers. For simplicity of presentation, the state vector q(t) for the compensator is considered to be (r+1), i.e., $q=[q_1,q_2']'$, where q_1 is a scalar and q_2 is a r-vector. The compensator dynamics is given by

$$\dot{q}(t) = Lq(t) + Ny(t) + DU(t) \tag{5}$$

where L, N and D are appropriate matrices to be determined. The sliding surface is considered a function of compensator variables, i.e.,

$$S = Pq(t) = 0 \tag{6}$$

in which P is partitioned into $P = [P_1, P_2]$. In the standard form of sliding mode control, P_2 is generally chosen to be an identity matrix. However, in the present study, we want the flexibility to choose P_2 appropriately. The block diagram for the entire augmented control system is shown in Fig. 1.

For simplicity, the compensator dynamics, Eq.(5), is considered to be in the regular form already, i.e., the first row of the D matrix is zero. Hence, Eq.(5) and (6) can be written as

$$\dot{q}_1(t) = L_{11}q_1(t) + L_{12}q_2(t) + N_1 y(t) \tag{7}$$

$$\dot{q}_2(t) = L_{21}q_1(t) + L_{22}q_2(t) + N_2 y(t) + D_2 U(t) \tag{8}$$

$$S = P_1 q_1(t) + P_2 q_2(t) = 0 \tag{9}$$

in which D_2 is a (rxr) matrix; $D = [0,\ D_2']'$; $N = [N_1',\ N_2']'$; and L is partitioned into L_{11}, L_{12}, L_{21}, and L_{22} submatrices. On the sliding surface, $\dot{S} = 0$, i.e.,

$$\dot{S} = P_1 \dot{q}_1(t) + P_2 \dot{q}_2(t) = 0 \tag{10}$$

From Eq.(9), one obtains $q_2(t) = -P_2^{-1} P_1 q_1(t)$. Substituting Eqs. (7)-(9) into Eq.(10), one obtains the so-called equivalent control, $U = U_{eq}$, on the sliding surface,

$$U_{eq} = Gy(t) + Hq_1(t) \tag{11}$$

in which

$$G = -(P_2 D_2)^{-1}(P_1 N_1 + P_2 N_2)\ ; \quad H = -(P_2 D_2)^{-1}[P_1(L_{11} - L_{12}P_2^{-1}P_1) + P_2(L_{21} - L_{22}P_2^{-1}P_1)] \tag{12}$$

Also, substitution of Eq.(9) into Eq.(7) leads to

$$\dot{q}_1(t) = (L_{11} - L_{12}P_2^{-1}P_1)q_1(t) + N_1 y(t) \tag{13}$$

Now, let us introduce an augmented state vector $\overline{x}(t)$ as

$$\overline{x}(t) = [Z', q_1]'\ ; \quad \overline{y} = [y', q_1]' \tag{14}$$

Then, Eq. (2) and Eqs.(11)-(13) can be cast into a (2n+1) vector equation after neglecting the earthquake excitation as follows

$$\dot{\overline{x}}(t) = \overline{A}\,\overline{x}(t) + \overline{B}\,\overline{U}(t) \tag{15}$$

$$\overline{y}(t) = \overline{c}\ \overline{x}(t) \tag{16}$$

in which

$$\overline{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix};\ \overline{B} = \begin{bmatrix} B & 0 \\ 0 & 1 \end{bmatrix};\ \overline{U} = \begin{bmatrix} U_{eq} \\ \dot{q}_1 \end{bmatrix};\ \overline{c} = \begin{bmatrix} C & 0 \\ 0 & 1 \end{bmatrix} \tag{17}$$

and Eqs. (11) and (13) become

$$\overline{U}(t) = \overline{G}\,\overline{y}(t) \tag{18}$$

where

$$\overline{G} = \begin{bmatrix} G & H \\ N_1 & L_{11} - L_{12}P_2^{-1}P_1 \end{bmatrix} \tag{19}$$

For the system equation given by Eq.(15) with the static output vector given by Eq.(16), the gain matrix $\overline{G}$ in Eq.(19) can be obtained by minimizing the quadratic performance index

$$J = E\left[\int_0^\infty (\overline{x}' Q\ \overline{x} + \overline{U}' R\,\overline{U})dt\right] \tag{20}$$

in which E[ ] denotes the statistical expectation and

$$Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix};\ R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \tag{21}$$

where Q_1 and Q_2 are state weighting matrices corresponding to Z(t) of the structure and $q_1(t)$ of the compensator, respectively. Similarly, R_1 and R_2 are control weighting matrices corresponding to U_{eq} and $\dot{q}_1(t)$, respectively.

Following the optimal output feedback control theory by Levine and Athans (1970), the gain matrix $\overline{G}$ in Eq.(19) is obtained by solving the following nonlinear matrix equations

$$\overline{G} = -R^{-1}\overline{B}'\,\overline{K}\,\overline{L}\,\overline{c}'\,(\overline{c}\,\overline{L}\,\overline{c}')^{-1} \tag{22}$$

$$\overline{M}\,\overline{L} + \overline{L}\,\overline{M}' + I = 0 \tag{23}$$

$$\overline{K}\,\overline{M} + \overline{M}'\,\overline{K} + Q + \overline{c}'\,\overline{G}'\,R\,\overline{G}\,\overline{c} = 0 \tag{24}$$

$$\overline{M} = \overline{A} + \overline{B}\,\overline{G}\,\overline{c} \tag{25}$$

Equations (22)-(25) can be solved for $\overline{G}$, $\overline{L}$, $\overline{M}$ and $\overline{K}$ iteratively as suggested by Srinivasa et al (1979). Note that when C (or $\overline{c}$) is full rank, i.e., the case of full-state feedback, $\overline{G}$ and $\overline{U}$ become, respectively, $\overline{G} = -R^{-1}\overline{B}'\overline{K}\overline{c}^{-1}$ and $\overline{U} = -R^{-1}\overline{B}'\overline{K}\overline{x}$. It follows from Eqs. (24) and (25) that $\overline{K}$ satisfies the well-known Riccati matrix equation

$$\overline{K}\,\overline{A} + \overline{A}'\overline{K} - \overline{K}\,\overline{B}\,R^{-1}\,\overline{B}'\,\overline{K} + Q = 0 \tag{26}$$

After the gain matrix $\overline{G}$ for the augmented system is obtained, parameters for the compensator are determined as follows. With the partitioning of $\overline{G}$, it follows from Eq. (19) that G, H, N_1 and $\overline{L}_{11} = L_{11} - L_{12}P_2^{-1}P_1$ can be determined. The nonsingular (rxr) matrix D_2 can be chosen appropriately through numerical simulations. The scalar P_1 and (rxr) matrix P_2 can be chosen arbitrarily. Then, the submatrix N_2 is computed from G in Eq. (12). Although L_{12} and L_{22} are also to be chosen arbitrarily, they should be chosen such that the augmented open-loop system is stable as will be described later. Finally, L_{11} is computed from $\overline{L}_{11} = L_{11} - L_{12}P_2^{-1}P_1$ whereas L_{21} is computed from H in Eq. (12).

Design of Controller

A controller is designed to drive the trajectory of the states of the compensator into the sliding surface S = 0 and maintain it there. To achieve this goal, a Lyapunov function $V = 0.5S'S$ is considered. The sufficient condition for the sliding mode S = 0 to occur as $t \to \infty$ is $\dot{V} \le 0$, i.e.,

$$\dot{V} = S'\dot{S} = S'\,[P_1\dot{q}_1(t) + P_2\dot{q}_2(t)] \tag{27}$$

For the design of the sliding surface and compensator described in the previous subsection, the external excitations to both the structure and compensator are neglected. However, they will be accounted for in the design of controller. The (r+1) excitation vector to the compensator, denoted by $E_c(t)$, is considered as

$$E_c(t) = [0, E_{12}']' \tag{28}$$

in which E_{12} = r-vector is a subset of the external excitation vector E(t) in Eq. (2), corresponding to the location of the controllers. In other words, elements of E_{12} are from the subset of elements of E which appear in those state equations where the control forces appear. Then, the equations for $q_2(t)$, Eq. (8), can be written as

$$\dot{q}_2(t) = L_{21}q_1(t) + L_{22}q_2(t) + N_2y(t) + D_2U(t) + E_{12} \tag{29}$$

Substituting Eqs. (7) and (29) into Eq. (27), one obtains

$$\dot{V} = S'(P_2D_2)[U(t) - U_{eq} + M_cq(t) + D_2^{-1}E_{12}] \tag{30}$$

in which U_{eq} is given by Eq.(11) and

$$M_c = (P_2D_2)^{-1}\left[(P_1L_{12}P_2^{-1}P_1 + P_2L_{22}P_2^{-1}P_1) \quad (P_1L_{12} + P_2L_{22})\right] \tag{31}$$

for $\dot{V} \le 0$, a possible continuous controller is given by

$$U(t) = U_{eq} - M_cq(t) - (P_2D_2)^{-1}\overline{\delta}S - D_2^{-1}E_{12} \tag{32}$$

in which $\bar{\delta}$ is a (rxr) diagonal matrix with the ith diagonal element $\delta_i \geq 0$. δ_i is referred to as the sliding margin. Substitution of Eq. (32) into Eq. (30) leads to $\dot{V} = -S'\bar{\delta}S \leq 0$. Note that U_{eq} has been determined in Eq. (11).

Stability of Augmented System

The state equation of the entire augmented system, including structure, compensator, and controller, in the original coordinate form, follows from Eqs. (2) and (5) as

$$\dot{X}_c = A_c X_c + B_c U(t) + E_s(t) \qquad (33)$$

in which the excitation to the compensator has been included and

$$X_c = \begin{bmatrix} Z \\ q \end{bmatrix}; \; A_c = \begin{bmatrix} A & 0 \\ NC & L \end{bmatrix}; \; B_c = \begin{bmatrix} B \\ D \end{bmatrix}; \; E_s = \begin{bmatrix} E(t) \\ E_c(t) \end{bmatrix} \qquad (34)$$

The controller in preceding subsection stabilizes the closed-loop system of Eq. (33) for any arbitrary choices of L_{12} and L_{22}. However, the open-loop system A_c may not be stable for any choices of L_{12} and L_{22}. Methods for the determination of L_{12} and L_{22} submatrices such that the open-loop augmented system A_c is stable are available. The stability of the open-loop system is important to guarantee the stability of the structure during controller saturation. It has been found that, for any system, the stability of the open-loop system and the closed-loop system with unsaturated controller are prerequisites for the stability of the closed-loop system with saturated controller. Due to space limitation, the proof of the above statement will not be presented.

REDUCED ORDER SYSTEM

Since civil engineering structures, e.g., high-rise buildings, long-span bridges, etc., are generally complex and consist of excessive degrees of freedom, the dimension of the system matrices in Eq.(2) will be very large. As a result, the design of a controller, either with full-state or static output feedback, may be quite difficult or even impossible. Hence, it is desirable to approximate a larger order system by a smaller order system numerically. One approach is to construct a reduced-order model by retaining only the dominant eigenvalues and eigenvectors of the full-order system. Since the response of the structure depends heavily on the dominant eigenvalues and eigenvectors of the system, the response behavior of the reduced-order system will be close to the original system. In the present study, a method proposed by Davison (1966) has been used to construct a reduced-order system for the original system in Eq.(2). With zero initial conditions, the solution of the linear system in Eq. (2) can be expressed as

$$Z(t) = \int_0^t \Gamma e^{\Lambda(t-\tau)} \Gamma^{-1} [BU(\tau) + E(\tau)] \, d\tau \qquad (35)$$

in which Λ is the diagonal eigenvalue matrix, and Γ is the eigenvector matrix. In Eq.(35), the eigenvalues of the system matrix A are assumed to be distinct, which is generally the case for civil engineering structures. Denoting $\Gamma = [\Gamma_1, \Gamma_2, .., \Gamma_n]$, $\Gamma^{-1} = S = [S_1', S_2', .., S_n']'$, and $q(\tau) = BU(\tau) + E(\tau)$, the solution in Eq.(35) can be written as

$$Z(t) = \xi_1 \Gamma_1 + \xi_2 \Gamma_2 + \ldots + \xi_n \Gamma_n \qquad (36)$$

where

$$\xi_i = \int_0^t e^{\Lambda_i (t-\tau)} S_i q(\tau) \, d\tau \qquad (37)$$

Let k < n be the total number of complex modes of the original system to be retained in the reduced-order system. Then, the response state vector $\tilde{Z}(t)$ of the reduced-order system is obtained from Eq.(36) as

$$\tilde{Z}(t) = \sum_{i=1}^{k} \xi_i \Gamma_i \qquad (38)$$

Without loss of generality, the first m elements of the reduced-order state vector, $\tilde{Z}(t)$, are chosen to be the m observations of the system, Eq.(4). Following the mathematical derivations presented by Davison (1966), a reduced-order system can be constructed by using Eq.(38) as

$$\dot{\tilde{Z}}(t) = \tilde{A}\tilde{Z}(t) + \tilde{B}U(t) + \tilde{E}(t) \qquad (39)$$

where $\tilde{A}$ is a (kxk) reduced-order system matrix, $\tilde{B}$ is a (kxr) reduced-order controller location matrix, $\tilde{E}(t)$ is the effective excitation vector for the reduced-order system, and $k \geq m$. Detailed descriptions for the derivation of these matrices and vectors can be found in Wu (1996). To obtain a reduced-order system with real coefficients, it is necessary to choose even numbers of complex modes, because the open-loop eigenvalues of a structural system generally occur in complex conjugate pairs.

The controllers for the reduced-order system in Eq.(39) can be designed following the procedures described in the previous section, including the full-state and static output feedback controllers.

SIMULATION RESULTS

The performance of the proposed sliding mode controller with compensator will be investigated for both the full-order and reduced-order systems through numerical simulations using a ten-story moment-resisting steel frame building considered in Shing, et al (1994). The fundamental frequency of the building is 2.59 rad/sec. The inherent structural damping ratio of the fundamental mode is 5%. A mass damper with a mass equal to 25% of the mass of the 10th floor (2.12% of the total mass of the building) will be installed on the top of the building. The El Centro NS (1940) earthquake with a peak ground acceleration scaled to 0.3g is used as the earthquake excitation.

For the structure without a damper, the peak interstory drifts and the peak absolute accelerations for 10 floors are shown in columns (2) and (3), respectively, of Table 1. Next, the mass damper described above is tuned to the fundamental frequency of the building and installed on the top of the building. With such a passive TMD, the peak interstory drifts and the peak absolute accelerations are shown in columns (4) and (5), respectively, of Table 1. The effect of TMD is obvious by comparing the results between columns (2)-(3) and (4)-(5). To reduce the response of the building further, an actuator is attached to the TMD, referred to as the active mass damper (AMD). A continuous sliding mode controller with a compensator (CSMC&C) is designed first for the full-order structure as follows: Q=diag.[10 10 10 10 10 10 10 10 10 10 0.025 1 1 1 1 1 1 1 1 1 1 0.001 0.001], R=[$1.0\text{x}10^{-3}$ 1] and $\bar{\delta} = 0.1$. The sliding surface for this case is chosen to be $P = [1,\ 1]$. The elements of the compensator matrix, L, are $L_{11} = -0.0316$, $L_{12} = 1.0$, $L_{21} = -0.968$ and $L_{22} = -1.0$. The matrix, D_2, for the compensator is selected as $D_2 = 1.0$. The peak interstory drifts and the peak absolute accelerations for this case are shown in columns (6) and (7), respectively, of the Table 1. It is observed from the table that the peak control force and the peak actuator stroke for this case are 295 kN and 85.60 cm, respectively.

The original building with TMD (or AMD) results in a (22x22) system matrix A. A reduced-order model is constructed by retaining the 3rd, 6th, 10th and 11th (mass damper) degrees-of-freedom of the original building. It has been determined from the Fourier spectrum of the El Centro earthquake that three modes of the original structure are required to be retained in the reduced-order system. A sliding mode controller with compensator (CSMC&C) is designed for the reduced-order system with Q=[10 10 10 0.001 1 1 1 0.001 0.001]; R=[$1.0\text{x}10^{-4}$ 1] and $\bar{\delta} = 0.1$. The sliding surface matrix, P, the matrix D_2 and the compensator matrix, L, for this case are the same as the previous case. The peak interstory drifts and the peak absolute accelerations for this case are shown in columns (8) and (9), respectively, of the Table 1. As observed from the table, the peak control force and the peak actuator stroke for this case are 266 kN and 84.4 cm, respectively. It is observed from Table 1 that the performance of the controller based on the reduced-order system is comparable to that of the controller using the full-order building. Hence, it is much easier to design the controller for the reduced-order system. Likewise, for the reduced-order system, sensors are installed only on the selected floors, whereas displacement and velocity sensors should be installed on every floor for the full-order system.

CONCLUSIONS

The design of continuous sliding mode controller with a fixed-order compensator using the theory of optimal control has been proposed. In addition to full-state feedback controllers, static output feedback controllers using only a limited number of sensors are proposed to facilitate practical implementations of the control systems. The advantages of introducing a compensator in the controller design have been demonstrated through extensive simulation results. For a tall building involving excessive degrees of freedom, static output feedback controllers, which require the installation of only a limited number of sensors, are feasible for practical implementations. However, it may not be easy to design static output feedback controllers for systems involving hundreds of degrees of freedom because of the high dimensions of system matrices. A method to construct a reduced-order system model, based on the retention of dominant modes of the original system, has been presented. Then, the controller is designed based on the reduced-order system. The main advantage of the control method based on the reduced-order system proposed herein over the modal control is that observers are not needed. This is because all the state variables of the reduced-order system are identical to selected state variables of the original structure. Simulation results indicate that the performance of the controllers designed using the reduced-order system is comparable to that of controllers designed using the full-order system. The performance of the reduced-order sliding mode controllers with a compensator has been shown to be quite remarkable.

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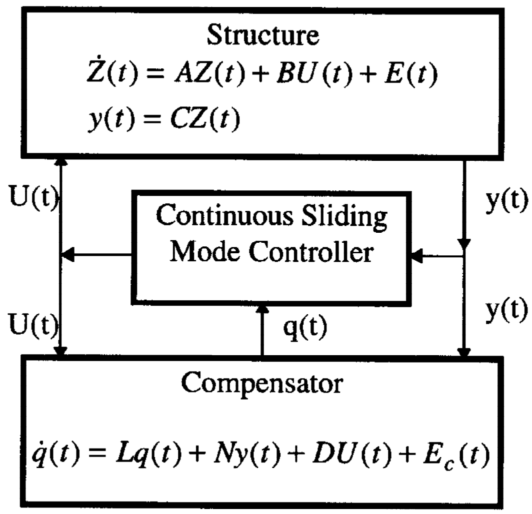

Fig. 1. Block Diagram of the Structure-Compensator-Continuous Sliding Mode Controller.

Table 1: Peak Response Quantities of a 10-Story Building Equipped with am Active Mass Damper.

	Original Structure		**Original Structure with TMD**		**Full - Order System U = 250 kN**		**Reduced - Order System U = 266 kN**	
Floor No. (1)	x_i (cm) (2)	$\ddot{x}_i$ (cm/sec^2) (3)	x_i (cm) (4)	$\ddot{x}_i$ (cm/sec^2) (5)	x_i (cm) (6)	$\ddot{x}_i$ (cm/sec^2) (7)	x_i (cm) (8)	$\ddot{x}_i$ (cm/sec^2) (9)
1	3.39	271	2.81	270	2.02	271	1.95	261
2	3.40	275	2.82	274	2.06	279	1.93	249
3	3.60	281	2.99	275	2.35	282	2.16	244
4	3.30	278	2.72	266	2.37	271	2.26	262
5	3.86	288	3.47	264	2.85	224	2.80	263
6	4.68	287	4.10	280	3.05	194	3.05	223
7	5.57	257	4.88	253	3.24	211	3.30	173
8	5.27	260	4.68	242	2.78	213	2.93	173
9	5.21	334	4.67	297	2.64	240	2.83	278
10	3.46	408	3.16	363	2.15	230	1.97	257
Stroke	----	---	53.17	350	85.60	1396	84.40	1198