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EFFECTS OF STOCHASTIC VARIABILITY OF SOIL PROPERTIES ON LIQUEFACTION RESISTANCE

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ABSTRACT

A more rational approach to geotechnical design is made possible by use of stochastic field based techniques of data analysis. The proposed methodology, aimed at more realistic liquefaction risk assessment, is based on data obtained from extensive field measurement programs, and emphasizes the importance of spatial variability exhibited by soil properties, within so called "stochastically homogeneous" soil layers. Stochastic input finite element simulations of the behavior of saturated soil deposits, as well as of structures founded on liquefiable soil and subjected to seismic loads are performed, using a multi–yield soil constitutive model. The Monte Carlo simulation results obtained using stochastic constitutive parameters are compared to deterministic input simulation results. It is concluded that a more realistic pattern of soil liquefaction occurrence, and higher pore water pressure build–up are predicted by stochastic input analyses. The influences on the numerical simulation results of: finite element mesh size, probability distributions of soil properties, correlation distances, and spatial cross–correlations between soil properties are also discussed.

KEYWORDS

spatial variability; soil liquefaction; in–situ tests; stochastic data analysis; non–Gaussian vector fields; finite element method; stochastic input.

INTRODUCTION

Proposed Approach

The proposed methodology is aimed at evaluating the effects of stochastic variability of soil properties on the system performance under dynamic loads. It uses the Monte Carlo simulation method in the sense that digital simulations of stochastic fields are combined with stochastic input finite element analyses. The different soil properties over the analysis domain are modeled as a multi–dimensional, multi–variate (nD-mV) stochastic field, each component of the multi-variate field representing one of the different soil properties (Popescu, 1995; Deodatis *et al.*, 1995). If the spatial trends (i.e. position dependent mean values of the various geomechanical soil properties over the analysis region) are removed, and if the resulting random fluctuations about the spatial trends are normalized, then these random fluctuations can be modeled as a zero–mean, unit–variance, homogeneous vector field. The resulting sample functions of the simulated vector field are then properly transformed to represent stochastic input parameters for the finite element analyses. Four steps are basically required for the analysis (Popescu, 1995):

1. Estimation of the probabilistic characteristics of spatial variability (spatial trends, spatially dependent variance, probability distribution functions, cross–correlation structure) based on results of extensive field measurement programs.

2. Digital generation of sample functions of an nD–mV, non–Gaussian stochastic field, each sample function representing a possible realization of relevant field measurement results (e.g. N_{SPT} blow–count, cone tip resistance, etc.) over the analysis domain.

3. Parameter calibration for the soil constitutive model employed for the finite element analysis, using correlations with in–situ soil test results.

4. Finite element analyses using stochastic parameter input obtained from the generated sample vector fields of soil properties; a sufficient number of finite element simulations has to be performed to predict the statistics of the response.

The first three steps are described in detail in Popescu (1995), Deodatis *et al.*, (1995), and Popescu *et al.*, (1996). Results of stochastic input finite element analyses, showing the influence of stochastic variability of soil properties on dynamically induced liquefaction, are presented hereafter. The computations are performed using the multi–yield plasticity soil model (Prevost, 1985, 1989) implemented in the computer code DYNAFLOW (Prevost, 1981). The values of soil constitutive parameters are derived from real field data – piezocone test results from an extensive soil testing program performed for the artificial islands in the Canadian Beaufort Sea (Gulf, 1984). Several issues related to the set–up of stochastic input finite element calculations, and to the effects on numerical predictions of finite element mesh size, probability distribution functions of various soil properties, spatial correlation distances, and spatial cross–correlations between soil properties are also discussed.

Field Data

The field test results selected for this study come from eight piezocone borings performed in a hydraulic fill deposit and placed on a straight line at 1 and 9m horizontal intervals between the centers of adjacent borings (Gulf, 1984; Jefferies *et al.*, 1988). The analyzed soil deposit is part of an artificial island, called Tarsiut P–45, and consisting of a steel caisson (Molikpaq) with a hydraulic fill sand core, supported by a sand berm (Jefferies *et al.*, 1985). In this study, only the field test results obtained in the core fill are considered. The results of piezocone tests are analyzed in terms of: (1) cone tip resistance, q_c, which is mainly dependent on the relative density of soil and on confining stress, and (2) soil classification index, I_c, which is related to grain size and soil type, and is a function of all three results of piezocone tests, namely: cone tip resistance, sleeve friction and dynamic pore pressure (Jefferies and Davies, 1993).

The main results of stochastic field data analysis, leading to the probabilistic characteristics of spatial variability of standardized (i.e. zero–mean, unit–variance) relevant soil properties, are listed in Table 1.

Table 1. Statistics of spatial variability of field data (after Popescu, 1995)

Soil property		cone tip resistance q_c	soil classification index I_c
Probability distribution		Beta distr. with $p = 3.5$, $q = 18.4$	Beta distr. with $p = 5.2$, $q = 120$
Correlation	vert.	Two Peak (Popescu, 1995) with correlation distance $\theta_z = 0.95$m	
struct. (separable)	horiz.	Two Peak with correlation distance $\theta_x = 12.1$m	
Cross–correlation structure		Point–wise cross–correlation structure with $\rho_{qI} = -0.58$	

STOCHASTIC INPUT FINITE ELEMENT ANALYSIS SET–UP

Transfer of Random Data

Sample functions of the stochastic field representing the spatial variability of material properties over the analysis domain are simulated using the spectral representation method. For a description of the methodology used to generate sample functions of this non–Gaussian vector field, the reader is referred to Popeşcu (1995), Deodatis *et al.* (1995). The sample functions are generated at predefined spatial locations on a so called "stochastic field mesh". The main criterion for selecting the stochastic field mesh size is the correlation distance. There are also limitations imposed by computational effort. However, different criteria are used for selecting the finite element mesh size. Consequently, when using the spectral representation method for stochastic field generation, it is very likely to have two different meshes: one for the random field discretization, and the other for the finite element analysis. A transfer of data is therefore necessary from one mesh to another. Two methods of data transfer are investigated (the reader is referred to Brenner, 1991, for a detailed literature review): (1) the ***midpoint*** method, in which the random field is represented by its values at the centroids of each finite element, and (2) the ***local averaging*** method, which assigns to each finite element a value obtained as an average of stochastic field values over the element domain.

For Gaussian random fields, the probability distribution function of the discretized field remains Gaussian for both midpoint and local averaging methods. In this case, the local averaging method seems to be a more logical approach, and was proven to provide better accuracy than the midpoint method for coarse meshes (Der Kiureghian and Ke, 1988). However, for non–Gaussian random fields the original distribution of the point field is difficult or impossible to obtain when using the local averaging method (Der Kiureghian and Ke, 1988). This is illustrated in Fig. 1, based on a numerical example (Popescu, 1995). The Beta distribution of the original field is reduced to the normal distribution by the local averaging process, as the normalized finite element area A/α increases. Consequently, for the case of non–Gaussian fields, the midpoint method, which preserves the original probability distribution as shown in Fig. 1, is deemed as more appropriate and used hereafter.

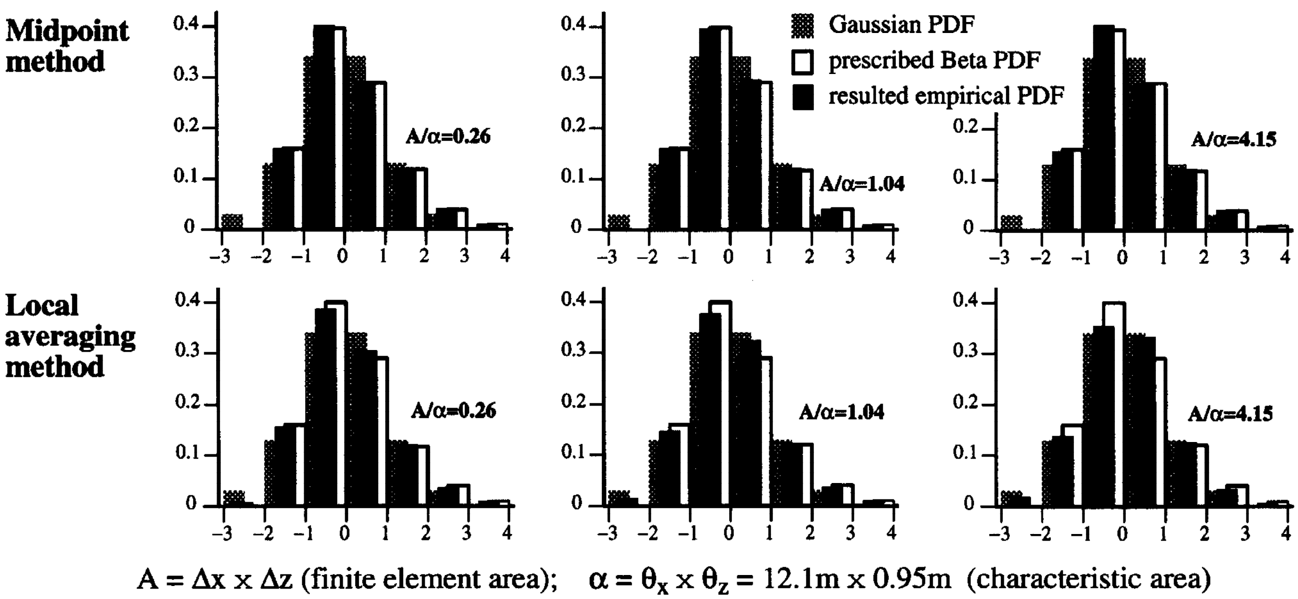

Figure 1: Influence of the data transfer method on resulted probability distribution function (results based on sample functions of a stochastic field describing the spatial variability of cone tip resistance recorded at Tarsiut P–45).

Selection of the Mesh Size for Stochastic Analysis

The mesh size used in deterministic finite element computations is usually dictated by the expected gradients in relevant results. However, when using stochastic input the size of the finite element mesh has to be small enough to capture the essential features of the random field (Brenner, 1991). Consequently, the upper bounds for the mesh size Δx_k depend on the correlation distances θ_k. Various values of these upper bounds have been reported for various correlation structures, e.g.: $\Delta x_k \leq (0.25 \ldots 0.5)\theta_k$ (Der Kiureghian and Ke, 1988) for an exponential correlation structure, $\Delta x_k \leq (0.7 \ldots 0.8)\theta_k$ (Popescu et.al., 1995) for a triangular correlation structure, $\Delta x_k \leq (0.4 \ldots 0.8)\theta_k$ (Popescu, 1995) for various "two peak" correlation structures. It is believed that the upper bounds depend on the type (shape) of correlation structure as well as on correlation distances. For more information on this issue the reader is referred to Shinozuka and Deodatis (1988).

Some results of a numerical experiment are presented in the following, in terms of predicted liquefaction index (the expression of liquefaction index is given in Fig. 3,b). A saturated soil deposit is subjected to seismic excitation, and the numerical predictions are performed using various mesh sizes. The finite element analysis set-up (dimensions of the analysis domain, materials, boundary conditions, input motion) is described in the next section. The results obtained for various mesh sizes are compared with the assumed "exact" solution, corresponding to a very fine mesh (1.50m × 0.25m). From the computational results presented in Fig. 2, it can be concluded that meshes up to 4.5m × 0.75m, corresponding to $\Delta x \approx 0.4\theta_x$ and $\Delta z \approx 0.8\theta_z$, provide acceptable results.

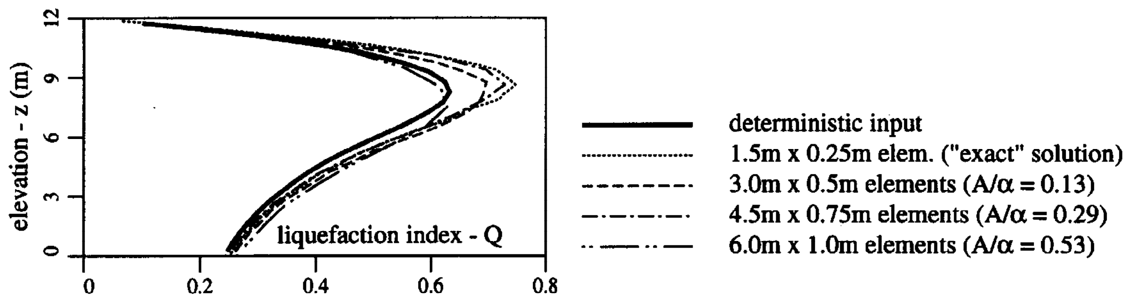

Figure 2: Influence of mesh size on predicted liquefaction index (field data from Tarsiut P–45).

EFFECTS ON LIQUEFACTION RESISTANCE

Finite Element Analysis Set – Up

The effects on the computational results of various factors related to stochastic variability of soil properties are investigated in the following based on dynamic analyses of saturated sandy soil deposits subjected to seismic excitation. The multi-yield plasticity model (Prevost, 1985, 1989) implemented in the computer code DYNAFLOW (Prevost, 1981) is used for this purpose.

A loose to medium dense soil deposit, with geomechanical properties as well as spatial variability characteristics corresponding to the hydraulic fill at Tarsiut P–45 is selected for the numerical examples. The analysis domain extends 60m in horizontal direction (Fig. 3,a). A 12m thick "statistically homogeneous" saturated sand deposit is overlaid by an 1.6m dry sand layer. A rigid, impervious bedrock is assumed at the base, where the input horizontal acceleration is applied. The base input motion is selected as the first 6 sec. of the N–S component of the 1964 Niigata Earthquake acceleration recorded at Akita Prefectural Office, scaled at 0.15 g. A refined discretization mesh (1920 1.5m × 0.25m finite elements for the saturated material, and 40 1.5m×1.6m finite elements for the dry sand) is used, to eliminate any effects induced by larger mesh sizes.

Stochastic vs. Deterministic Input Parameters

A deterministic analysis is performed for comparison, for the same soil deposit described in the previous section. The soil constitutive parameters for the deterministic input analysis are variable with depth, corresponding to the linear variation exhibited by the spatial trends of the field measurement results (Popescu, 1995). Consequently, the deterministic input analysis is performed using the same average values of soil parameters as for the stochastic input computations.

A comparison between results of deterministic and stochastic parameter input computations performed using field data from Tarsiut P–45 is presented in Fig. 3,a in terms of predicted excess pore pressure ratio with respect to the initial effective vertical stress, u/σ_{v0}. Areas with significant excess pore pressure build–up ($u/\sigma_{v0} \geq 0.6$) are only emphasized. Four realizations of the vector field of material properties are employed to derive four sets of stochastic input soil parameters. Consequently, four different responses in terms of excess pore pressure ratio are obtained by the stochastic input analysis. There are however a series of common features of the stochastic input computational results, as compared to the results of the deterministic input analysis: (1) more pore pressure build–up is predicted by the stochastic than by the deterministic input model; (2) in the case of stochastic input results, there are patches with large predicted excess pore pressure, corresponding to the presence of loose pockets in the material; this predicted pattern of excess pore pressure build–up better explains the phenomenon of sand boils observed in areas affected by soil liquefaction.

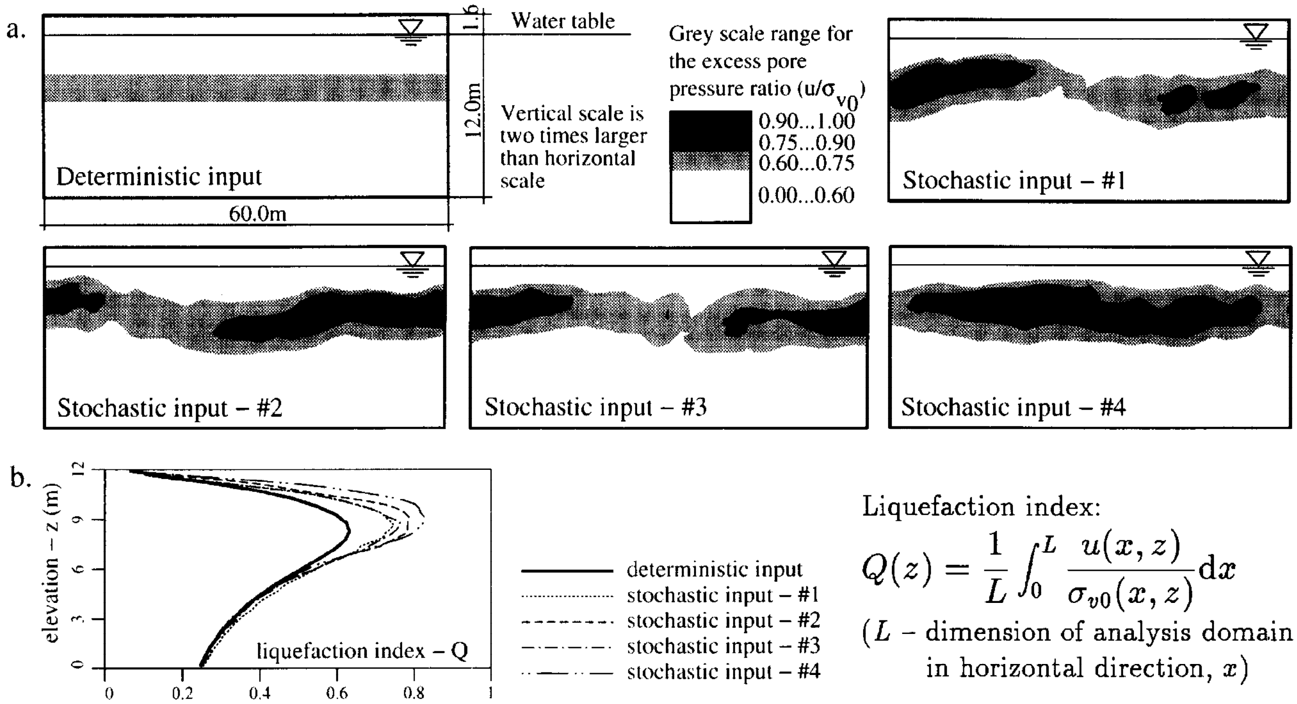

Figure 3: Comparison between deterministic and stochastic parameter input computations in terms of: a. predicted excess pore pressure ratio, and b. liquefaction index (field data from Tarsiut P–45).

Average predicted pore pressures are also compared in Fig. 3,b in terms of liquefaction index, computed by averaging the excess pore pressure ratio in horizontal direction. The resulted values clearly indicate higher excess pore pressure build–up predicted by the stochastic than by the deterministic input analysis, for the loose hydraulic fill deposit.

Influence of Probability Distribution of Soil Properties

Some results of a study on the influences of probability distribution functions of soil properties on the amount of pore water pressure build–up are presented in Figure 4. The stochastic input parameters are obtained starting from three realizations of the stochastic field simulated using the same seed number for the generation of the random phase angles (Popescu, 1995), but different target distribution functions (case 1 – Beta distribution, as derived from field data analysis, case 2 – Normal distribution, and case 3 – Lognormal distribution). The average values of input soil parameters, as well as the variance of their spatial variability are the same for all cases. Consequently, a similar pattern of pore water pressure build–up is predicted for all three cases. However, the resulted amount of excess pore pressure is clearly different for the three situations analyzed, and is apparently dependent on the left tail of the distribution function (the cumulative distributions of the standardized field test results are shown in Figure 4): larger pore pressure build–up is predicted as the left tail of the probability distribution is longer. (The left tail of the probability distribution of cone tip resistance corresponds to loose pockets in the soil mass).

This important conclusion also indicates an explanation for predicting more pore pressure build–up in case of stochastic input computations: the increase in pore pressure is facilitated by the presence of looser zones, and more variability in soil parameter spatial distribution leads to increased likelihood of loose pockets. Of course, denser zones of material are correspondingly represented, however their contribution seems to be negligible in the case of dynamically induced soil liquefaction.

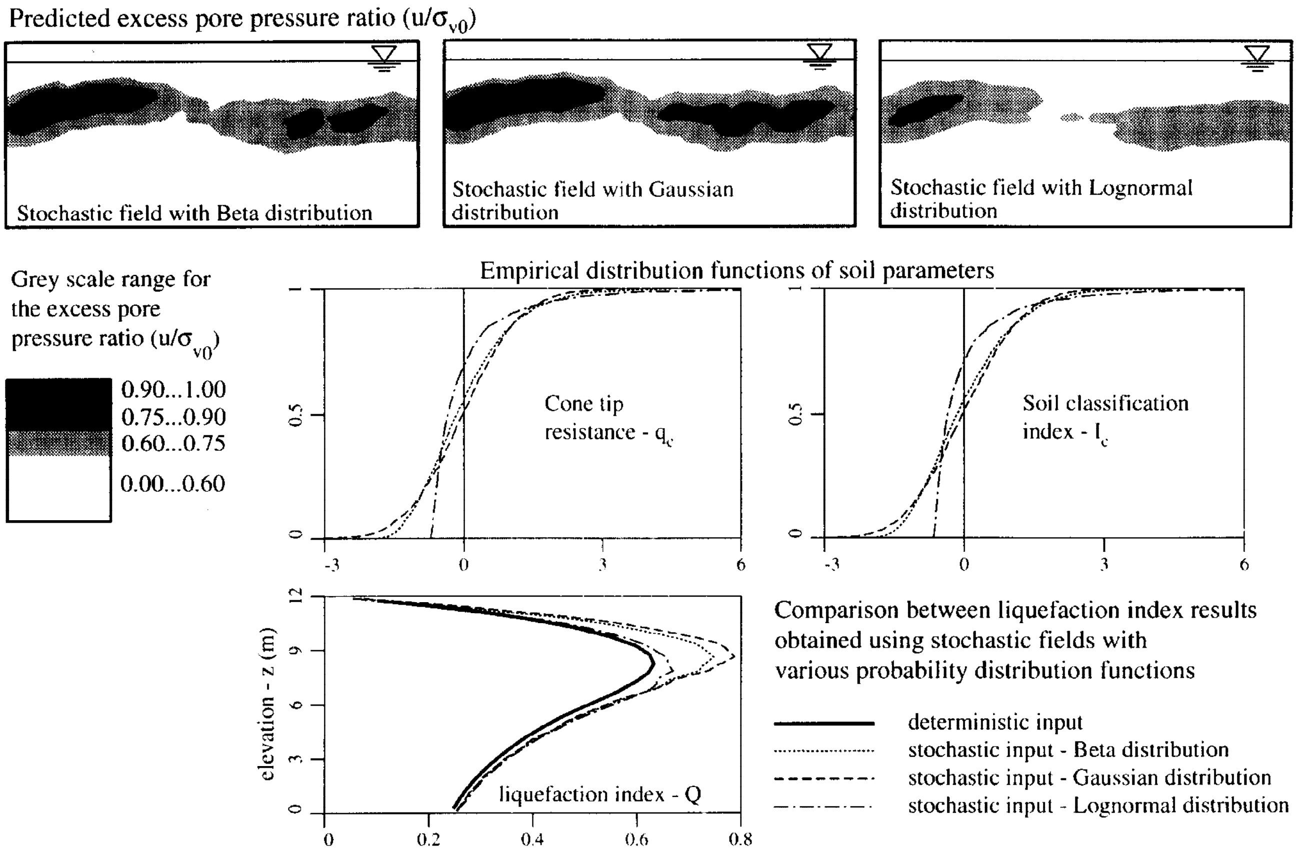

Figure 4: Influence of assumed probability distribution functions on predicted excess pore pressure build–up (field data from Tarsiut P–45).

Other factors, like assumed spatial correlation distances and cross–correlation between various soil properties were found to have less influence on the numerical simulation results, at least for the type of field data used in this study (Popescu, 1995).

APPLICATION – STRUCTURE FOUNDED ON SATURATED SOIL DEPOSIT

A structure representing a four story building is founded on a relatively loose sand deposit, as shown in Fig. 5. The saturated soil layer is discretized into 840 finite elements (1.5m×0.5m). Semi–infinite soil deposit conditions are prescribed at the lateral boundaries of the analysis domain. Impervious soil–structure interface is assumed. The base input acceleration is selected as the strong motion recorded at Akita Prefectural Office during the 1964 Niigata Earthquake, and scaled at 0.15 g.

The same four realizations of the stochastic field of soil properties generated based on results of piezocone tests performed at Tarsiut P–45 are employed to derive the soil constitutive parameters. Some of the results, in terms of predicted excess pore pressures, are presented in Fig. 5. Computational results obtained using deterministic parameter input, with soil property values corresponding to the average values of the stochastic soil parameters, are shown for comparison. The same conclusions can be inferred: more pore pressure is predicted by stochastic than by deterministic analyses, especially in the free field. More important, however, seems to be here the predicted pattern of pore pressure build–up which, for some possible realizations of the spatial distribution of soil properties, may lead to relatively large differential settlements of the structure, as shown in Fig. 6. It can be observed that, for some realizations of the stochastic field, the stochastic input analysis predicts displacements of the structure which are considerably larger than those predicted by deterministic analysis.

CONCLUSIONS

A Monte Carlo type simulation procedure for evaluating the effects of stochastic variability of soil properties on soil behavior under dynamic loads is proposed. The main steps are: (1) stochastic

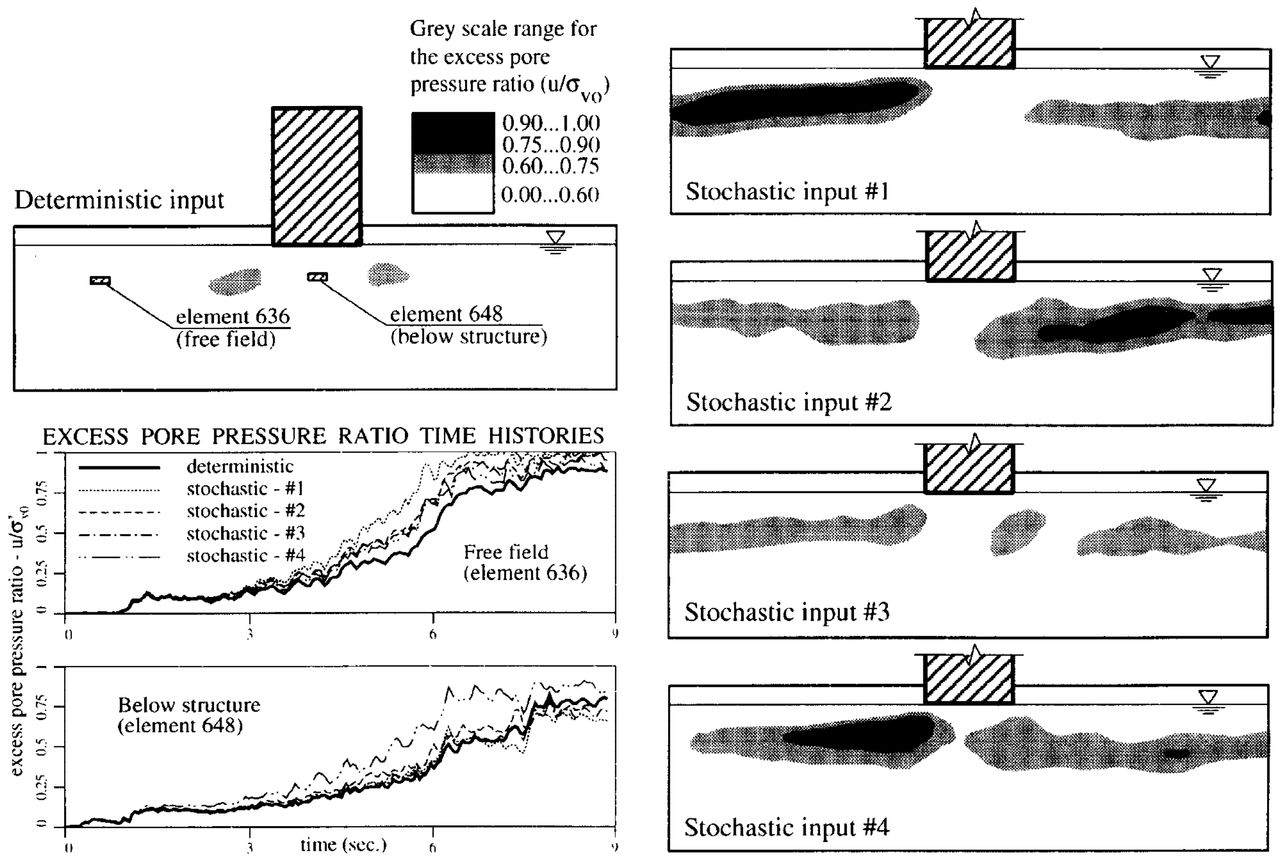

Figure 5: Structure founded on liquefiable sand deposit: predicted excess pore pressure ratio u/σ'_{v0}: contours, at time $T = 6.0$sec., and time histories (field data form Tarsiut P–45).

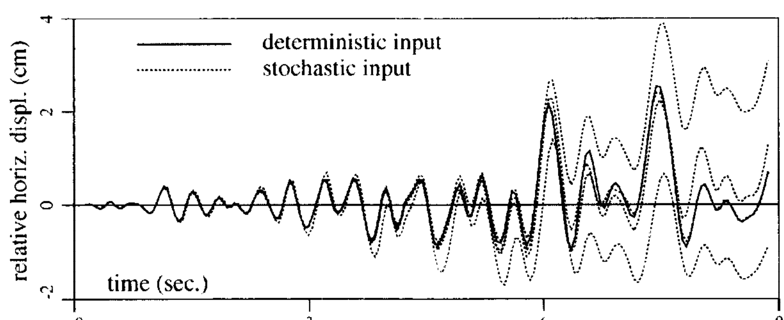

Figure 6: Predicted horizontal displacements of the top of structure, relative to the base of structure ($\delta^x_{\text{node 950}} - \delta^x_{\text{node 938}}$).

analysis of field data, (2) digital simulation of non–Gaussian vector fields representing the spatial distribution of various soil properties, and (3) stochastic parameter input finite element analyses.

The effects of spatial variability of soil properties on soil liquefaction are investigated by comparisons between results of deterministic input and stochastic input finite element analyses, performed for saturated soil deposits subjected to seismic excitation. From numerical examples based on real in-situ soil test results, it is concluded that both the pattern and the amount of dynamically induced pore water pressure build-up are strongly affected by spatial variability of soil properties. Moreover, deterministic analyses which account for the average soil strength, and therefore cannot simulate the presence of loose pockets in the soil mass, are deemed to provide non-conservative results.

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