Paper No. 475. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

SEISMIC COLLAPSE ANALYSIS OF STEEL FRAME STRUCTURES

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ABSTRACT

Current code provisions for aseismic design rely on the capacity of the structure to resist deformations beyond its elastic limit. Notwithstanding, a severe ground motion may induce large lateral displacements and, hence, partial or total collapse by lateral instability. An analytical tool is thus needed to check a structure's safety margin against collapse, and to identify the structural components that may be in need of reinforcement. This paper presents an algorithm to perform this type of analysis for three-dimensional steel frame structures with numerical tests illustrating its effectiveness. The algorithm is formulated on the basis of a step-by-step finite element analysis and utilizes nonlinear large-deformation three-dimensional shell elements to account for material and geometric nonlinearities and spread of plasticity. It detects a partial or total collapse when a zero or negative eigenvalue is found, and operates on the updated stiffness matrix at the time the zero or negative eigenvalue is found to identify the buckled regions that lead to collapse. The structures considered in the tests are a cantilever beam for which experimental results have been reported in the literature and a two-story frame, designed for the purpose of this study, which meets 1991 AISC specifications.

KEYWORDS

Collapse Analysis, Steel Structures, Nonlinear Finite Element Analysis, Buckling

INTRODUCTION

The past several decades have seen earthquakes of moderate to large magnitude damage or destroy structures of modern aseismic design. As it is well known, the foremost objective of seismic codes is to insure a design which will resist severe shaking without collapse (SEAOC, 1990). However, the damage and many collapses in Mexico City during the Michoacan earthquake of September 19, 1985, demonstrated that current code provisions for steel structures, which allow inelastic deformations, but do not require a nonlinear analysis, may not be sufficient to prevent collapse (Villaverde, 1991). The Northridge earthquake of January 17, 1995 has also called into question the fundamental design of moment-resistant connections and their ability to reliably achieve the ductility for which they were designed. Both the building codes of the United States (ICBO, 1988) and Mexico (Garcia-Ranz and Gomez, 1987) incorporate the same ductility based design philosophy to resist collapse. This philosophy dictates that critical areas within a structural member, whose yield strength may be exceeded by a severe earthquake, be designed to achieve a large level of ductility and produce stable hysteretic behavior so that the entire structure will also be ductile and display stable hysteretic behavior (Bertero *et al.*, 1991). The main concern when designing for inelastic behavior is to prevent brittle fracture or severe buckling in or adjacent to the zone of inelasticity (AISC, 1992).

In view of the possibility of a partial or total collapse in structures subjected to earthquakes, a method is needed to check a structure's safety margin against collapse and to identify before hand those components whose reinforcement may help impede collapse. At present however, there are no tools or analytical

CONSTRUCTION OF ALGORITHM

To delineate the buckled region responsible for the collapse of a structure, it is possible to manipulate the effective stiffness matrix of the structure and record the degrees of freedom associated with the first zero or negative pivot encountered in the factorization process. That is, this first zero or negative pivot may be used to bracket the portions of the structure undergoing collapse. Consequently, if several consecutive effective stiffness matrices at different time steps are stored during the time-history analysis, and if all of these matrices include zero or negative pivots when factored, a collapse mode may be identified and a sequence of failure determined. The construction of the proposed collapse algorithm is built upon the following three premises:

1. The occurrence of one of more zero or negative pivots within the factorization of a system of equilibrium equation indicates instability.

2. The first zero or negative pivot encountered in the factorization of the effective stiffness matrix of the structure, corresponding to the i th degree of freedom, is sufficient to delineate an unstable section of the structure within degrees of freedom 1 to i.

3. Within the factorization of the system of equilibrium equations, degree of freedom i is the last degree of freedom located in that unstable portion of the structure.

The validity of the first premise has already been discussed in the preceding section. The second premise may be shown to be valid within the factorization process as follows: Within the Gaussian elimination, the value of any pivot i is calculated from the coefficients of rows and columns 1 to i. Therefore, for all practical purposes all of the degrees of freedom from i+1 to n could be fixed for the purposes of calculating pivots 1 to i without affecting the calculation of pivots 1 to i. Demonstrating the validity of the third premise involves calculation of the mode shape for the rigid-body motion induced by an unstable structure. It may begin by stating that for a structure containing a rigid body mode, the corresponding frequency is equal to zero. Its free-vibration equation of motion may be written as

$$\left(K - \omega_i^2 M\right)\phi_i = 0, \tag{2}$$

where the frequency is denoted by ω, and the mode shape vector by ϕ. For the rigid-body mode shape this equation may be reduced to

$$K\phi_i = 0, \tag{3}$$

since its frequency is equal to zero. One can then make one of the values in the mode shape vector equal to one, partition the stiffness matrix to separate the portion corresponding to this element of the mode shape, and solve for the subvector containing the rest of the mode shape values. If the selection of the degree of freedom assigned to unity corresponds to a degree of freedom for which its amplitude tends to infinity, upon the structure reaching instability, this subvector, together with the value made equal to one, form a vector of only ones and zeros. For example, in the two beam element case presented in the previous section, the degree of freedom with the assigned value of unity may be either degree of freedom 3 or 4. The calculated rigid-body mode shape,

$$\phi = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \tag{4}$$

indicates thus an infinite displacement for degrees of freedom 3 and 4, and a finite displacement for degrees of freedom 1 and 2 which show mode shape values of 0. This is expected as visual inspection dictates the end of the cantilever is unstable. It is instructive to notice the position of the last value equal to one in the mode shape vector. It corresponds to the position of the zero pivot encountered in the factored coefficient matrix. Both correspond to the fourth degree of freedom. If the columns and rows of the coefficient matrix are rearranged so that the degrees of freedom are ordered according to the sequence 3,4,1,2, the reordered mode shape and factored coefficient matrix are, methods whereby one can identify the location of failed members, or areas of local or lateral-torsional buckling within a member or connection, after it is detected that a structure undergoes collapse under a specific earthquake ground motion. There is also no tool to identify the failure path that leads to such a collapse mechanism. The current technique for detecting the collapse of a structure is to monitor its response, within a finite element time-history analysis, for a sudden increase in displacement. This technique of collapse detection may not always be accurate, and can be cumbersome to identify problem areas within a structure. Furthermore, it does not allow the precise delineation of buckled areas within the structure, and requires the graphical display of a characteristic response, typically lateral story displacement or maximum ductility demand, over many time steps.

The purpose of this paper is to introduce a proposed algorithm for the detection of collapse in steel frame structures and the identification, through the precise delineation of their unstable degrees of freedom, the factors that lead to such a collapse. To this end, use is made of the finite element method and a detailed modeling with shell elements to capture local buckling, the spread of plasticity, and its effects on global deformation. The finite element analysis accounts for both material and geometric nonlinearities. Use is also made of a time history numerical solution on the nonlinear equations of motion and of a technique within the solution process to detect the collapse of the structure and to locate afterwards the buckled areas within the structure.

THEORETICAL BACKGROUND

In a nonlinear finite element analysis, collapse is identified within the factorization of the stiffness matrix, which is required to solve a linearized system of simultaneous equations. In this research, the factorization of the stiffness matrix involves a form of Gauss elimination. This elimination reduces the coefficient matrix to triangular form, and the triangular form allows the calculation of the unknown displacement vector through back-substitution. The onset of a partial or total collapse is indicated when at least one of the diagonals in the factored matrix, a pivot, becomes equal to or less than zero.

Such a technique can be illustrated using as an example the two-element cantilever beam shown in Fig. 1, which has a hinge in its second element.

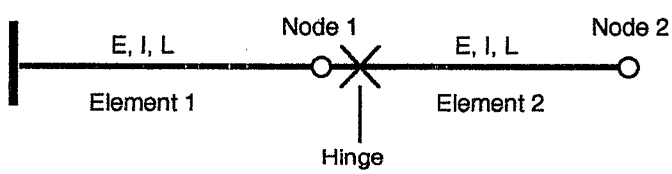

Fig. 1 Two-Element Cantilever With Hinge

This cantilever is verifiably unstable through visual inspection or factorization of the global stiffness matrix. For this beam, the global stiffness matrix and factored global matrix in upper triangular form are given by:

$$K_g = \frac{EI}{L^3}\begin{bmatrix} 15 & -6L & -3 & 3L \\ -6L & 4L^2 & 0 & 0 \\ -3 & 0 & 3 & -3L \\ 3L & 0 & -3L & 3L^2 \end{bmatrix} \text{ and } K_{fac} = \frac{EI}{L^3}\begin{bmatrix} 15 & -6L & -3 & 3L \\ 0 & 8/5\,L^2 & -6/5\,L & 6/5\,L^2 \\ 0 & 0 & 3/2 & -3/2\,L \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (1),$$

where E, I and L are the modulus of elasticity, moment of inertia and length, respectively. As it can be seen, the factored stiffness matrix indicates the last pivot is equal to zero. In turn, this zero pivot value indicates an unstable structure.

The presence of a zero, or negative, pivot indicates instability, and may be thus used to define stability. Bazant and Cedolin (1991) write "a structure is stable if a small change in the initial conditions (input) leads to a small change in solution" or "the structure is stable if a finite change in initial conditions (input) does not cause an infinite change in the solution (output, response)." It can be seen the two beam element example does not fit the criteria for a stable structure. For any load vector the factored stiffness matrix in eq. (1) produces an infinite displacement for the last degree of freedom. This is may be easily observed in the attempted back-substitution, which starts at the last equation, involving a division by zero to produce an infinite displacement. A more general check for stability involves a check of the coefficient matrix for positive definiteness.

$$\phi_{reordered} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \tag{5}$$

and

$$K_{fac,reordered} = \frac{EI}{L^3}\begin{bmatrix} 3 & -3L & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & . & . & . \\ 0 & . & . & . \end{bmatrix}, \tag{6}$$

respectively. The factorization can not be completed beyond the second equation as Gaussian elimination breaks down with the advent of a zero pivot. It can now be observed that the position of the zero pivot and the last value of one in the mode shape vector both correspond to eq. 2 of the reordered system, or degree of freedom 4. We may constrain degrees of freedom 1 and 2 and the structure remains, as it can be seen whether through inspection or numerical calculation, unstable.

It can now be stated the third premise is true. It was shown in the previous section a zero pivot implies an infinite displacement for any load vector. Therefore, this degree of freedom corresponding to the first zero pivot must have a value of 1 in the rigid body mode shape and a displacement which is infinitely large in comparison to stable portions of the structure, i.e., those degrees of freedom with mode shape values of zero. As was derived from the second premise, all degrees of freedom i+1 to n may be constrained without affecting the relative displacements of degrees of freedom 1 to i and implies those degrees of freedom, i+1 to n, assuming only one unstable region within the structure, correspond to those mode shape values of zero. This means that degree of freedom i is the last degree of freedom encountered in a particular order of equations which encompasses all the degrees of freedom within the unstable region, and that some or all of the degrees of freedom prior to i correspond to those degrees of freedom within that unstable portion of the structure. It must be made clear, the first zero pivot may not encompass all unstable portions in the frame, it just delineates the first unstable region encountered in the frame following that particular order of equations.

APPLICATION OF ALGORITHM

The basic aspects of the collapse algorithm presented above will be demonstrated using the frame shown in Fig. 2. This frame is an unstable two-dimensional, three story, single bay frame, with 18 degrees of freedom. The double hinged beam elements form the second story columns clearly producing an unstable structure. These hinged beam elements supply no resistance to lateral or rotational loads, but do resist axial loads. There are three degrees of freedom per node. The degrees of freedom correspond to an axial, lateral and rotational displacement, in that order. From the premises one would expect the factored stiffness matrix to have a zero pivot when the first collapse mode is encompassed. So the question to be asked, to verify the zero pivot position, as one moves incrementally from degrees of freedom 1 to 18 is, "does the inclusion of this degree of freedom allow this portion of the structure to form a rigid body mode", or "can this structure resist a load at that degree of freedom in question, assuming the degrees of freedom beyond that in question are constrained?" If one begins with node 1, clearly degrees of freedom 1 to 3 are stable. Progressing to node 2, and to degrees of freedom 4 to 6, and the same question is asked, it is tempting to think degree of freedom 5 produces a zero pivot, but it does not, as that translation is constrained by the beams from nodes 2 to 5 when all the degrees of freedom at node 5 are constrained. By a similar reasoning, degrees of freedom 6 through 13 do not produce a zero pivot, or encompass a collapse mode. A zero pivot is produced when degree of freedom 14 is included. This is because with the inclusion of that translation, the nodes 2, 3, 4 and 5 are free to translate. It is free to translate even if degrees of freedom 15 through 18 are constrained. From the position of the zero pivot, it can then be said that a collapse mode is found within degrees of freedom 1 to 14 and that degree of freedom 14 represents the boundary of the collapse mode. To further bracket the collapse mechanism, factor the stiffness matrix, but begin at equation 18, and proceed in the Gaussian elimination to equation 17 and so on, until a zero pivot is found. This approach will show degree of freedom 5 forming a zero pivot. This forms a new collapse mode forming within degrees of freedom 18 through 5, therefore bracketing the collapse mechanism between degrees of freedom 5 and 14. This completes the analysis of analysis of this structure and demonstrates the basics of the collapse algorithm and the bracketing of unstable regions within a structure.

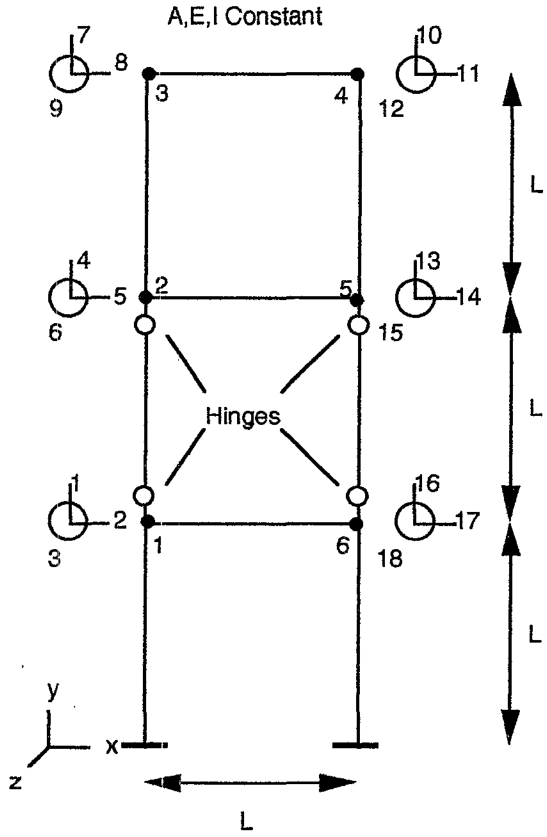

Fig. 2 Three-Story Frame

FINITE ELEMENT APPLICATIONS

The first step in a collapse analysis is to model steel frames or sections with a collection of shell elements using a nonlinear finite element program, such as NIKE3D (Maker et al 1991), the program used in this study. These shell elements constitute the flanges and webs of the beams, columns and connections of the frame. The use of a sufficiently large number of nonlinear shell elements in the constitution of a frame member has been shown to accurately model the spread of plasticity, local buckling and their effects on the response of its steel sections and subassemblages. Two models are discussed, a cantilever beam and a two-story steel frame. More details on these examples along with a more extensive listing and application of the algorithm may be seen in work by Martin (1995).

Cantilever Beam

The analysis of the cantilever beams is based upon and compared against experimental testing conducted by Povov and Stephen (1972). This study concentrates on the testing of the W24x76 beam sample, with an all welded connection. The goal in this analysis is not to reproduce the experimental result, as there is insufficient information for a dynamic analysis, but to induce a buckling failure similar to that found in the experimental test. The loading in the finite element analysis for this study is a pushover one; that is, a linearly increasing load until failure. This load is increased quickly, on the order of the time rates recorded in earthquakes. The state of deformation at time 0.066 seconds is shown in Fig. 3. This figure clearly shows the buckled flange and web. After this time the cantilever enters collapse. In the plot of this mesh, buckling in the expected location is clearly shown. The collapse algorithm also performs well and identifies the unstable nodes, which in the figure are identified by the black dots within the encircled area .

Three-dimensional Frame

This section describes the design, modeling and analysis of a two story, three-dimensional, frame of over 5,700 nodes, 5,300 shell elements and 34,000 degrees of freedom. The purpose for creating and analyzing this frame is two fold. The primary purpose is to investigate the utility of the collapse algorithm in much larger and complex problems than described previously. This type of modeling would be of the order required for laboratory prototypes in shake table experiments. The second purpose is to investigate collapse behavior in a structure meeting UBC and AISC specifications. The time history analysis performed in this test covers two slightly different variations of the frame. In the first version, the basic frame is considered, with continuity plate thicknesses matching that of the beam flanges framing into the column. In addition, no doubler plates are used in the panel zones. In the second version, the corner W10x54 column's continuity plates and panel zones thicknesses are increased to that of the column flanges. The frame considered for this part of the study is shown in Fig. 4. The frame is a two-story, two bay, frame with a setback and a simple geometry.

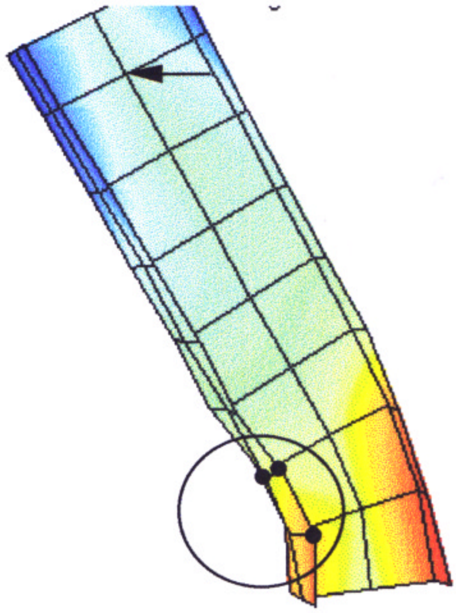

Fig. 3 State of Deformation in Cantilever Beam Model at Time 0.066 s, with Δt=0.001 s

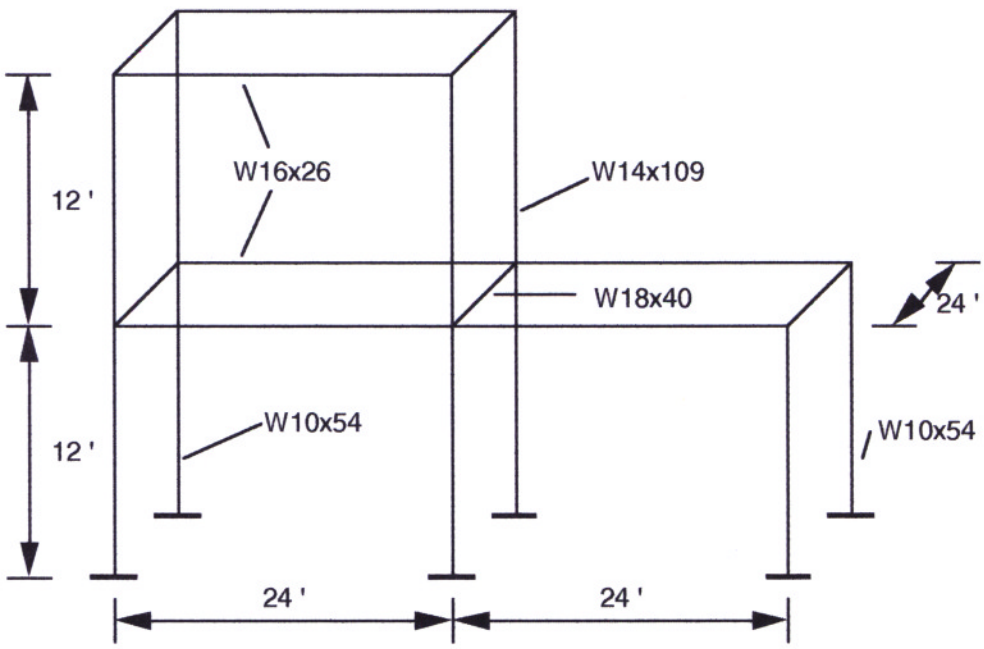

Fig. 4 Three Dimensional Frame

Although the frame meets AISC and UBC requirements, it is designed with the intention of producing buckling-initiated collapse. As such, all the chosen sections possess width-to-thickness ratios approaching AISC (1992) seismic provision limits for compression elements. Thus, this frame is designed to meet four main criteria. The first is the required minimum base shear, the second is the column-beam moment capacity ratio, the third, the interstory drift limit, and the fourth criteria, the minimum shear strength for the panel zones of beam-to-column connections. Both frame versions are subjected to a base excitation consisting of a portion of the three components of the Newhall - LA County Fire Station (Darragh *et al.,* 1994) ground motions recorded during the Northridge earthquake of January 17, 1994. The time-history analysis of the models were run with the horizontal components of the selected ground motion multiplied by 1.25, with the vertical component unmodified. This excitation induced a collapse in the structure after 1.70 seconds, or 86 time steps. The CPU time required per time step was approximately 50 seconds on a Cray C90.

The final time step, in which two stiffness arrays were recorded for later collapse algorithm processing, included eight stiffness reformations and ten nonlinear iterations. The last two factored stiffness reformations include negative pivots and indicate the onset of collapse. Fig. 5 shows the results of the collapse algorithm, and thus the unstable regions within the frame. These unstable regions are located within the bars and triangles and are contained within the joint and surrounding areas of the corner W10x54 column in the rear, first story, portion of the frame. The first stiffness array processed by the collapse algorithm, i.e., the initial factored stiffness matrix indicating negative pivots, encloses the unstable areas within the bars, and the second processed stiffness array encloses the region within the triangles. These regions do not include any degrees of freedom within the deck, as they are stable. This area, however, does include some stable degrees of freedom within the beams, column and connection zone. The bars and triangles are placed to bracket the entire unstable region and are drawn at the extreme degrees of freedom within the unstable area.

The bracketing of the unstable regions from the first unstable stiffness array identifies the connection zone and surrounding lengths of the adjoining beams and column. The second array identifies, as additionally unstable, the base of the column and a small region on the right-hand side of the beam. These results are not unexpected for this frame, and seem to present a good "snapshot" of the onset of collapse with the formation of further unstable regions forming "hinges." Most importantly, unstable degrees of freedom are calculated within the panel zone and continuity plates from both the first and second recorded stiffness arrays. It is these continuity plates and panel zones which are reinforced to help offset collapse in a subsequent analysis (not repeated here).

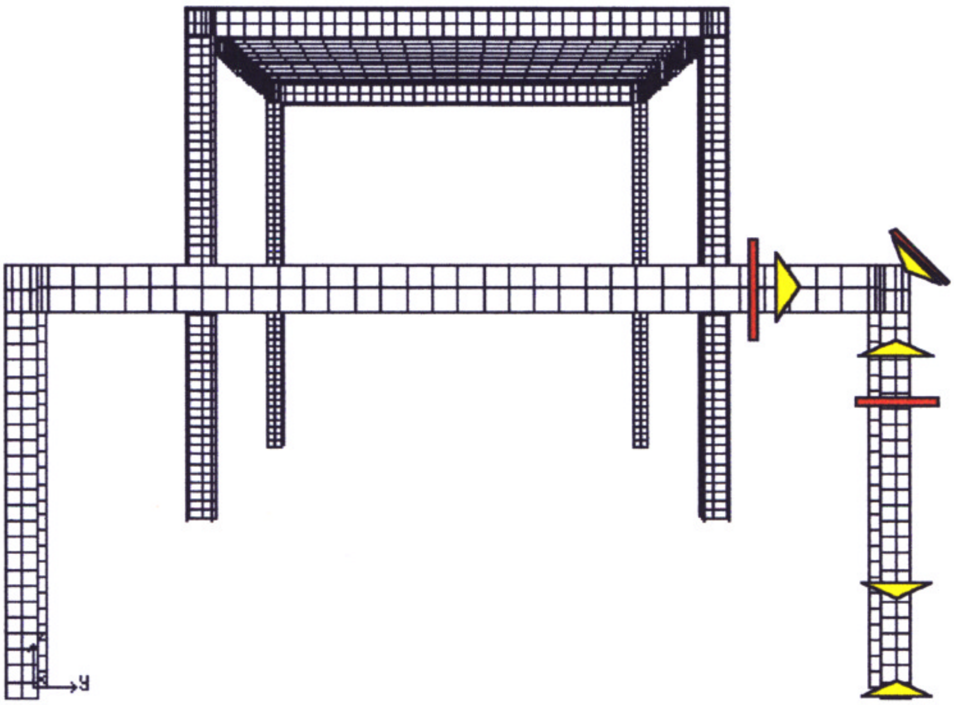

Fig. 5 Unstable Degrees of Freedom

CONCLUSIONS

An algorithm has been introduced to detect ensuing collapse in steel frame structures and to identify those unstable degrees of freedom responsible for collapse, serving to bound unstable regions within the members and joints. The algorithm is based upon a step-by-step nonlinear finite element analysis and operates on the updated stiffness matrices, which when factored indicate zero or negative eigenvalues, dictating imminent collapse. A collection of nonlinear, large deformation shell elements in a sufficiently fine mesh is used to appropriately capture local buckling and spread of plasticity. In addition, a numerical study has been performed with three structures to assess the effectiveness and demonstrate the application of the proposed algorithm.

The main conclusion that can be drawn from this work is that with the finite element formulation used, and the proposed algorithm, it is possible to detect the onset of a partial or total collapse in steel frames and identify the location of the areas of instability where this collapse is initiated. The algorithm succeeds in providing a tool for the delineation of buckled regions without the need to monitor a characteristic displacement to define collapse, as it detects and identifies problematic areas amid the crucial time step in which collapse occurs. It also obviates the need to plot a deformed mesh, which in any case may miss local instabilities not readily identifiable. It provides thus an elegant and precise tool that may be used to check a structure's safety margin against collapse and identify the critical factors that may affect this safety margin.

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