



A STUDY ON THE PRACTICAL METHODS FOR GROUP FACTOR OF PILES

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ABSTRACT

Dynamic response analysis of a structure supported by a large number of piles requires the group effect of a pile group to be estimated. The group effect is primarily dependent on frequency, but it can also be represented by the group factor derived from the static value of impedance of a pile group, as fundamental natural frequencies of the soil-pile-structure system are generally restricted to low frequencies. Thus, two simple methods for evaluating group factors are proposed in this paper. These methods are based on a parametric study using thin-layer formulation. The first method is to estimate group factors of a large number of piles based on a small number of piles. This method is derived from the characteristic that group factors can be represented by the power function of the number of piles. The second method is to utilize the design equations of group factors for homogeneous and two-layered soil. These equations are based on the distinct relationships between group factors and interaction factors. These two methods produce results that correspond well with analytical solutions.

KEYWORDS

Group factor; thin-layer formulation; homogeneous soil; two-layered soil; interaction factor.

INTRODUCTION

Recently, many structures supported by a large number of piles have been constructed. To evaluate the dynamic responses of such structure, using a sway-rocking model, the soil-pile system should be expressed by the impedance functions defined at the pile head. These impedances of a pile group can be evaluated by rigorous methods such as thin-layer formulation developed by Wass *et al.* (1984), but such methods can only be applied when there are a small number of piles due to restrictions on computer memory capacity and computational expenses.

Therefore, some simplified ideas for a large number of piles have been proposed. For example, a method using the interaction factor, which means the ratio of displacement at the head of a passive pile to that of an active pile, has been developed by Dobry and Gazetas (1988) and Hijikata *et al.* (1994a). Another method based on the similarity of impedances between a large number of piles and a small number of piles has been developed by Hijikata *et al.* (1994b).

The impedances determined by both the rigorous and simplified methods mentioned above are primarily dependent on frequency. On the other hand, the fundamental natural frequencies of the soil-pile-structure

system are generally restricted to low frequencies. Thus, the impedances can be approximated by static springs with viscous dampers whose damping coefficients are constant for all frequencies.

Note also that this static spring for a pile group can be estimated from the static value of impedance of a single pile multiplied by the group factor and the number of piles. To develop this practical idea, rational methods for the group factor are necessary. Indeed, this group factor is basically dependent on the number of piles, pile distance, soil properties, and so on. However, rational methods considering the above parameters for group factor have not been determined. Thus, this research proposes two practical methods to evaluate the group factor of a large number of piles through parametric studies carried out to investigate the characteristics of pile groups using thin-layer formulation.

CALCULATION OF GROUP FACTORS

Definition of Group Factors

In this study, the group factors are defined as shown below:

$$\text{Horizontal Group Factor} \quad \alpha_{HH} = K_{HH}^G / (N_p K_{HH}^S) \quad (1)$$

$$\text{Vertical Group Factor} \quad \alpha_{VV} = K_{VV}^G / (N_p K_{VV}^S) \quad (2)$$

$$\text{Rotational Group Factor} \quad \alpha_{RR} = K_{RR}^G / (N_p K_{RR}^S + \sum_{i=1}^{N_p} l_i^2 K_{VV}^S) \quad (3)$$

where K is the static value of impedance, G is the index for pile group, S is the index for single pile, N_p is number of piles, and l_i is the distance between the i th-pile and the center of the pile group.

Parametric Studies for Group Factors

Parametric studies are carried out to investigate the characteristics of pile groups based on thin-layer formulation. Parameters employed are the number of piles (which is varied from 3×3 to 30×30), pile distance, pile length, soil properties, and so on, as shown in Table 1.

In this study, piles are buried in homogeneous soil or two-layered soil consisting of a surface layer and infinite space, and they are arranged in a square plan with pile heads fixed to rigid caps. In addition, the method for this parametric study is a superposition scheme (Kaynia and Kausel, 1982) based on interaction factors derived from 2-pile analysis using thin-layer formulation. Also the group factors are calculated according to equations (1)~(3), assuming the values of real parts of the impedances on frequency of 0.1 Hz to be static values.

Group factors in cases of A-1, B-1, B-4 and C-1 are indicated by the solid lines in Fig. 1. The solid lines in Fig.2 show group factors in cases with different pile rigidities and where the ratio of the distance of the piles(S) to the diameter of the piles(B) is fixed at 4.0. The dotted lines and symbols in these figures are defined in the subsequent sections. These results indicate some important points concerning the characteristics of group factors:

- (1) The group factor increases with the ratio of S/B .
- (2) The group factor decreases against pile rigidity.
- (3) The group factor increases with the stiffness of infinite space, and this characteristic is remarkable especially for vertical and rotational group factors.
- (4) The group factor of pile groups in two-layered soil varies with the length of pile.
- (5) In all cases, the rotational group factor of a 9-pile group is almost 1.0.

Therefore, the S/B ratio, pile rigidity, pile length, and soil properties should be considered in addition to the number of piles when evaluating group factors.

Table.1 Cases for parametric study

Case No.	Length of pile (m)	Diam-eter of pile (m)	Rigid-ity of pile	Shear wave velocity (m/sec.)
A-1	20	1	1.0	100/100
2	20	1	10.0	100/100
3	20	1	0.1	100/100
4	10	1	1.0	100/100
5	10	1	10.0	100/100
6	10	1	0.1	100/100
B-1	20	1	1.0	100/300
2	20	1	10.0	100/300
3	20	1	0.1	100/300
4	10	1	1.0	100/300
5	10	1	10.0	100/300
6	10	1	0.1	100/300
C-1	20	1	1.0	100/500
2	20	1	10.0	100/500
3	20	1	0.1	100/500
4	10	1	1.0	100/500
5	10	1	10.0	100/500
6	10	1	0.1	100/500
D-1	20	0.5	1.0	100/500
2	20	2	1.0	100/500
3	5	1	1.0	100/500
4	30	1	1.0	100/500
5	20	1	1.0	70/500
6	20	1	1.0	150/500
7	20	1	1.0	250/500

(properties of piles)
 Young's modulus $E_0^* = 2.1 \times 10^6 \text{ t/m}^2$
 Density $\gamma = 2.4 \text{ t/m}^3$
 Poisson's ratio $\nu = 0.167$

(surface layer)
 Shear wave velocity $V_S = 100 \text{ m/s}$
 Density $\gamma = 1.8 \text{ t/m}^3$
 Poisson's ratio $\nu = 0.4$

* The cases of $0.1 \times E_0$ and $10 \times E_0$ are also considered.

(infinite space)
 $V_S = 100, 300, 500 \text{ m/s}$ diameter of pile
 $\gamma = 1.8 \text{ t/m}^3$: $B = 0.5, 1, 2 \text{ m}$
 $\nu = 0.4$ ratio of distance(S) to B : $S/B = 2.5, 4, 6$

Note

- 1) Rigidity of pile means the ratio of Young's modulus to the standard value as shown in this figure.
- 2) Shear wave velocity in the format of 100/300 means values for the surface layer and infinite space.
- 3) In cases A, B and C, pile groups of 3×3 , 5×5 , 7×7 , 10×10 , 20×20 and 30×30 are calculated.
- 4) In case D, only 20×20 pile group is calculated.

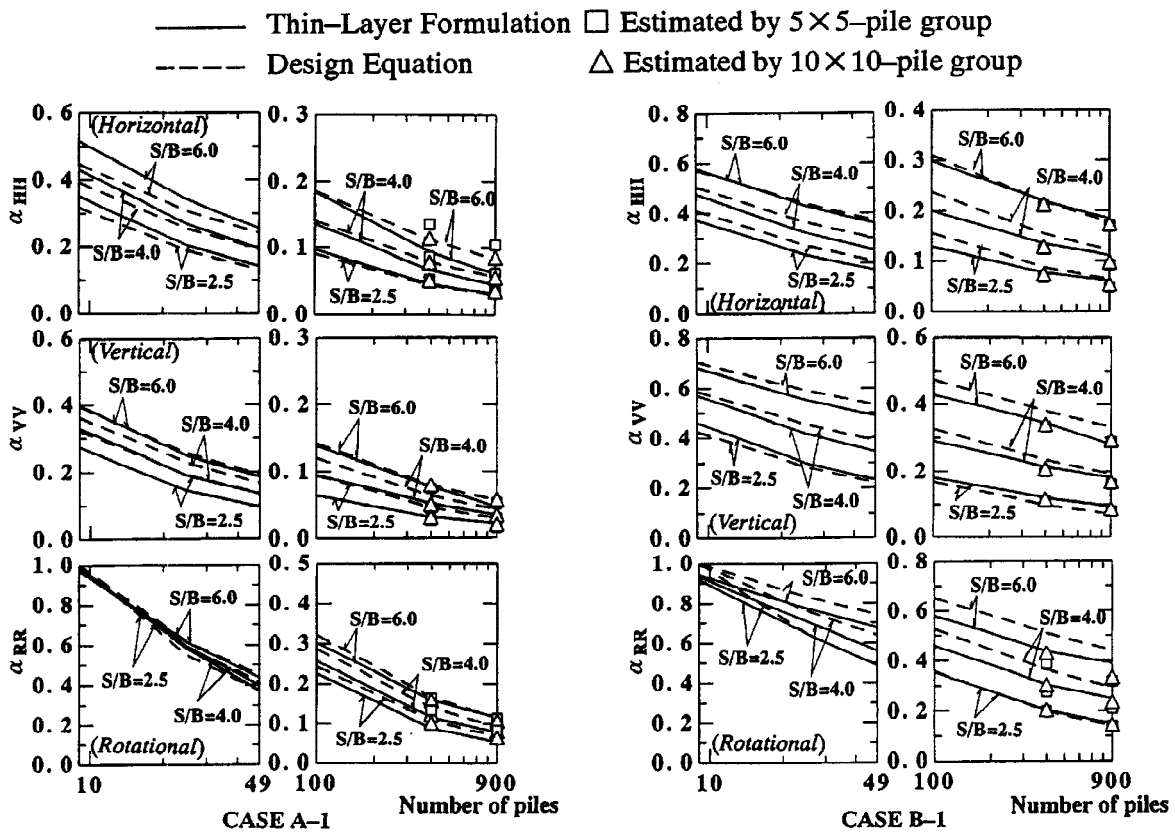


Fig.1a Group factors by thin-layer formulation to investigate the impact of the distance of piles, the length of piles, and soil properties on group factors.

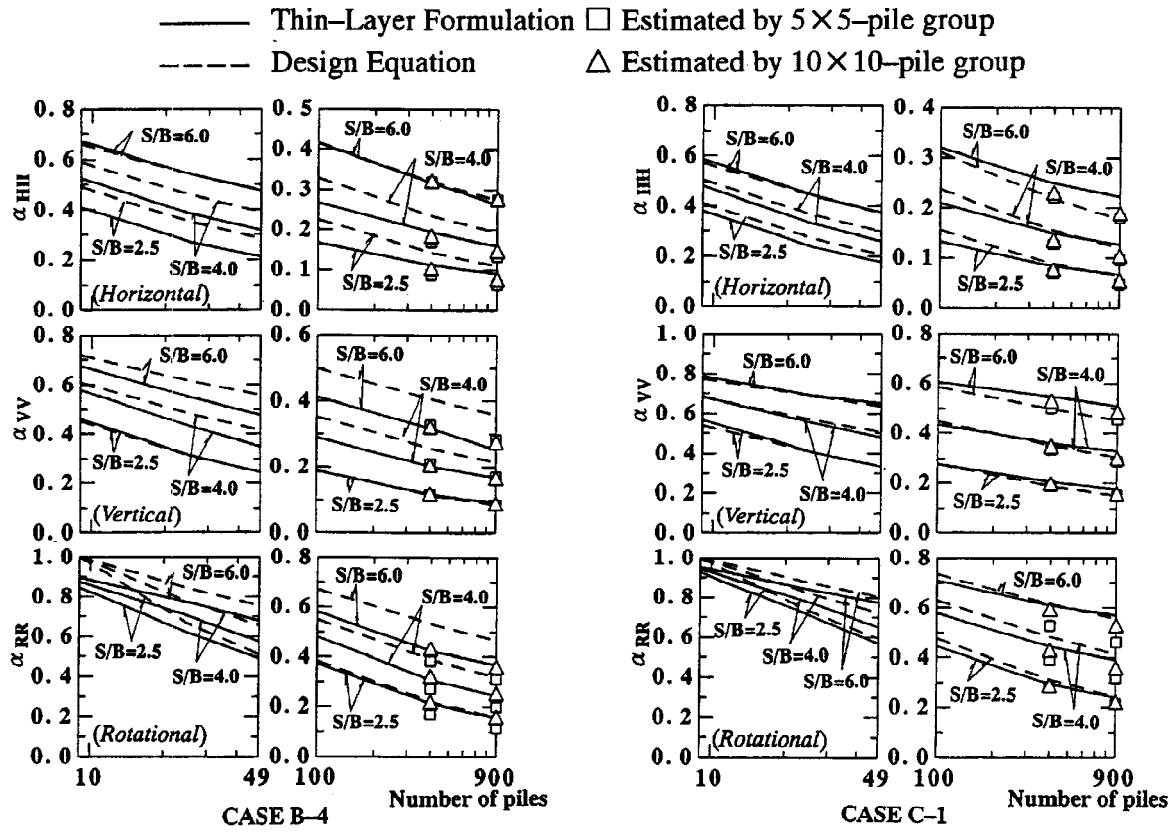


Fig.1b Group factors by thin-layer formulation to investigate the impact of the distance of piles, the length of piles, and soil properties on group factors.

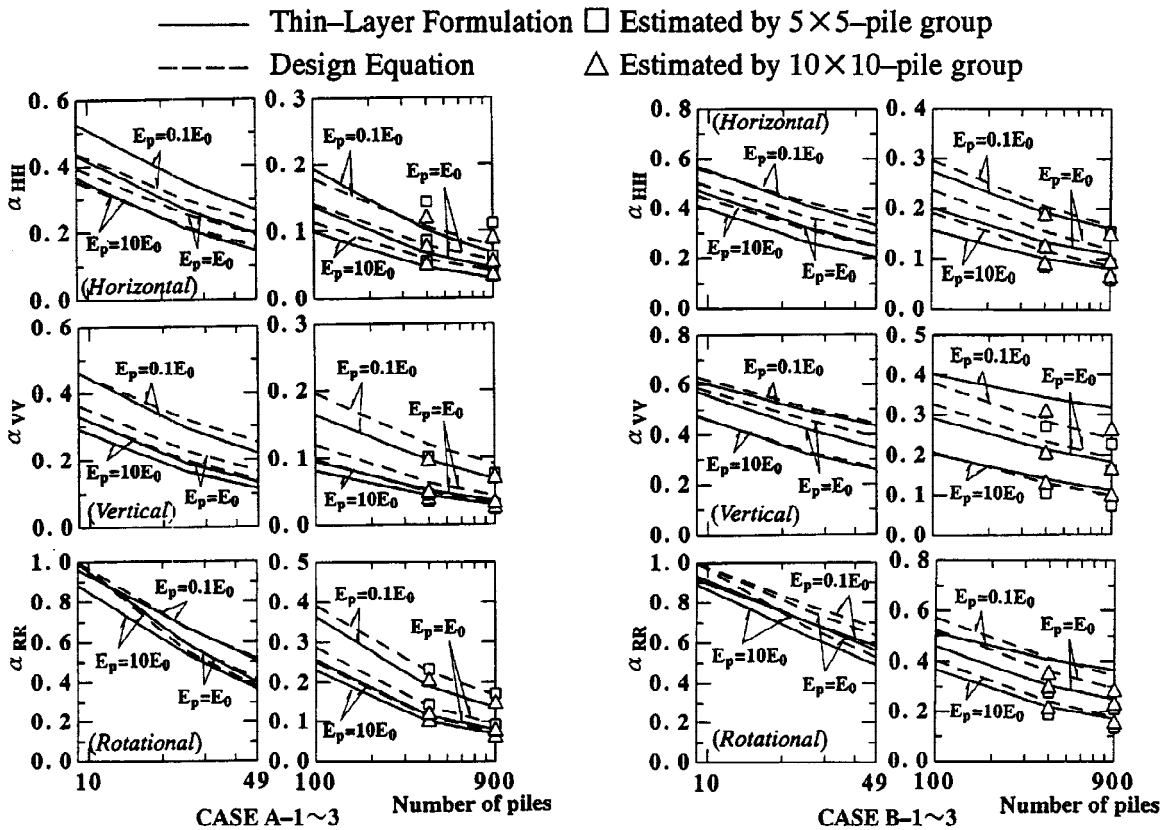


Fig.2 Group factors by thin layer formulation to investigate the impact of pile properties on group factors in cases with the S/B ratio fixed at 4.0.

GROUP FACTORS BASED ON ANALYSES FOR A SMALL NUMBER OF PILES

It is well known that horizontal group factor can be approximated by $N_p^{-0.5}$ (Hijikata, 1990). Accordingly, group factors in this study are represented by the power functions of the number of piles as follows:

$$\alpha_{HH} = N_p^{-a} \quad (4)$$

$$\alpha_{VV} = N_p^{-b} \quad (5)$$

$$\alpha_{RR} = (N_p/9)^{-c} \quad (6)$$

In equation (6), the number of piles N_p is divided by 9, as the rotational group factor of a 9-pile group becomes almost 1.0, irrespective of the soil and pile conditions, as was mentioned in the previous section. Then the values of $a(=-\log(\alpha_{HH})/\log(N_p))$, $b(=-\log(\alpha_{VV})/\log(N_p))$ and $c(=-\log(\alpha_{RR})/\log(N_p/9))$ are calculated from the group factors by thin-layer formulation as described in the previous section and from the number of piles. Here, $\log(\cdot)$ means an ordinary logarithm. Fig.3 shows the relationship between these values and the number of piles. According to Fig.3, values of a , b and c can be assumed to be almost constant, irrespective of the number of piles. Based on these characteristics, the relationship between the group factor of a large number of piles and that of a small number of piles can be described as shown below:

$$-a = \log(\alpha_{HH}^M)/\log(N_p^M) = \log(\alpha_{HH}^N)/\log(N_p^N) \quad (7)$$

$$-b = \log(\alpha_{VV}^M)/\log(N_p^M) = \log(\alpha_{VV}^N)/\log(N_p^N) \quad (8)$$

$$-c = \log(\alpha_{RR}^M)/\log(N_p^M/9) = \log(\alpha_{RR}^N)/\log(N_p^N/9) \quad (9)$$

where superscripts M and N denote indexes for a large number of piles and a small number of piles, respectively. Using equations (7)~(9), the group factors of a large number of piles can be evaluated by those of a small number of piles calculated by a rigorous method such as thin-layer formulation. In Figs. 1 and 2, the group factors for a 20×20 -pile group and a 30×30 -pile group estimated using a 5×5 -pile group and a 10×10 -pile group are indicated by symbols. The group factors estimated using this method approximately agree with more rigorous solutions. On close investigation it is found that the group factors based on a 10×10 -pile group are more precise than those based on a 5×5 -pile group as compared with analytical solutions. This phenomenon results from the fact that the values of a , b and c vary slightly with the number of piles.

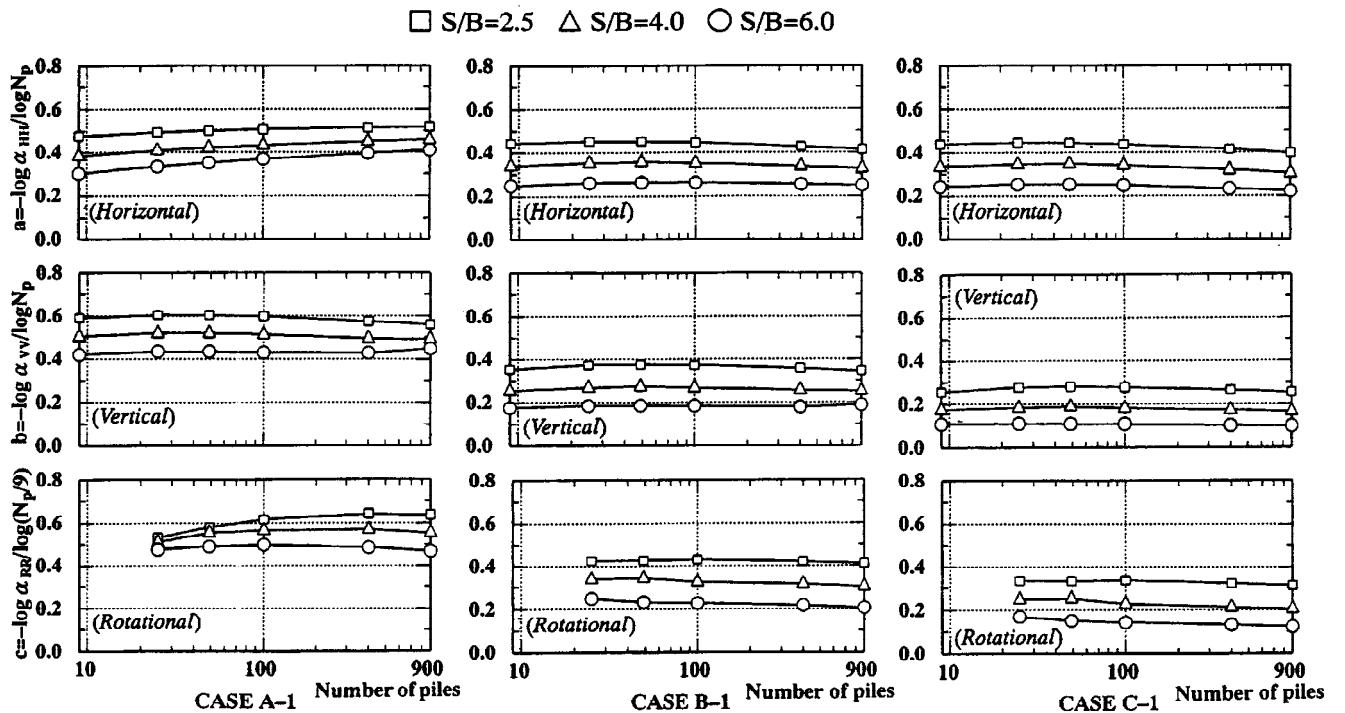


Fig.3 Relationship between group factors and the number of piles

DESIGN EQUATIONS FOR GROUP FACTORS

Relationships between Group Factors and Interaction Factors

Analyses by a rigorous method for a small number of piles are necessary to estimate group factors of a large number of piles, when using the method described in the previous section. Thus, the design equations are proposed in this section. According to these equations, group factors of a large number of piles can be calculated without using a rigorous method. Moreover, these equations can be derived from relationships between group factors and interaction factors. The interaction factor means the ratio of displacement at the head of a passive pile to that of an active pile. Fig.4 shows the relationships between the values of a , b and c for a 20×20 -pile group and interaction factors. In this figure, interaction factors are calculated for adjacent piles using thin-layer formulation; horizontal group factors are related to horizontal interaction factors parallel to the force on an active pile; and vertical and rotational group factors are related to vertical interaction factors.

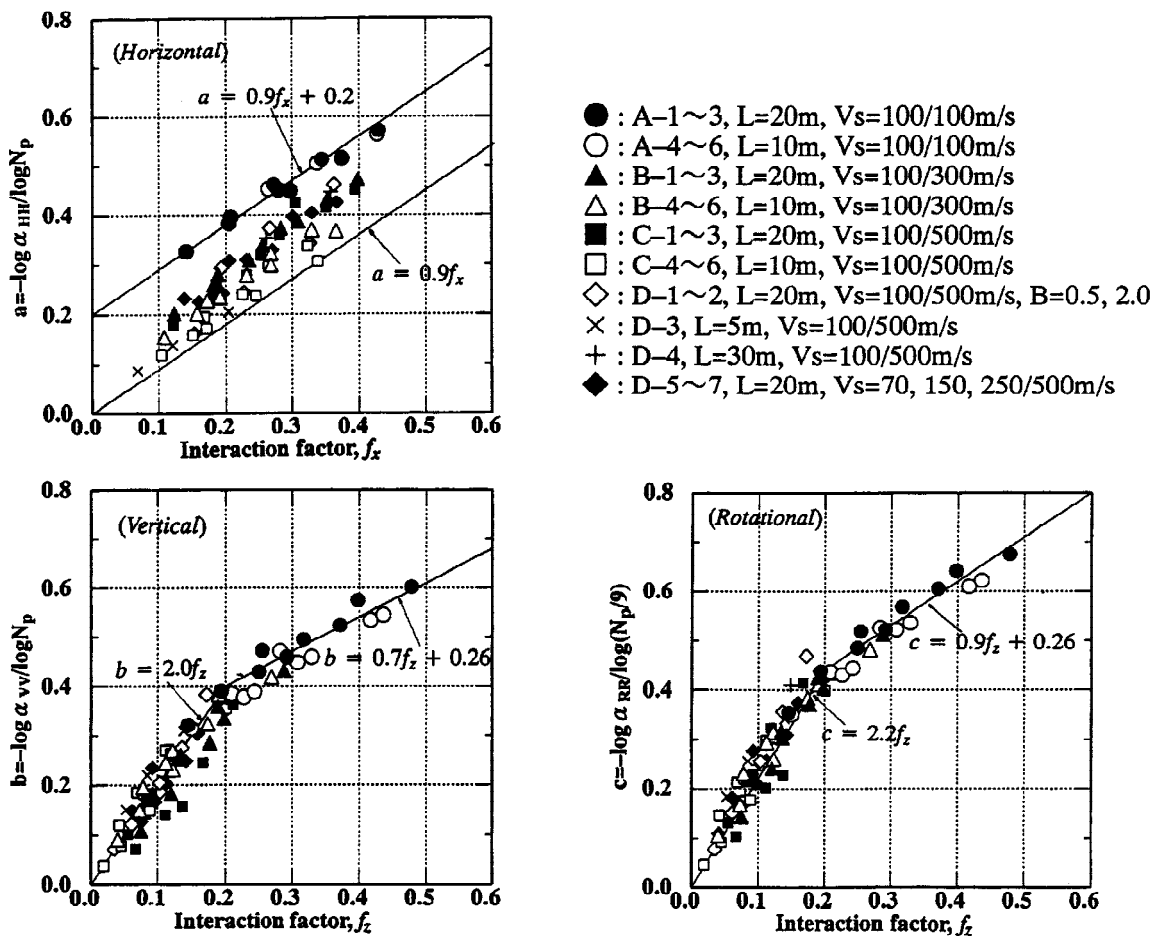


Fig.4 Relationship between values of a , b and c and interaction factors

Design Equation for Horizontal Group Factor

Regarding horizontal group factors in Fig.4, it seems that they do not correspond to the interaction factors. However, taking note of cases using the same piles and soil, the relationship between the value of a and the horizontal interaction factor f_x can be represented as follows:

$$a = 0.9f_x + z \quad (10)$$

where, z is almost 0.2 for homogeneous soil and less than 0.2 for two-layered soil. In addition, z for two-layered

soil varies with pile properties and soil properties, growing larger with the length of pile. Fig.5 shows the relationship between z and B/L . Here, L denotes the pile length and the values of z in this figure are averages of three values corresponding to the S/B ratio. These three values are calculated by f_x and a using equation (10). Moreover, in this relationship, the length of a pile for homogeneous soil is assumed to be infinite. According to this figure, the value of z can be regressed as below:

$$z = 1/\{5 + (65B/L)^{1.5}\} \quad (11)$$

It should be noted that the following equation for the interaction factor f_x for piles in homogeneous soil has been proposed by Hijikata *et al.* (1994a):

$$f_x = \{0.3 + 0.16 \log(E_P/E_S)\}(B/S)^{0.75} \quad (12)$$

where, E_P and E_S are Young's modulus of the pile and soil, respectively. Following the above representation, the interaction factor for two-layered soil is determined as follows:

$$f_x = \{0.3 + W \log(E_P/E_S)\}(B/S)^{0.75} \quad (13)$$

Fig.6 indicates the relationship between W and B/L . The value of W can be regressed in the same way as in equation (11) below:

$$W = 0.16 - 4(B/L)^2 \quad (W \geq 0.0) \quad (14)$$

As a result, the design equation for horizontal group factor is proposed as equations (4), and (10)~(14).

- | | |
|---------------------------------|---|
| ● : A-1~3, L=20m, Vs=100/100m/s | □ : C-4~6, L=10m, Vs=100/500m/s |
| ○ : A-4~6, L=10m, Vs=100/100m/s | ◇ : D-1~2, L=20m, Vs=100/500m/s, B=0.5, 2.0 |
| ▲ : B-1~3, L=20m, Vs=100/300m/s | × : D-3, L=5m, Vs=100/500m/s |
| △ : B-4~6, L=10m, Vs=100/300m/s | + : D-4, L=30m, Vs=100/500m/s |
| ■ : C-1~3, L=20m, Vs=100/500m/s | ◆ : D-5~7, L=20m, Vs=70, 150, 250/500m/s |

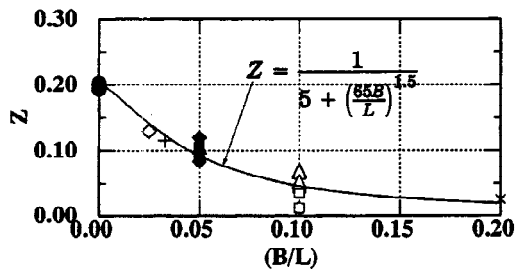


Fig.5 Relationship between z and B/L

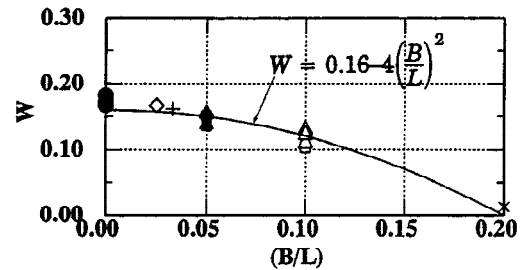


Fig.6 Relationship between W and B/L

Design Equations for Vertical and Rotational Group Factors

The vertical and rotational group factors in Fig.4 correspond well with the vertical interaction factor f_z , irrespective of the conditions of pile and soil. Therefore, their relationships can be approximated by the linear relationships as shown below:

$$\text{For vertical group factor} \quad b = 2.0f_z \quad (f_z \leq 0.2) \quad (18)$$

$$b = 0.7f_z + 0.26 \quad (f_z \geq 0.2) \quad (19)$$

$$\text{For rotational group factor} \quad c = 2.2f_z \quad (f_z \leq 0.2) \quad (20)$$

$$c = 0.9f_z + 0.26 \quad (f_z \geq 0.2) \quad (21)$$

Again, the following equation for the interaction factor f_z for piles in homogeneous soil has been proposed by Hijikata *et al.* (1994a):

$$f_z = \{0.22 \log(E_P/E_S)\}(B/S)^{0.5} \quad (\log(E_P/E_S) \leq 3.18) \quad (22)$$

Following the above representation, the vertical interaction factor for two-layered soil is determined as follows:

$$f_z = \{0.3 \log(E_P/E_S)\lambda + 0.5(1-\lambda)\delta\}(B/S) \quad (23)$$

where λ is the distribution of stress in the surface layer, δ is the ratio of displacement at the pile head to that at the pile end (both of them are defined in Fig.7). In equation (23), the first term represents the effect of the surface layer on the interaction factor, and the second term represents the effect of infinite space. In addition, λ and δ can be evaluated simply by the beam-on-Winkler foundation model for a single pile.

As a result of the investigations described above, the design equations for vertical and rotational group factors are proposed as equations (5), (6) and (18)~(23).

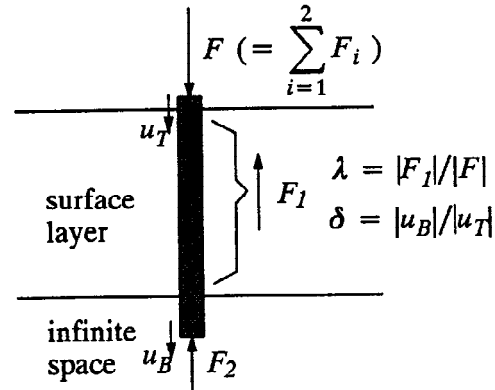


Fig.7 Definitions of λ and δ

The group factors estimated by the design equations are indicated by the dotted lines in Figs.1 and 2. According to these figures, the group factors determined by the design equations show close agreement with those determined by thin-layer formulation for groups of up to 1,000 piles.

CONCLUSIONS

Based on this research, the characteristics of the group factors can be summarized as follows:

- (1) The group factor varies not only with the number of piles but also with the distance of piles, pile properties, soil properties, and so on. Therefore, such elements must be considered in order to evaluate the group factor precisely.
- (2) The group factor normalized by the number of piles can be assumed to be almost constant, irrespective of the number of piles. Accordingly, the group factor of a large number of piles can be estimated from that of a small number of piles.
- (3) The group factor is related to the interaction factor, which is defined as the ratio of displacement at the head of a passive pile to that of an active pile. Based on the relationship between these factors, design equations can be proposed to evaluate the group factor of a large number of piles directly.

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