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AN OVERALL PROBABILISTIC APPROACH TO EVALUATE SOIL LIQUEFACTION

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ABSTRACT

This paper presents an overall probabilistic approach to evaluate seismic liquefaction potential for saturated sandy deposits during earthquakes. The approach is derived based on the elementary notion of sets. As input parameters, SPT values and expective maximum accelerations of ground are used. The approach can give a liquefaction probability both at any depth and a given layer in ground. The data base of 39 historical earthquakes was employed for formulating the model proposed in this study. Comparation of the approach with Seed simplified procedure is performed to check the validity of the approach by case studies.

KEYWORDS

Liquefaction probability; notion of sets; overall probabilistic; soil liquefaction.

INTRODUCTION

Various procedures for estimating liquefaction probability for saturated sandy soils have been developed in the past twenty years. Most of them are to base it either on Seed's simplified procedure (Atkinson and Finn, 1983, 1985, 1987), combined with Cornell's seismic assessment model, or on the laboratory cyclic stress test (Fardis and Veneziano, 1981). These procedures are all based on the concept of the stress ratio proposed by Seed firstly. Some empirical methods have been extended to estimate the probability of liquefaction from SPT value, N, and maximum acceleration of ground surface (Christian and Swiger, 1975).

Probabilistic versions of more sophisticated approaches have also been developed (Donovan, 1971; Faccioli, 1973; Haldar and Tang, 1979, 1981; Chou, 1983).

The cornerstone of an overall probabilistic approach is a rational stochastic model in which the relation between seismic stress caused soil liquefaction and resistance against liquefaction can be well described in concept of statistics consistently. It is the purpose in this study to attempt to establish such a model based on the elementary notion of sets. The word "overall" means that the major emphasis will be on the consistency of the used concepts in probability theory and that to except some empirical factors in the new probabilistic model as much as possible, because the empirical relations can be considered to be deterministic in fact.

BASIC CONSIDERATIONS

The evolution of the simplified procedure for evaluating liquefaction potential of level deposits saturated using field data obtained from SPT was reviewed by Seed and his coworkers (Seed and Idriss et al., 1971, 1981, 1983). They presented the field data for sites which are known to have liquefied or not liquefied during earthquakes in the United States, Guatemala, Argentina, Japan, China and other parts of the world to establish a criterion for evaluating liquefaction potential in earthquakes of magnitude 7.5. This empirical procedure is believed to provide the most useful and deterministic method available at the present time. It is difficult, however, to handle some uncertainties associated in the famous expression of the stress ratio:

$$\tau_{av}/\sigma_v = 0.65\, \sigma_v/\sigma'_v a_{max} r_d \qquad (1)$$

where τ = seismic stress; σ and σ' = total and effective overburden pressure respectively; a = horizontal acceleration of ground surface; and r = depth reduction coefficient. The numeric factor of 0.65, for example, assumes empirically that the equivalent uniform shear stress is equal to 65 percent of the absolute maximum shear stress. In fact, the actual time history of ground motion will have an irregular and random form. The acceleration correction factor is used to reduce the surface acceleration for depth since the soil is a deformable body rather than a rigid one assumed. But it is well-known that there is a significant scatter of the values for this factor and increases with depth. The analysis of such uncertainties in this approach reveales that the uncertainties associated with the load parameters exceed those of in the resistance parameters (Haldar and Tang, 1979).

Obviously, these uncertainties associated with some secondary and unperfect, or unkown, factors in such an empirical definition are not expected to negligible and further research is needed.

Christian and Swiger (1975) utilized discriminant techniques to analyze directly the data base from 39 earthquakes and have defined a parameter, A, as following:

$$A = a_{max} (\sigma_v/\sigma'_v) \qquad (2)$$

The factor, A, can be regarded as an alternative earthquake-induced action considered the depth effects. Relative density, Dr, determined from SPT values by Gibbs & Holtz relation was used as resistance parameter against liquefaction. This statistical procedure has given the statistics of the data set of 39 earthquakes and can give the corresponding confidence level lines. These lines separate liquefiable from nonliquefiable cases but can not give the probability that liquefaction will occur. Nevertheless, the parameters, Dr and A, are two convenient and practical variables in probabilistic model. Statistics of the data set are also used in this paper.

OVERALL PROBABILISTIC MODEL

Many of the characteristics of probabilistic problems in engineering can be defined formally and modeled succinctly using the elementary notion of sets (Ang and tang, 1975). This study is an attempt to apply this philosophy to the evaluation of liquefaction potential.

Let R and S denote statistically independent random variables representing the natural logarithms of Dr and A, with mean values of $\mu(R)$ and $\mu(S)$ and standard deviations of $\sigma(R)$ and $\sigma(S)$, respectively. A sample space of Z, then, can be defined:

$$Z = \{R, S\}. \qquad (3)$$

Step-1 of the Approach

First assumption is that there exist two prospective limit values, α and β, in the sample space consisting of all cases in which liquefaction have been certified and that liquefaction is expected to occur whenever both R is smaller than or equal to α at given $S=\beta$ and S is greater than or equal to β at given $R=\alpha$ at a point of depth in soil layer. There are two mutually exclusive and collectively exhaustive events, E1 and E2, in the sample space:

$$E1 = \{R \le \alpha | S=\beta\} \text{ and}$$
$$E2 = \{S \ge \beta | R=\alpha\}. \tag{4}$$

As events can be combined to obtain other new events via the operational rules of sets and subsets, the liquefaction potential denoted by a new event of E[L] will be the union of the events E1 and E2:

$$E[L] = \{E1 \cup E2\}. \tag{5}$$

This means that the given point in deposits will be liquefied by the occurrence of either or both of the events E1 and E2. Since

$$p\{E[L]\} = 1, \tag{6}$$

the liquefaction condition can be expressed by

$$p\{R \le \alpha | S=\beta\} - p\{S \le \beta | R=\alpha\} = 0. \tag{7}$$

Second assumption is that there exist other two prospective limit values, θ and δ, and

that liquefaction is not expective to occur whenever both R is greater than or equal to θ at given $S=\delta$ and S is smaller than or equal to δ at given $R=\theta$ at the same point in deposits. Then, there are other two mutually exclusive and collectively exhaustive events, E3 and E4, in the sample space:

$$E3 = \{R \ge \alpha | S=\beta\} \text{ and}$$
$$E4 = \{S \le \beta | R=\alpha\}. \tag{8}$$

Similarly, the condition for nonliquefiable cases can be expressed by

$$p\{S \le \delta | R=\theta\} - p\{R \le \theta | S=\delta\} = 0. \tag{9}$$

Considering that (7) and (9) can be written into the forms of the conditional density functions and that they are also normal distributions with the same expected values and variances as those of R and S, an overall probabilistic model for liquefaction potential limits can be develped:

$$\beta = \mu(S)+\{\sigma(S)/\sigma(R)\}\{\alpha-\mu(R)\} \tag{10}$$

$$\delta = \mu(S)'+\{\sigma(S)'/\sigma(R)'\}[\theta-\mu(R)'\} \tag{11}$$

in which the note of apostrophe represents the cases of R and S where liquefaction did not occur in fact.

Employing the statistical properties of the data set, as shown in Tab.1, into (10) and (11), the model can be formulated as

$$\beta = 1.8948\alpha - 8.6467 \tag{12}$$

$$\delta = 1.8412\theta - 9.1409 \tag{13}$$

These lines are shown in Fig.1 and Fig.2. Figures show that liquefaction will definitely occur for a given point falling to the left of the α-β line (p=1) and will definitely not occur for a point falling to the right of the δ-θ line (p=0). Obviously, the area enclosed by the two lines hints that liquefaction will be possible with a probability. This is the important feature of the model which is different from the other statistical expressions essentially. From the formulation above, the other form of the model, by use of Gibbs and Holtz's relation, can be derived:

$$\tilde{N}(L) \le 0.21(\sigma'_v+70)\exp[1.06\ln(a_{max}\sigma_v/\sigma'_v)] \quad (14)$$

and

$$\tilde{N}(N) \ge 0.46(\sigma'_v+70)\exp[1.08\ln(a_{max}\sigma_v/\sigma'_v)] \quad (15)$$

in which $\tilde{N}(L)$ and $\tilde{N}(N)$ = the critical number of blowcounts from SPT for liquefiable and nonliquefiable cases respectively; a max = peak horizontal acceleration (g); σ and σ'= total and effective overburden pressures in kN/m² respectively.

Table 1. Statistics of data set (Christian and Swiger, 1975)

Description	All data	Nonliquefied	Liquefied
μ of lnDr	4.07164	4.27580	3.94404
σ of lnDr	0.28567	0.23367	0.23962
μ of lnA	-1.20982	-1.26815	-1.17336
σ of lnA	0.44178	0.43024	0.45406

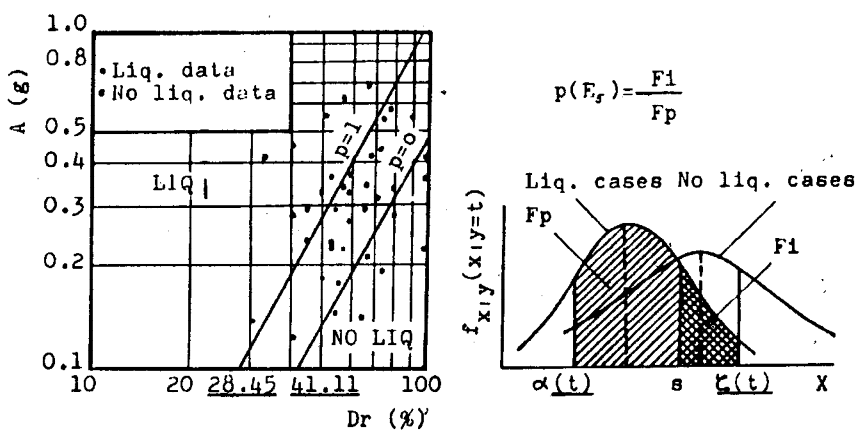

Fig. 1. Data points, critical lines (Logarithmic) and the third assumption

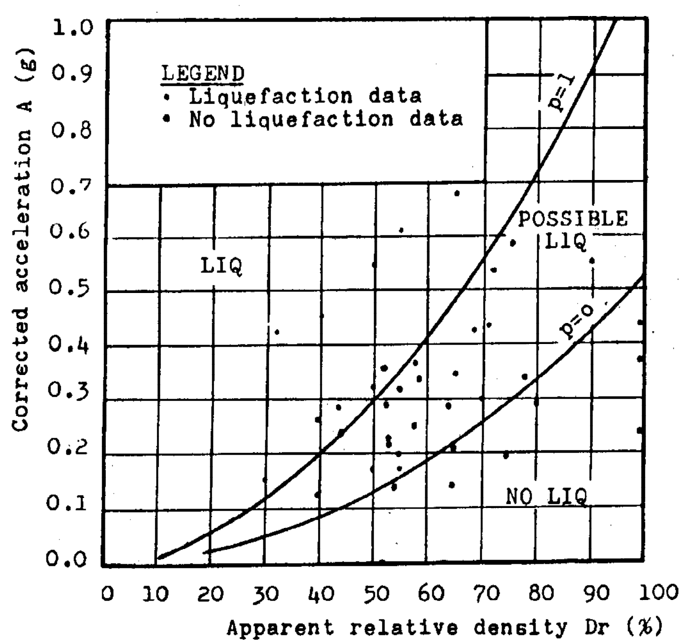

Fig. 2. Data points and critical lines (Arithmetic)

Step-2 of the Approach

Thus far, we have found the critical line between possible liquefaction and certain liquefaction and the other critical line between possible liquefaction and nonliquefaction. It is also of interest to evaluate the probability that liquefaction will occur likely for those points located in the area enclosed by the two critical lines.

Consider here that another sample space consists of all possible liquefaction cases and that a given point, (R=r,S=s), is located this space:

$$\alpha(s) \le r \le \theta(s) \quad \text{and} \quad \delta(r) \le s \le \beta(r) \tag{16}$$

in which $\alpha(s)$ and $\theta(s)$ are the variation limits of r; $\beta(r)$ and $\delta(r)$ are those of s. Then, there are two statistical independent events of E5 and E6:

$$E5 = \{S=s|R=r\} \text{ and } E6 = \{R=r|S=s\}. \tag{17}$$

The event defined by actual liquefaction potential, E[P], can be derived from the intersection of E5 and E6:

$$E[P] = \{E5 \cap E6\}. \tag{18}$$

Third assumption is that p[E5] and p[E6] can be indicated by the ratios between partial and entire domains of the integrals of the conditional density functions. The integration limits, $\alpha(s)$ and $\beta(r)$, is obtained by using of eq.(12) , letting β=s and α=r, respectively. By the same way, $\theta(s)$ and $\delta(r)$ can be calculated from eq.(13). Thus, the liquefaction probability for the given point of [r=lnDr;s=lnA] will be:

$$p\{E[P]\} = \frac{[\Phi(a)-\Phi(b)][\Phi(c)-\Phi(d)]}{[\Phi(a)-\Phi(c)][\Phi(b)-\Phi(d)]} \tag{19}$$

in which Φ = distribution function of standard normal variates; and

a = 4.25 + 2.26 lnA
b = 4.17 lnDr - 16.64
c = 2.58 + 2.20 lnA
d = 4.06 lnDr - 17.55

This expression has been programed for practical use, as shown in Fig.3 for example.

Step-3 of the Approach

According to de Morgan rule, calculation for several points in different depths of a given site can give the liquefaction probability for the whole layer under study, P(L), as follows:

$$P(L) = 1 - \prod_{i=1}^{m} (1 - p\{E[P]\}_i) \tag{20}$$

where m = the total number of calculated points in the whole layer of soil.

Of cause, a three dimensional calculation for liquefaction potential of site can be performed more easily by use of eq.(20).

CASE STUDIES

To investigate the validity of the approach proposed, the equations mentioned above were tested on data of past earthquakes from 1975 to 1979. These data were not used in formulating the model. The results of the analysis are in good agreement with field observation.

Gatemala Earthquake of February 4, 1976 (Seed et al., 1981)

According to step-1 of the approach, it is seen that liquefaction would occur in the site represented by the boring #1 but would not occur in the site represented by the borings #3 and #4. It is of very interest that the boring #2 falls just inside the lower limit of the possible liquefaction as shown in Tab.2. And according to the step-2 of the approach, the probability that the site of boring #2 will liquefy is only 4.7%. Obviously, such a small value means that liquefaction was impossible in fact (Chou, 1983).

Table 2. Results of analysis for Guatemala earthquake of February 4, 1976

Boring number	By the Step-1	By the Step-2, p{E[E]}	Field observation
#1	Liquefied	100%	Yes
#2	Possible	4.7%	Just No
#3	No	0	No
#4	No	0	No

Monte Negro Earthquake of April 14, 1979 (Talaganov, 1980)

It is found that liquefaction would occur surely under the depth of 3m and that a border line between sure liquefaction and possible liquefaction exists at 2-3m according to the step-1. By the step-2, the probability that liquefaction did occur above 2m is estimated to be 0.40. Hence, it can be seen that a major part of the profile would be liquefied, as shown in Tab.3.

Table 3. Results of analysis for Monte Negro earthquake of April 14, 1979

Depths (m)	2	3	4	5	6	7	8
SPT (N-value)	9	9	8	7	6	6	7
P{E[L]}	0.4	1	1	1	1	1	1

Miyagiken-oki Earthquake of April 12, 1978 (Tohno, 1978)

The results of the analysis are shown in Tab.4 (Chou, 1983).

Table 4. Probabilities versus depth in Miyagiken-oki Earthquake

Site	Depths (m)	p{E[L]} (%)	P[L] (%)	Field Observation
A	2.3	42	92	Yes
	3.3	24		
	4.3	46		
	5.3	21		
	6.3	56		
	7.3	0.09		
	8.3	0		
B	2.0	0	0	No
	4.0	0		
	6.0	0		
	8.0	0		
	10.0	0		

COMPARATION WITH SEED'S SIMPLIFIED PROCEDURE

Comparation of the approach with Seed's simplified procedure was performed to discuss the approach in depth.

Consider for example, a sand deposit at a depth of 6m for which the water table is 1.5m below the ground surface and which is subjected 10 cycles of ground shaking. A total saturated density of 21.0 N/cubic cm, a buoyant density of 18.9 N/cubic cm above the water table and a buoyant density of 11.0 N/cubic cm for this deposit are known.

The following values of maximum acceleration required to cause initial liquefaction for given values of relative density can be determined by use of Seed's procedure as shown in Tab.5. But, in the approach proposed here, the relative densities corresponding to the values of maximum acceleration listed in Tab.5 can be obtained by use of eq.(19) or Fig.3 (Tab.6).

Table 5. Required peak accelerations by Seed's procedure

Dr (%)	a max (g)
40	0.116
50	0.145
60	0.174
70	0.203

Table 6. Corresponding relative densities converted by the approach here

a max (g)	A (g)	Dr (%) p=1	Dr (%) p=0
0.116	0.212	33	52
0.145	0.265	38	63
0.174	0.318	42	70
0.203	0.412	53	89

It is obvious that the values of the relative density in Tab.5 are located in the middle of those values in Tab.6 between cases of p=1 and p=0. And the liquefaction probabilities determined by use of the approach proposed are very low. This fact just hints that it is initial liquefaction as in Seed's simplified procedure.

The observed cases of liquefaction from Seed and Peacock (1970) are summarized in in Tab.7. It is seen that the results of the two methods both are coincided with the field observation.

Table 7. Observed cases from Seed

a max (g)	Liquefy Very Likely	Depends on Soil & Magnitude	Liquefy Very unlikely
0.10	Dr $<$ 33%	33% $<$ Dr $<$ 54%	Dr $>$ 54%
0.15	Dr $<$ 48%	48% $<$ Dr $<$ 73%	Dr $>$ 73%
0.20	Dr $<$ 60%	60% $<$ Dr $<$ 85%	Dr $>$ 85%
0.25	Dr $<$ 70%	70% $<$ Dr $<$ 92%	Dr $>$ 92%

CONCLUSIONS

An alternative for analysing statistically uncertainties in the data from historical cases where liquefaction was or was not observed during earthquakes is presented, using of the elementary notion of sets. This is an attempt to develop a overall probabilistic model for evaluating soil liquefaction.

The overall probabilistic approach, consisting of three steps, was formulated based on the new model in which deterministic and empirical factors are excepted for the consistency of the used concepts in probability theory.

The approach allows us to assess simply whether a given site is to liquefy or not and what the probability is that liquefaction will occur, if expected maximum acceleration of ground surface and SPT value, N or Dr, are known.

The expressions in this approach have been programed for practical convenience and examined with some case studies. Further research is needed to consider more cases in which seismic liquefaction hazards and geotechnical characteristics are well investigated recently.

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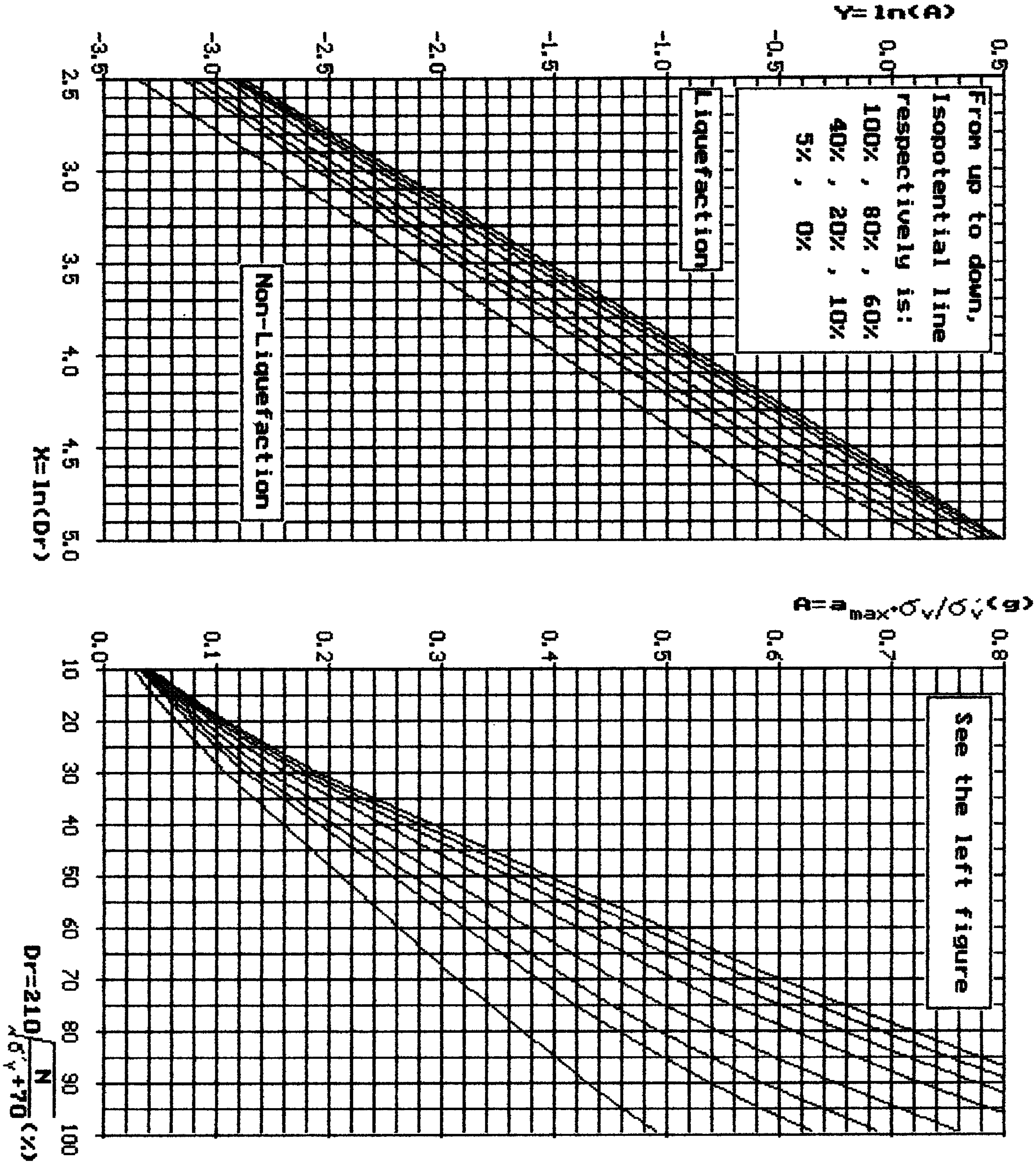

Fig. 3. Liquefaction potential chart by probabilities