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HYSTERETIC DAMPING OF REINFORCED CONCRETE ELEMENTS

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ABSTRACT

The energy dissipation in post-yield cyclic loading is an important characteristic of reinforced concrete members, which significantly affects the global seismic response of the structural system to strong ground motions. For post-yield cycles of given amplitude, this energy dissipation may be conveniently expressed as an effective hysteretic damping ratio ζ, i.e. as $\zeta = E_h/4\pi E_{el}$, in which E_h is the energy dissipated in a full cycle of loading-unloading-reloading and E_{el} the elastic strain energy, i.e. $F_{max}\delta_{max}/2$, at the peak force and displacement of the cycle. In this way one can quantify: a) the effective damping ratio implicit in the hysteresis rules of the cyclic nonlinear model used for the phenomenological description of the inelastic hysteretic behavior of the member, as well as b) the "actual" effective damping ratio of the members, as this is revealed in cyclic testing. For a realistic representation of the energy dissipation in the members of a system in the course of its nonlinear dynamic response analysis, the hysteretic damping ratio implicit in the member model should correspond to the value experimentally measured at the same amplitude of loading. To this end, in this paper the effective damping ratio ζ implicit in the various frequently used hysteresis models is quantified as a function of the ductility ratio μ and the model parameters. In addition, the results of cyclic tests of reinforced concrete members are used to quantify the "actual" effective damping ratio as a function of μ and the geometric and mechanical characteristics of the member.

KEYWORDS

Cyclic loading, damping ratio, hysteretic damping, hysteretic models, reinforced concrete members.

EFFECTIVE DAMPING IMPLICIT IN AVAILABLE MEMBER HYSTERETIC MODELS

Most nonlinear dynamic response analyses of RC structures performed today utilize at the member level empirical nonlinear hysteretic relations between the bending moment M and a corresponding deformation measure, δ, such as the curvature Φ at the same section, or the total rotation of a plastic-hinge or the chord-rotation of the shear span, θ, if M is the moment at the end of the member, etc. For monotonic or virgin loading in one direction, the M-Φ or M-θ relation is conveniently taken multilinear: A bilinear relation, with the corner point signifing yielding, is the simplest and most common choice, while trilinear relations, in which the cracking point is also included, are used when a realistic description of the behavior prior to yielding is necessary. The hysteretic behavior is described through a set of unloading-reloading rules for large or small load reversals. Unloading from a peak deformation $\delta = \mu\delta_y$ on the post-yield branch of the monotonic curve (with δ_y denoting the yield deformation and μ the corresponding ductility factor) is typically taken linear up to a point on the horizontal (δ-) axis at a deformation equal to $\varepsilon\delta_y$ (Fig. 1(a)). In the early Clough and Johnston (1966) model, as modified by Anagnostopoulos (1972) to unload at a slope equal to that of the elastic branch divided by μ^a (a is a parameter between 0 and 1), ε equals:

$$\varepsilon=\mu\left(1-\frac{1+p(\mu-1)}{\mu^{1-a}}\right) \tag{1}$$

in which p is the post-yield hardening ratio. In the widely used Takeda et al (1970) model, as modified by Litton (1975) to unload to permanent deformation (1-α) times that for elastic unloading, ε is:

$$\varepsilon=(1-\alpha)(1-p)(\mu-1) \tag{2}$$

In the Park et al (1987) and the Reinhorn et al (1988) models, in which unloading is directed towards a point on the elastic branch of the monotonic curve in the opposite direction, at a moment M α-times the yield value, M_y, (α>1), ε equals:

$$\varepsilon=\frac{\alpha(1-p)(\mu-1)}{\alpha+1+p(\mu-1)} \tag{3}$$

For the deviration of (3) the distinction between pre- and post-cracking stiffness of the Park et al (1987) model was neglected. Finally in the Roufaiel and Meyer (1987) model, ε is independent of any model parameter:

$$\varepsilon=\frac{(1-p)(\mu-1)}{1+2p(\mu-1)} \tag{4}$$

(1) holds also for the Costa and Costa (1987) and the Coehlo and Carvalho (1990) models. According to all the above mentioned models, the continuation of unloading as first loading in the opposite direction is directed to the yield point in this latter direction and then continues on the post-yield hardening branch of the monotonic curve (Fig. 1(a)). Unloading from this latter branch follows the same rules, so that if the reversal is from a deformation $-\mu\delta_y$ it intersects the δ-axis at a point $-\varepsilon\delta_y$ with ε given by (1) to (4). It can be shown that in this first full cycle of force at peak ductility μ the hysteretic damping ratio ζ equals:

$$\zeta_1=\frac{2(\mu-1)(1-p+\varepsilon p)+3\varepsilon}{4\pi\mu(1+p(\mu-1))} \tag{5}$$

Contrary to the above, first loading in the opposite direction according to the Q-hyst model by Saiidi and Sozen (1981) goes from the point at deformation $\varepsilon\delta_y$ on the δ-axis directly to the point at deformation $-\mu\delta_y$ on the monotonic curve in the opposite direction (Fig. 1(a)), giving a first-cycle hysteretic damping ratio of:

$$\zeta_{1Q}=\frac{(\mu-1)(1-p+3\varepsilon p)+3\varepsilon}{4\pi\mu(1+p(\mu-1))} \tag{6}$$

with ε given by (1) with a=0.5.

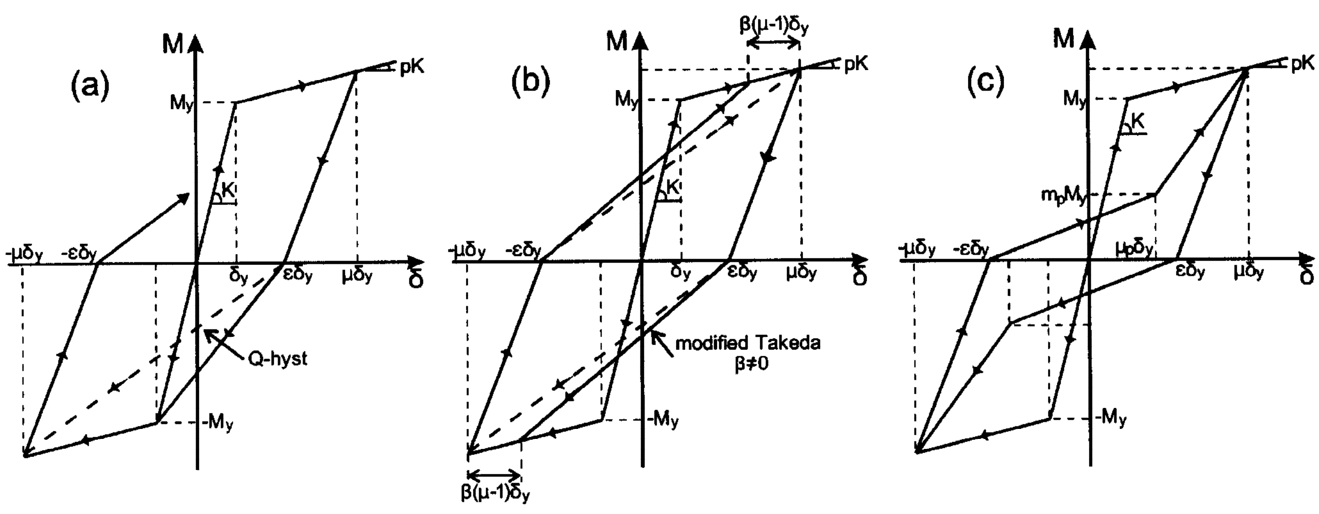

Fig. 1. Hysteretic models: (a) First full load cycle; subsequent cycle: (b) without, and (c) with pinching.

A major feature of the cyclic behavior is the inverted-S shape and the pinching of subsequent hysteresis loops. Early and still widely used models (Clough and Johnston, 1966, Takeda et al, 1970, Saiidi and Sozen, 1981) do not consider pinching and direct reloading straight to a point on the monotonic branch at the extreme deformation $\mu\delta_y$ that has ever taken place in the direction of reloading, or in any of the two directions in Saiidi and Sozen (1981). In the Litton (1975) modification of the Takeda model, the straight reloading branch is directed towards a point on the monotonic curve at maximum deformation $(\mu-\beta(\mu-1))\delta_y$ instead of $\mu\delta_y$ (Fig. 1(b), $\beta<1$ is a parameter). With the exception of this model, the hysteretic damping ratio in a full unloading-reloading cycle of both force and deformation, at deformation amplitude $2\mu\delta_y$, is according to all these models:

$$\zeta_{n>1}=\frac{1}{\pi}\frac{\varepsilon}{\mu} \tag{7}$$

with ε given by (1) and (2). For the Litton (1975) modification of the Takeda model, $\zeta_{n>1}$ equals:

$$\zeta_{Litton,n>1}=\frac{(1-p)(\mu-1)}{\pi\mu}\left(1-\alpha+0.5\beta(1-p)\frac{1-p(\mu-1)(1-\alpha)}{1+p(\mu-1)}\right) \tag{8}$$

According to the other models mentioned above, pinching of the hysteresis loops is achieved by introducing bilinear reloading, directed first to a point at moment m_pM_y and corresponding deformation $\mu_p\delta_y$, and then to the point of extreme previous deformation $\mu\delta_y$ on the monotonic branch (Fig. 1(c)). It is easy to show that the hysteretic damping ratio in a full unloading-reloading cycle is in this case:

$$\zeta_{pinch,n>1}=\frac{1}{2\pi}\left(\frac{\varepsilon-\mu_p}{\mu}+\frac{m_p(\varepsilon+\mu)}{\mu(1+p(\mu-1))}\right) \tag{9}$$

In the Roufaiel and Meyer (1987) model, reloading is directed first to a point on the elastic branch at a moment mM_y; then $\mu_p=m_p=m$. In the Coelho and Carvalho (1990) model, reloading takes place first at a slope m-times the one to the end-point of the reloading branch at the extreme previous deformation $\mu\delta_y$ ($m<1$ is a model parameter), until the M-axis is reached; then $m_p=m(1+p(\mu-1))\varepsilon/(\varepsilon+\mu)$ and $\mu_p=0$ and the right-hand side of (9) equals $(1+m)\varepsilon/2\pi\mu$. In the Costa and Costa (1987) model the first reloading branch has a slope μ^β-times less than that to the end-point of reloading on the monotonic curve (μ^β, with $\beta<1$ replaces m of the Coelho and Carvalho (1990) model) until the line connecting the origin to this latter point is reached; then $m_p=\varepsilon/(\varepsilon\mu^{\beta-1}+\mu^\beta-1)$, $\mu_p=m_p(1+p(\mu-1))/\mu$, and the right-hand side of (9) becomes $\varepsilon(1+\varepsilon/\mu(\varepsilon\mu^{\beta-1}+\mu^\beta-1))/2\pi\mu$. Finally in the Park et al (1987) and the Reinhorn et al (1988) models $\mu_p=\varepsilon$, while in Park et al (1988) $m_p=2\varepsilon\gamma(1+p(\mu-1))/(2\varepsilon(1+p(\mu-1))+\gamma(\mu-\varepsilon))$ and in Reinhorn et al (1988) $m_p=2\varepsilon\gamma/(\varepsilon+\gamma)$, with γ being a parameter of these models and denoting the ordinate of a point (as a fraction of M_y) towards which the first reloading branch is directed. This point lies on the unloading branch from the extreme previous deformation of $\mu\delta_y$ in the first model, or on the first branch of the monotonic curve in the second.

(5) and (6) for the first post-yielding cycle and (7) to (9) for the subsequent ones, give the hysteretic damping ratio implicit in each model, as a function of the ductility ratio μ and the model parameters. Typical results are plotted in Fig. 2, for p=2%. Such results, along with the experimental evidence in the next section, can serve as a guide for the selection of the model parameter values.

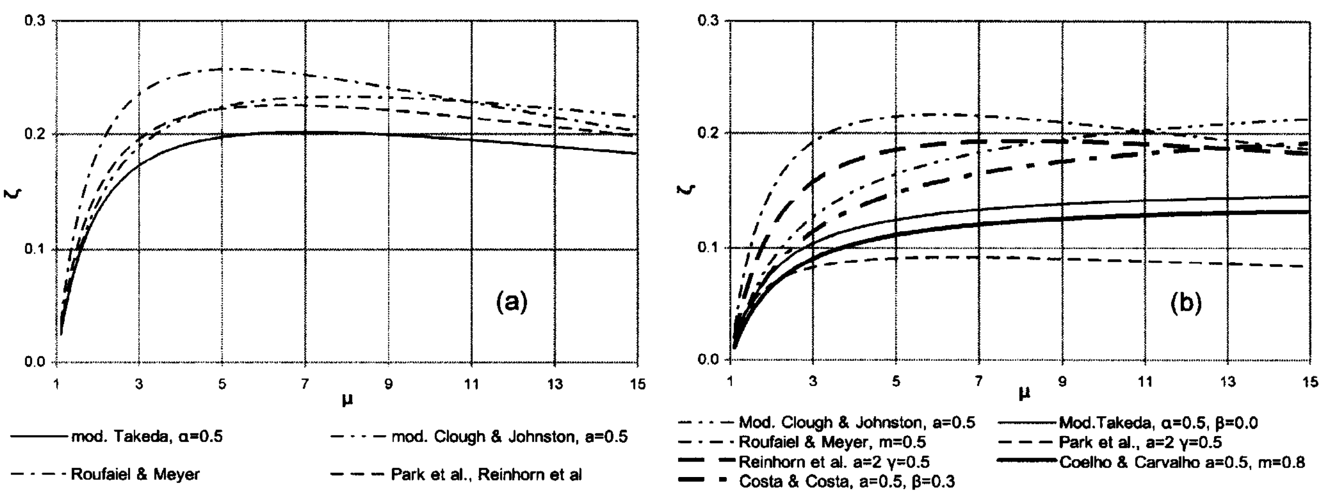

Fig. 2. Damping ratio vs. ductility ratio of various hysteretic models: (a) first load cycle; (b) subsequent cycles.

In the second part of this paper a databank of 187 cyclic uniaxial tests on RC members, each with several cycles of pre- and post-yield loading, are utilized to determine pairs of ζ vs. μ. From the digitized test results the energy dissipated in each half-cycle of loading to peak deformation ductility μ, E_h, is computed, along with the maximum force-deformation product, $(F\delta)_{max}$. The tests are drawn from: Abrams (1987), Ang et al (1989), Atalay and Penzien (1975), Bertero et al (1974), Bousias et al (1995), Brown and Jirsa (1971), Building Research Institute (1976, 1978), Burns and Siess (1966), Celebi and Penzien (1973), Chronopoulos and Vintzileou (1995), Darwin and Nmai (1986), Garstka (1993), Hanson and Conner (1972), Hwang and Scribner (1984), Krätzig et al (1989), Ma et al (1976), Mander (1983), Otani and Cheung (1981), Otani et al (1980), Park et al (1982), Popov et al (1972), Rabbat et al (1986), Saatcioglou and Ozcebe (1989), Scribner and Wight (1978), Takizawa and Aoyama (1976), Tegos (1984), Umehara and Jirsa (1984), Viwathanatepa et al (1979), Woodward and Jirsa (1984), Zagajeski et al (1978) and Zahn et al (1989). The databank covers a range of shear span values l/h from 1.0 to 6.5, of axial load ratio $v=N/A_cf_c$ from 0 to 0.475, of mechanical reinforcement ratios in tension, ω_1, from 0.035 to 0.275 and in compression, ω_2, from 0.035 to 0.21, of cylindrical concrete strength at testing from 14MPa to 46MPa and of mechanical confining steel ratio, ω_w, from 0.07 to 0.81. In 14 tests the specimens had a T cross-section and in few a circular, polygonal, or hollow rectangular. In 30 tests steel was of Grade 500, while in the rest it was a milder, more ductile steel.

A first examination and a prelimanary statistical treatment of the data show that: a) There is very large scatter of individual data about a mean ζ vs. μ relation, even within a family of specimens with the same geometric and mechanical proporties, or even in a single test. This scatter obscures the difference of damping between the first post-yield cycle and the subsequent ones reflected in (5), (6) on one hand and in (7) to (9) on the other. b) There is significant energy dissipation in post-cracking, pre-yield load cycles, equivalent to a damping ratio of about 8% of critical, almost independently of the amplitude of loading and of specimen characteristics. c) The average damping ratio at given $\mu>1$ increases with shear span ratio l/h, decreases with axial load ratio v, increases slightly with the mechanical ratio of confining steel, ω_w, and decreases with the mechanical ratio of tension reinforcement, ω_1. d) The values of l/h, v, ω_w and ω_1 in the 187 tests are statistically independent, whereas ω_2 is strongly positively correlated to ω_1, as in most tests $\omega_1=\omega_2$. Therefore, only one of these two variables can be considered as statistically independent of all others. ω_1 is selected for that purpose.

To quantify the dependence of the ζ-μ relation on l/h, v, ω_w and ω_1, regression analyses of the parameters of those of the hysteretic models refered above, in which hysteresis is controlled by a single parameter, on l/h, v, ω_w and ω_1 are performed. The expressions for damping in post-yield load cycles after the first, (7)-(9), are used for this purpose. For the modified, according to Litton (1975), Takeda et al (1970) model with $\beta=0$, (8), the regression parameter is α. In the modified, in Anagnostopoulos (1972), Clough and Johnston (1966) model, for which (7) gives $\zeta=(1-(1+p(\mu-1))/\mu^{1-a})/\pi$, the regression parameter is the unloading exponent a and in the Roufaiel and Meyer model, for which (7) gives $\zeta=(1-p)(\mu-1)\,(1+m+4p(\mu-1))/2\pi\mu(1+2p(\mu-1))^2$, the regression parameter is m, which controls pinching. As damping is not very sensitive to the hardening ratio, p, its value is taken equal to 2%.

Regressions are first performed considering all 3228 data points with $\mu>1$ in common. The "overall" regressions of parameter α of the modified by Litton (1975) Takeda model with $\beta=0$ (2353 α values between 0 and 1) and of exponent a of the modified by Anagnostopoulos (1972) Clough and Johnston (1966) model (2313 a values greater than 1.0), on ω_w and ω_1 turn out to be statistically insignificant. So the regression of these two parameters on l/h and v suffices, with little loss in predictive capability. Only l/h seems to have some limited statistical significance for parameter m of the Roufaiel and Meyer (1987) model (1247 m values between 0 and 1), the other three variables, v, ω_w and ω_1, being completely insignificant statistically. The best "overall" regressions of these three model parameters are given by:

$$\alpha \text{ (modified Takeda)} = 0.586-0.0444\,\frac{l}{h}+0.249v \tag{10}$$

$$a \text{ (modified Clough \& Johnston)} = 0.606-0.0392\,\frac{l}{h}+0.213v \tag{11}$$

$$m \text{ (Roufaiel \& Meyer)} = 0.432+0.0098\,\frac{l}{h} \tag{12}$$

Due to the nonuniform distribution of data in the range of l/h and v values, another series of regressions is performed, after dividing the range of l/h in 6 sectors (from 1.0 to 1.6, 1.6 to 2.6, etc. Fig. 3) and the range of v values into 4 sectors (0 to 0.05, 0.05 to 0.15, 0.15 to 0.25 and 0.25 to 0.475). With the exception of two of

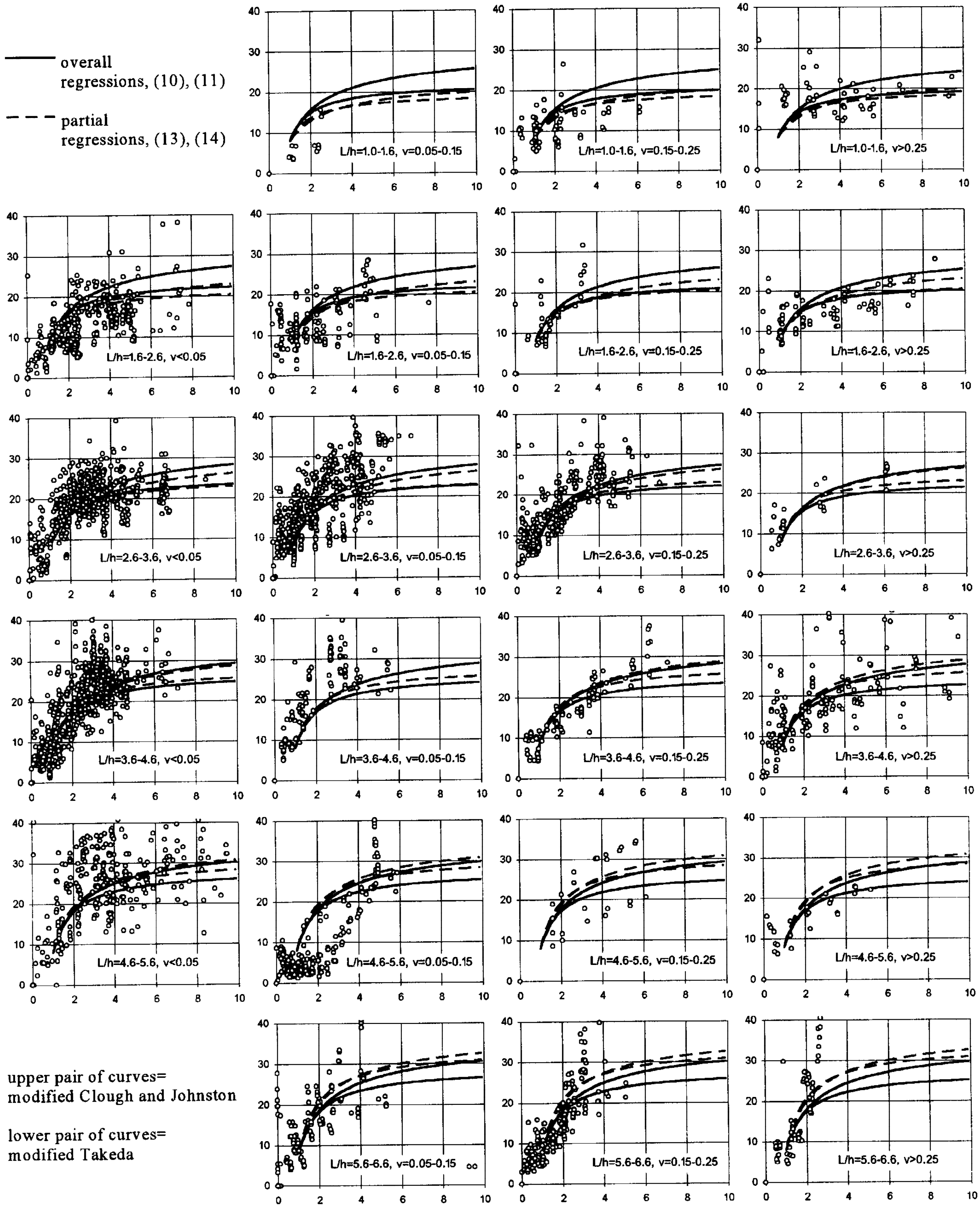

Fig. 3. Damping ratio $\zeta(\%)$ vs. ductility ratio μ in uniaxial tests. Experimental data compared to "overall" regression lines of (10), (11) (solid lines) and "partial" regression lines of (13), (14) (dashed lines). Upper solid and dashed line : modified to Clough and Johnston (1966) model, lower solid and dashed line : modified Takeda (1970) model.

them, the 24 cells of l/h and ν combinations contain a few tenths to several hundreds of ζ-μ pairs with μ>1, which allow a partial nonlinear regression of ζ on μ of the form suggested by the modified Takeda or the modified Clough and Johnston model, with parameters α and a respectively. The so-determined 22 α and a values for the 22 (l/h, ν) cells are regressed then on l/h and ν. The regression of these α and a on ν turns out to be statistically non-significant. The final result of these "partial" regressions on l/h alone is:

$$\alpha \text{ (modified Takeda)} = 0.755\text{-}0.0936\ \frac{l}{h} \tag{13}$$

$$a \text{ (Clough \& Johnston)} = 0.842\text{-}0.09213\ \frac{l}{h} \tag{14}$$

"Partial" regressions of this type for the Roufaiel and Meyer parameter m are not meaningful, as they do not provide a statistically more significant regression than a constant value of 0.46 to 0.47. (13) and (14) make use of all 3228 data points with μ>1 and provide a better fit to the ζ vs. μ relation for the individual (l/h, ν) cells, but do not reflect any dependence of α and a on ν. Moreover, the estimates of their coefficients are statistically less robust (more uncertain) than those of (10) and (11). The ζ vs. μ relations resulting from the modified Takeda and Clough and Johnston models for the parameters in (10), (13) and (11), (14) respectively (after adding the 8% damping associated with pre-yield cycles) are shown in Fig. 3, separately for each (l/h, ν) cell and compared with the corresponding test results. The predictions of all models coincide for μ between 1 and 2, but the modified Clough and Johnston model seems to provide better fit to the data for higher μ values.

The ζ vs. μ relations implicit in the modified Takeda and Clough and Johnston models, with parameters resulting from the "overall" and the "partial" regressions, (10), (11) and (13), (14) (after the addition of the 8% damping ratio of pre-yield cycles), is compared in Fig. 4 to experimental results derived from 46 transverse force-deformation components of truly biaxial tests, with non-proportional loading in the two orthogonal directions. The data are drawn from Bousias et al (1995), Li et al (1987), Low and Moelhe (1987), Otani et al (1980), Saatcioglou and Ozcebe (1989), Takizawa and Aoyama (1976) and Woodward and Jirsa (1984). As most data points lie above the regressions fitted to the uniaxial test results, the comparison demonstrates the higher hysteretic damping associated with biaxial loading and response. The higher hysteretic damping is attributed to the coupling between the two transverse directions (Bousias et al, 1995).

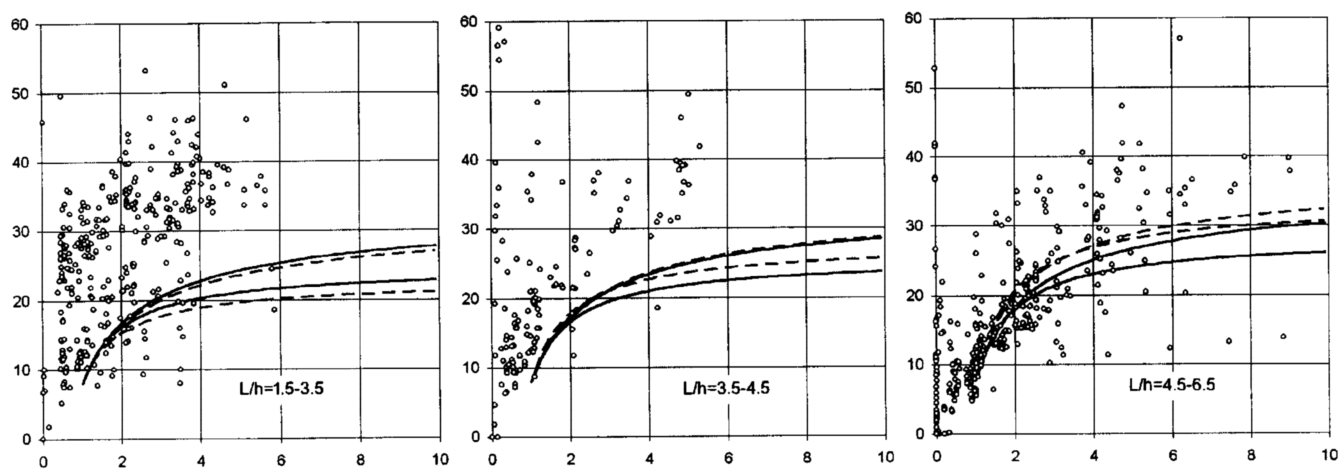

Fig. 4. Damping ratio ζ(%) vs. ductility ratio μ from biaxial tests, compared to the four uniaxial regression lines of Fig. 3.

CONCLUSIONS

The hysteretic damping ratio ζ, implicit in the frequently used phenomenological cyclic nonlinear force-deformation models of RC members, can be expressed analytically in terms of the ductility ratio μ and the model parameters, separately for the first full load cycle, in which post-yield loading follows the monotonic curve in both directions, and for subsequent symmetric unloading-reloading cycles at ductility μ. Such analytical expressions show a tendency of ζ to level off with increasing μ faster than suggested by available test results. For non-zero hardening these expressions even result in a reduction of damping with ductility, beyond a certain value of μ. Despite the large scatter of the experimental ζ vs. μ results, there is a clear tendency of damping to increase with increasing shear span ratio l/h and to decrease with axial force ratio ν. However, hysteretic damping seems to be nearly independent of the amount of longitudianal and confining steel. Accordingly, expressions have been derived statistically for the parameters of some of the single-parameter hysteretic models, such as the Clough and Johnston (1966) model, as modified by Anagnostopoulos (1972), and the Takeda et al (1970) model, as modifiel by Otani (1974) and Litton (1975), in terms of l/h and v. These expressions enable these models to represent on the average the energy dissipation of RC members in post-yield load cycles. A nearly constant damping ratio of about 8% seems to be associated with the post-cracking, pre-yield cyclic behavior. Finally, biaxial non-proportional loading of RC members seems to be associated with much higher damping than unidirectional cyclic loading, due to the coupling between the two transverse directions.

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