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MODIFIED BANG-BANG CONTROL LAW FOR STRUCTURAL CONTROL IMPLEMENTATION

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ABSTRACT

Linear quadratic regulator (LQR) has been used extensively in many control systems designed for structural control applications due to its stability and robustness. However, recent results obtained from simulation, laboratory experiments, and full-scale structural applications show that it is difficult to employ linear feedback control laws to produce a significant peak response reduction when the peak response occurs during the first few cycles of the time history. On the other hand, although another class of well-known optimal control laws, so called "Bang-Bang" control, has been investigated for several decades, their potential in civil engineering structural control has yet to be exploited. The purpose of this paper is to introduce a new implementable control law for response control of civil engineering structures based on the optimal bang-bang control principle. Through a series of simulation studies and experimental verification in the laboratory using a model structure, it is shown that the proposed new control law can significantly improve peak response reduction under the same constraints imposed on the control resources as in the linear quadratic regulator case.

KEYWORDS

Active control; seismic protection; Bang-Bang control law; numerical simulation; experimental verification.

INTRODUCTION

Research and development in active control of civil engineering structures has an approximately 20-years history (Yao, 1972). In recent years, remarkable progress has been made. It is now at the stage where full-scale active systems have been installed in actual structures and have performed well for the purposes intended (Soong *et al.*, 1991; Reinhorn *et al.*, 1993). While significant progress has been made, the true potential of active vibration control of structures has not been fully exploited (Housner *et al.*, 1994). For example, most of the current operating systems are designed based primarily on the classical linear quadratic regular (LQR) formulation, which may not be effective in producing significant peak response reductions (Soong and Reinhorn, 1993). Recent research has demonstrated that a proper modification of the control laws can provide large dividend in control effectiveness for a set of more relevant control objectives (Wu *et al.*, 1995). The purpose of this paper is to introduce a new implementable control law for response control of structures under seismic loads and to verify it by manes of simulations and model structural experiments.

In optimal control theory, a class of well-known control laws, so called "Bang-Bang" control, has been investigated for several decades (Bellman *et al.*, 1956). Recently, Indrawan and Higashihara (1993) utilized a Bang-Bang control law to control a single-degree-of-freedom structure with an active mass damper subjected to seismic loads. Simulation results show that, keeping the same maximum control force. the LQR control gives a maximum displacement of 1.99 *mm*, while the Bang-Bang control yields 1.11 *mm*. Thus, remarkable control efficiency can be achieved by using the proposed Bang-Bang control law. But unfortunately, as stated in this paper, since Bang-Bang control laws lead to a singular control requirement, servo-hydraulic actuators, which are popular control force delivery devices in actual structural control implementation, are not suitable for this kind of control laws due to high-speed switching of control forces. Some modifications are therefore necessary for practical applications of Bang-Bang control laws to civil engineering structural control.

The work presented in this paper is focused on the development of an implementable control law which can provide improved peak response control performance under the same constraints imposed on the control force and other resources as in the LQR control case. First, general formulations of Bang-Bang control are briefly reviewed, and their advantages and disadvantages are discussed. Then necessary modifications of Bang-Bang control law are made based on a series expansion of singular functions. The efficiency of the modified control law is examined extensively by means of numerical simulations. Finally, in order to evaluate implementability of the new control law, a series of comprehensive control experiments are carried out in the laboratory using a 1/3-scale three-story model structure with ground excitations supplied by a shaking table. The experimental results indicate that the implementation of the new control law has no inherent difficulties and its design can be carried out following the same procedure as in the linear feedback control case.

BANG-BANG CONTROL LAWS

Consider a general linear building structure modelled by an n-degree-of-freedom lumped mass-spring-dashpot system. The matrix equation of motion of the structural system, subjected to a horizontal earthquake ground acceleration $\ddot{x}_0(t)$, can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{D}\mathbf{u}(t) + \mathbf{m}\ddot{x}_0(t) \tag{1}$$

where $\mathbf{x}(t) = [x_1, x_2, \cdot \cdot \cdot, x_n]^T$ is a relative displacement vector, $\mathbf{u}(t)$ is a control force vector, $\mathbf{D}$ is an matrix denoting the location of the controllers, $\mathbf{M}$ is a diagonal matrix with jth diagonal element m_j, and $\mathbf{C}$ and $\mathbf{K}$ are tri-diagonal damping and stiffness matrices, respectively. In the above, the superscript T indicates vector or matrix transpose. In the state-space representation, Eq. (1) becomes

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{w}\ddot{x}_0(t) \tag{2}$$

where

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{bmatrix} ; \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} ; \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{D} \end{bmatrix} ; \quad \mathbf{w} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{m} \end{bmatrix} \tag{3}$$

In bang-bang control, the control objective is to minimize the following performance index

$$J = \frac{1}{2}\int_0^{t_f} [\mathbf{z}^T(t)\mathbf{Q}\mathbf{z}(t)]\, dt \tag{4}$$

subjected to the control force constraint

$$|\mathrm{u}(t)| \leq \mathrm{u}_{max} \tag{5}$$

Here, without loss of generality, only one control force is considered. According to Pontryagin Maximum Principle, one can derive the optimal control effort as

$$\mathrm{u}(t) = -\mathrm{u}_{max} \cdot \mathrm{sgn}\,[\mathbf{B}^T\boldsymbol{\lambda}(t)] \tag{6}$$

where $\boldsymbol{\lambda}(t)$ is the co-state vector which can be obtained by solving the first-order differential equation

$$\dot{\boldsymbol{\lambda}}(t) = -\mathbf{A}^T\boldsymbol{\lambda}(t) - \mathbf{Q}\mathbf{z}(t) \tag{7}$$

It is noted from Eq. (6) that the optimal control effort is of a typical bang-bang type: $\mathrm{u}(t)$ switches from one to the other of its extreme values when $\mathbf{B}^T\boldsymbol{\lambda}(t)$ changes its sign.

The advantage of the bang-bang control law is that maximum control efforts can be exploited by this kind of control action since the control force always takes its maximum values. Simulation results indicate that the bang-bang control can offer much better control efficiency than that provided by LQR control (Indrawan and

Higashihara, 1993). On the other hand, the disadvantages of the bang-bang control law are also clear when one wants to apply this kind of control laws to real structural control. First, it is almost impossible for servo-hydraulic actuators, which are popular control force delivery devices in current structural control implementation, to follow this kind of high-speed switching control command and apply the required control force. Second, Eq. (7) has to be solved on line during the control process, which will increase time delay significantly, and sometimes may lead to instability due to the accumulated error in on-line numerical evaluation of Eq. (7).

Instead of minimizing the performance index Eq. (4), a suboptimal bang-bang control law can be derived by minimizing the time derivative of a Lypunov function of the system. First, define a quadratic function of the state variable as

$$V(\mathbf{z}) = \mathbf{z}^T\mathbf{S}\mathbf{z} \tag{8}$$

where matrix $\mathbf{S}$ is the solution of the Lyapunov matrix equation

$$\mathbf{A}^T\mathbf{S} + \mathbf{S}\mathbf{A} = -\mathbf{Q} \tag{9}$$

Since the open-loop system matrix $\mathbf{A}$ is stable, and the weighting matrix $\mathbf{Q}$ can be selected to be a symmetric and positive semi-definite matrix, solution matrix $\mathbf{S}$ is also symmetric and positive. Thus the function $V(\mathbf{z})$ is a Lyapunov function of the open-loop system. The time derivative of the Lyapunov function is

$$\dot{V}(\mathbf{z}) = \dot{\mathbf{z}}^T\mathbf{S}\mathbf{z} + \mathbf{z}^T\mathbf{S}\dot{\mathbf{z}} \tag{10}$$

Substituting the closed-loop state equation into Eq. (10), one obtains

$$\dot{V}(\mathbf{z}) = -\mathbf{z}^T\mathbf{Q}\mathbf{z} + 2\mathrm{u}\mathbf{B}^T\mathbf{S}\mathbf{z} \tag{11}$$

It is clear that, if the control force takes the form

$$\mathrm{u}(t) = -\mathrm{u}_{max} \cdot \mathrm{sgn}\,[\mathbf{B}^T\mathbf{S}\mathbf{z}(t)] \tag{12}$$

$\dot{V}(\mathbf{z})$ will be minimum. The control law given by Eq. (12) is called a suboptimal bang-bang control, which avoids on-line evaluation of the differential equation.

MODIFIED BANG-BANG CONTROL LAW

Consider a general absolute function $|\mathrm{x}(t)|$, which can be expressed as

$$|\mathrm{x}(t)| = \{\mathrm{x}^2(t) - \alpha^2 + \alpha^2\}^{1/2} = \alpha\{1 + [\mathrm{x}^2(t) - \alpha^2]/\alpha^2\}^{1/2} \tag{13}$$

Define

$$\mathrm{y}(t) = [\mathrm{x}^2(t) - \alpha^2]/\alpha^2 \tag{14}$$

Then Eq. (13) becomes

$$|\mathrm{x}(t)| = \alpha\{1 + \mathrm{y}(t)\}^{1/2} \tag{15}$$

which can be expanded as

$$|\mathrm{x}(t)| = \alpha\{1 + \frac{1}{2}\mathrm{y}(t) - \frac{1}{2}\cdot\frac{1}{4}\mathrm{y}^2(t) + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\mathrm{y}^3(t) - \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\cdot\frac{5}{8}\mathrm{y}^4(t) + \cdots\} \tag{16}$$

if the condition $|\mathrm{y}(t)| \le 1$ is satisfied, i.e.,

$$\alpha \ge \frac{max|\mathrm{x}(t)|}{\sqrt{2}} \tag{17}$$

Therefore, a discontinuous function $|\mathrm{x}(t)|$ can be approximated by a continuous function Eq. (16) under condition Eq. (17).

Let $\nu(t) = \mathbf{B}^T\mathbf{S}\mathbf{z}(t)$ in Eq. (12), then the suboptimal bang-bang control force becomes

$$\mathrm{u}(t) = -\mathrm{u}_{max} \cdot \frac{\nu(t)}{|\nu(t)|} \tag{18}$$

Substituting Eq. (16) into Eq. (18), the modified bang-bang control can now be written as

$$\mathrm{u}(t) = -\mathrm{u}_{max} \cdot \frac{\nu(t)}{\alpha\{1 + \frac{1}{2}\mathrm{y}(t) - \frac{1}{8}\mathrm{y}^2(t) + \frac{3}{16}\mathrm{y}^3(t) - \frac{5}{128}\mathrm{y}^4(t) + \cdots\}} \tag{19}$$

in which $\alpha \geq [max|v(t)|]/(\sqrt{2})$ and $y(t) = [v^2(t) - \alpha^2]/\alpha^2$. The control law, Eq. (19), overcomes both singular control requirement and on-line computation of the differential equation in real control implementation.

NUMERICAL EXAMPLE

The numerical example considered here is a one-story structure, which can be idealized as a single-degree-of-freedom (SDOF) system. The structural parameters are: mass = 2.942 tons, natural frequency = 4.1 Hz, and damping ratio = 2.62%. An active tendon control system with 438 *kN/m* tendon stiffness and 36° tendon angle is selected to provide the control force to the structure. The base excitation is the N-S component of the 1940 El Centro acceleration record, whose intensity is scaled down by 2/3 to prevent the structure from exceeding the elastic limit in the uncontrolled case. In the simulation, different cases consisting of uncontrolled, linear (LQR) control, bang-bang control as well as modified bang-bang control (only 3rd order approximation) are considered. Figure 1 illustrates the peak response reduction for linear, bang-bang and modified bang-bang control cases, respectively, as compared with the uncontrolled case. It is seen that the bang-bang control law clearly produces better control performance in term of peak response reduction under the same maximum control force requirement as in the linear control case. Furthermore, the difference in peak response reduction between bang-bang control and modified bang-bang control is less than 5%. For example, under the maximum control force of 1.59 *kN*, the maximum relative displacement reduction is 40.9% by employing the linear control law, while for bang-bang and modified bang-bang, the reductions are 62.2% and 57.6%, respectively. For the peak absolute acceleration, the reductions are 36.5%, 48.6% and 46.7%, corresponding to linear, bang-bang and modified bang-bang control cases, respectively. In order to show more detailed control performance, Fig. 2 illustrates a set of typical relative displacement and control force time histories in different control cases. Obviously, the high-speed switching control force required by the bang-bang control law is not suitable for hydraulic actuators although this kind of control law can offer best control performance. On the other hand, the modified bang-bang control law provides smooth control force requirement and achieves almost the same control efforts as in the bang-bang control case, particularly during the peak response period.

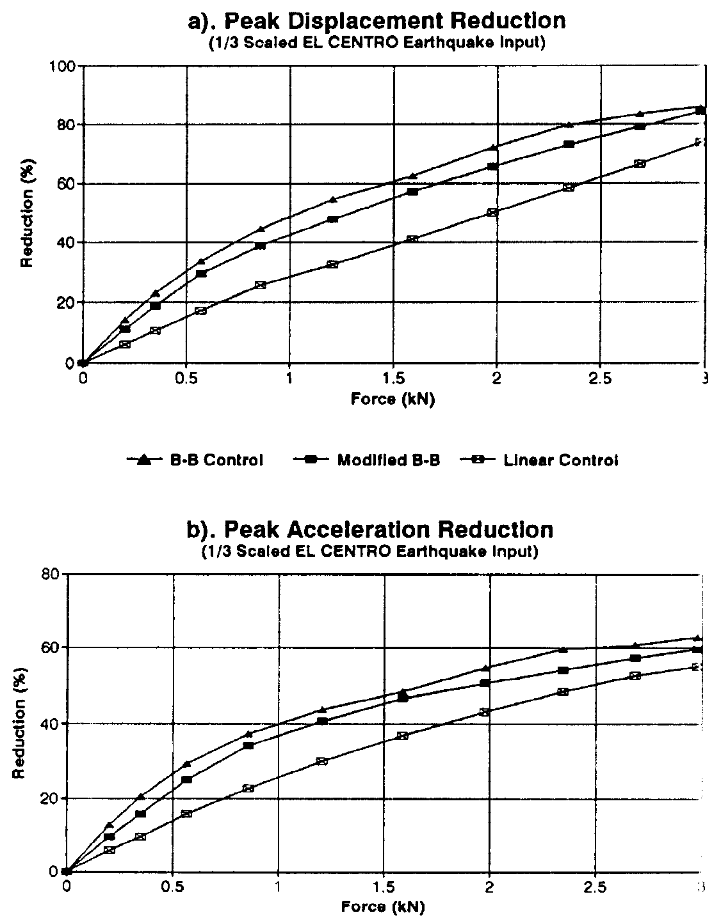

Fig. 1. Peak Response Reductions under Different Control Force Constraints

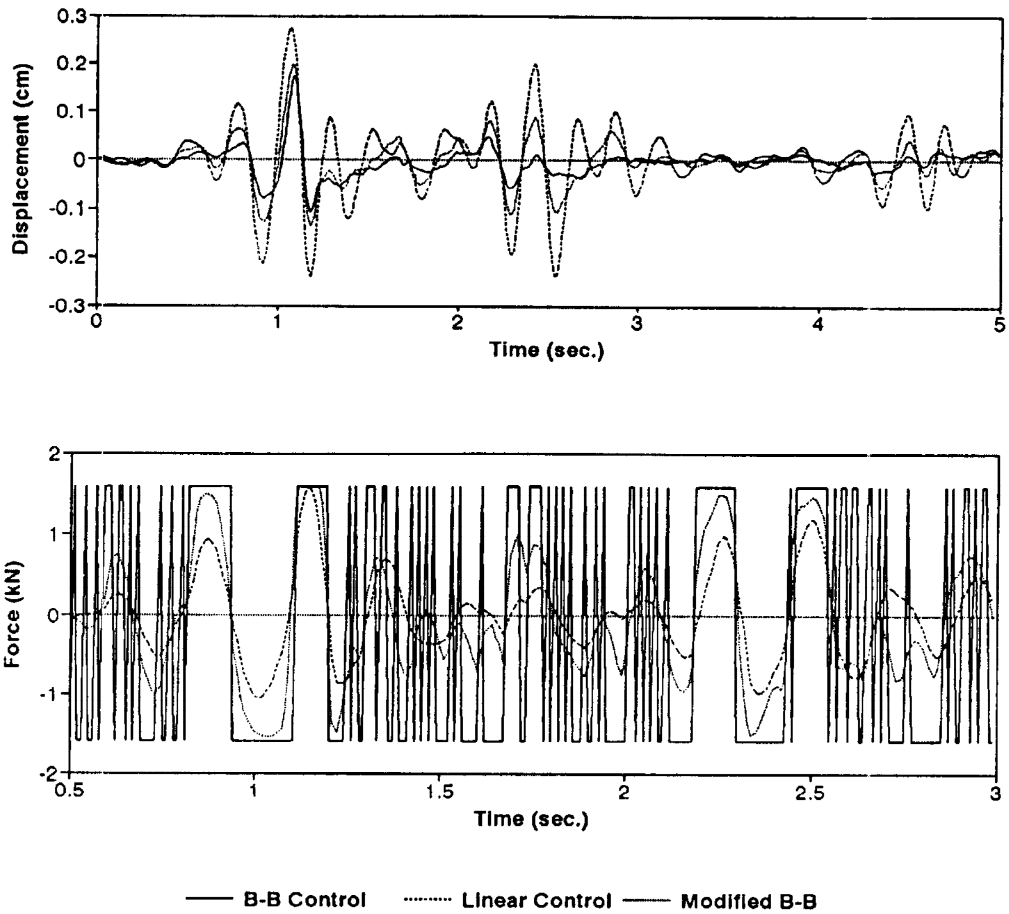

Fig. 2. Response and Control Force Time Histories from Simulation

EXPERIMENTAL VERIFICATION

In order to evaluate implementability of the modified bang-bang control law, a series of comprehensive experimental investigation was carried out on a 1/3-scaled model structure in the laboratory. Figure 3 shows a diagram of the model structure and control system. In the experiment, the state variable measurements were made by means of displacement transducers (Temposonic-TM) installed on each floor and at the base. The measured relative displacements were obtained from the differences between the measured absolute floor displacements and the base displacement. The transducer signals were low-frequency filtered first to eliminate the high-frequency noise, and were further passed through analog differentiators to yield the measured relative velocities. The base acceleration and absolute accelerations of the masses were directly measured by accelerometers installed at the base and on each floor slab. The control force was obtained from measured displacements of the actuator piston or from the load cells installed on each tendon. These measurements also provided information for control operations and for performance evaluation of the system.

Three control experiments consisting of uncontrolled, linear (LQR) control and modified bang-bang control were carried out under the 1/2-scaled Taft earthquake input and the 1/4-scaled El Centro earthquake input. Table 1 summaries the maximum responses at every floor and the maximum control forces at the first floor under both earthquake excitations. Correspondingly, Figs. 4 shows the top-floor response time histories and the first floor control force time histories under the Taft earthquake excitation. It can be seen from Table 1 that, by employing the modified bang-bang control law, the peak response reductions for both relative displacement and absolute acceleration can be more than 19% over those produced by the linear control law under the same maximum control force constraint. At the top floor, for example, the relative displacement and absolute acceleration reductions are 29.6% and 31.3% for the linear control case, while for the modified bang-bang control, the reductions are 54.3% and 50.8%. Furthermore, the higher the series order in the modified bang-bang control law, the better the control performance. However, the experiment indicates that, once the series order is higher than four, the hydraulic actuator control system may become unstable. Actually, only the first-order approximation can provide significantly improved control performance as compared to the LQR control case. In real structural control implementation, therefore, the first order approximation of bang-bang control may be a safe and effective control solution. It is also of interest to note from Figs. 4 that the larger peak response reductions offered by the modified bang-bang control law during the initial period are accompanied by a larger control force applied to the structure. This is just our expected control objective by using nonlinear control laws instead of LQR algorithm. Finally, the response time histories illustrate that not only the peak response but the overall response is also reduced by employing the modified bang-bang control law.

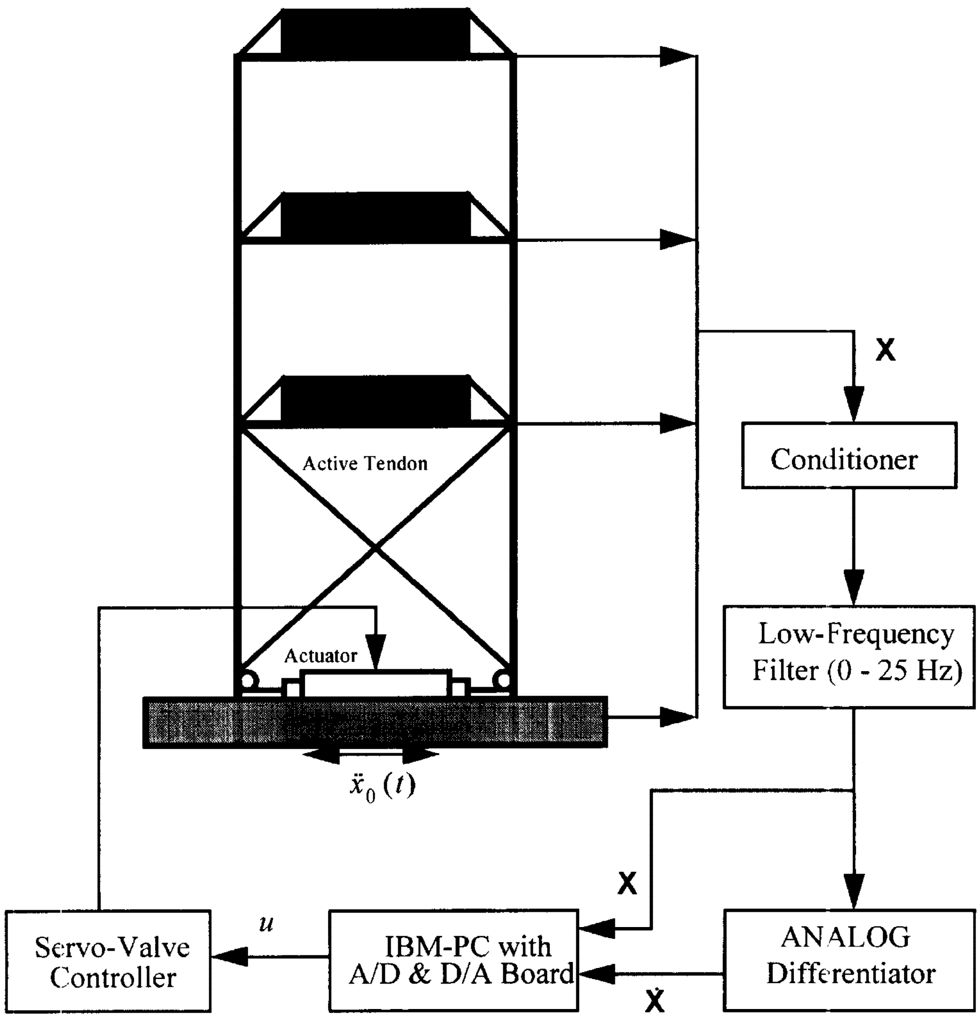

Fig. 3. Model Structural and Control System

Apart from the above experimental verification, analytical simulations were also performed under the same conditions as in the experiments. The maximum responses and control forces are also listed in Table 1. It is seen that the differences between experiment and simulation are generally within 10%, not only for the responses but also for the control forces. The reductions from experiments are somewhat larger than those from simulations since the applied control forces in the experiments are slightly larger than those in simulations. The major reason is that the hydraulic actuator used in the laboratory was not able to generate the required control forces precisely, and in general a small part of control force overshoot would occur at the peak of control actions.

Table 1. Maximum Response Verification for Model Structure

Control Algorithms	No.	Relative Displacement				Absolute Acceleration				Max. Actuator Force (kN)	
		Values (cm)		Reduction (%)		Values (g)		Reduction (%)			
		Simu.	Exp.	Simu.	Exp.	Simu.	Exp.	Simu.	Exp.	Simu.	Exp.
1/2-Scaled Taft Earthquake Input											
without Control	1	0.335	0.396			0.223	0.229				
	2	0.900	0.953			0.253	0.259				
	3	1.343	1.346			0.401	0.376				
Linear Control	1	0.228	0.216			0.143	0.159			1.000	1.010
	2	0.658	0.612			0.169	0.154				
	3	0.984	0.947	26.7	29.6	0.270	0.259	32.7	31.1		
Modified B-B Control 1st Order	1	0.176	0.147			0.166	0.168			1.000	1.023
	2	0.536	0.452			0.166	0.154				
	3	0.794	0.678	40.9	49.6	0.233	0.190	41.9	49.5		
Modified B-B Control 2nd Order	1	0.167	0.145			0.183	0.179			1.000	1.050
	2	0.505	0.432			0.170	0.169				
	3	0.747	0.630	44.4	51.2	0.229	0.191	42.9	49.2		
Modified B-B Control 3rd Order	1	0.164	0.135			0.193	0.187			1.000	1.054
	2	0.488	0.425			0.172	0.154				
	3	0.720	0.615	46.7	54.3	0.226	0.185	43.6	50.8		
1/4-Scaled El Centro Earthquake Input											
without Control	1	0.451	0.452			0.217	0.187				
	2	1.096	1.146			0.316	0.290				
	3	1.528	1.671			0.400	0.369				
Linear Control	1	0.274	0.244			0.158	0.142			0.861	0.916
	2	0.728	0.643			0.196	0.166				
	3	1.045	0.965	31.6	42.3	0.262	0.221	34.5	40.1		
Modified B-B Control 3rd Order	1	0.191	0.180			0.155	0.156			0.861	0.943
	2	0.533	0.462			0.162	0.147				
	3	0.787	0.721	48.5	56.9	0.228	0.195	43.0	47.2		

CONCLUDING REMARKS

In civil engineering structural applications, peak response control is of practical importance due to its close tie with safety. The work presented in this paper is focused on the development of implementable new control laws which can provide improved peak response control performance under the same constraints imposed on the control force and other resources as in the linear control case. Based on an evaluation of advantages and disadvantages of the traditional LQR control law and the bang-bang control law, a modified bang-bang control algorithm has been proposed. The simulation and experimental results presented in this paper show that the new control law can be more effective than LQR in peak response reduction, and more suitable than bang-bang control laws in real structural control application. The successful accomplishment of experiments indicates that implementation of the modified bang-bang control law is feasible in practice and presents no inherent difficulties. Its design can be carried out following the same procedure as in the linear control case. Good agreement between experimental and simulation results makes it possible to extrapolate this new control law for potential full-scale structural applications. The modified bang-bang control law suggested herein can thus provide an effective means for enhancing structural control effectiveness.

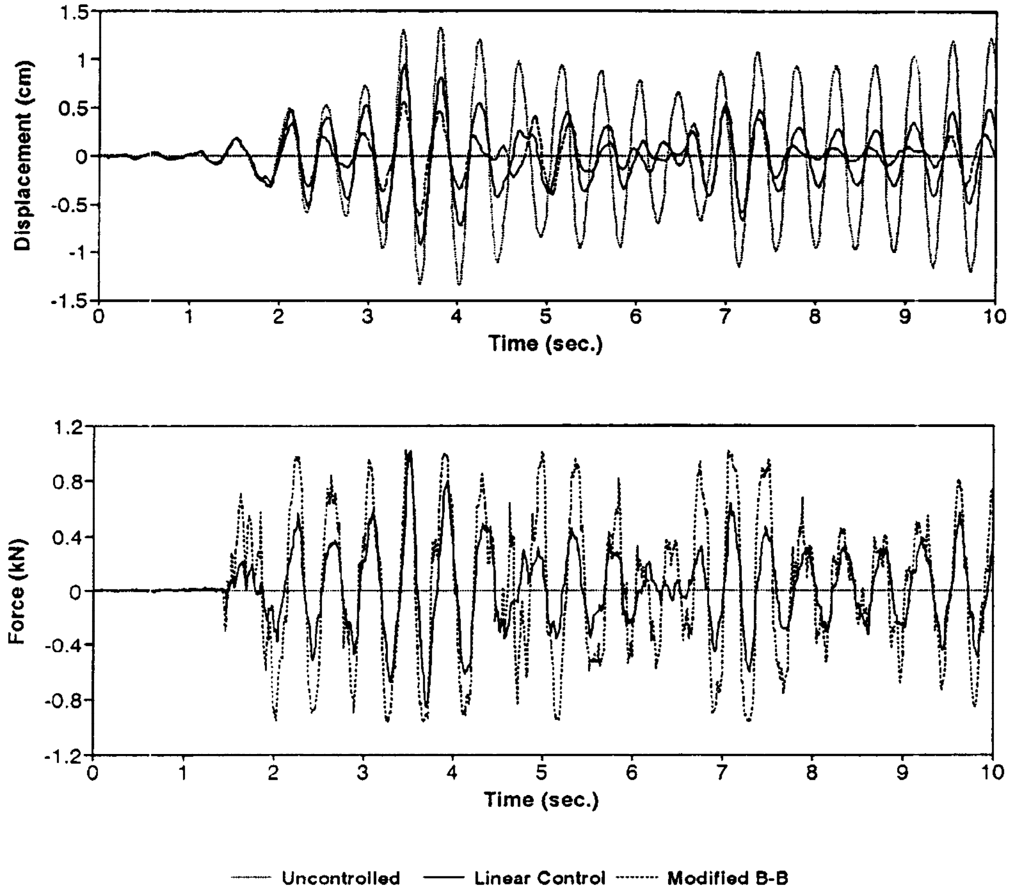

Fig. 4. Response and Control Force Time Histories from Experiment

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