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RESPONSE CONTROL OF STRUCTURES BY TUNED MASS DAMPERS AND THEIR GENERALIZATIONS

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ABSTRACT

This paper presents the results of a study performed on tuned mass dampers (TMD) and their generalizations - multi-tuned mass dampers (MTMD) and active tuned mass dampers (ATMD). Correspondence between the design of a TMD for a SDOF structure and a certain mode of a MDOF structure is drawn to simplify TMD design to control a single mode of a damped multi-modal structure. An example is given to illustrate the design procedure. Investigations are made for controlling multiple structural modes using MTMD. Regarding ATMD, a recently proposed acceleration- and velocity-feedback algorithm is simplified with practical modifications.

KEYWORDS

Tuned mass dampers; Multi tuned mass dampers; Active tuned mass dampers; Acceleration feedback; Velocity feedback; Parametric study; Design.

INTRODUCTION

The concept of tuned mass dampers (TMD) dates back to year 1909, when Frahm invented a vibration control device called *dynamic vibration absorber.* Ormondroyd and Den Hartog (1928) showed that by introducing damping in Frahm's absorber, its performance can be significantly improved. Den Hartog derived closed form expressions for damper parameters which optimize the steady-state response of an undamped single-degree-of-freedom (SDOF) structure-absorber system subjected to harmonic excitation acting on the structure.

TUNED MASS DAMPERS

A schematic representation of a *damped vibration absorber* or so-called *tuned mass damper* (TMD) is shown in Fig. 1. The equations of motion of this SDOF structure-TMD mechanism are given as:

$$M\ddot{X}(t) + KX(t) - [c\{\dot{x}(t) - \dot{X}(t)\} + k\{x(t) - X(t)\}] = P(t) \tag{1}$$

$$m\ddot{x}(t) + c\{\dot{x}(t) - \dot{X}(t)\} + k\{x(t) - X(t)\} = p(t) \tag{2}$$

where μ: Damper mass to main mass ratio, $\mu = \frac{m}{M}$.
$P(t)$: Force acting on main mass. For base excitation with acceleration $\ddot{x}_g(t)$, $P(t) = -M\ddot{x}_g(t)$.
$p(t)$: Force acting on damper mass. It is given as:

$$p(t) = \begin{cases} \frac{m}{M}P(t) & \text{for base (earthquake-type) excitation} \\ 0 & \text{for main mass (wind-type) excitation} \end{cases}$$

To facilitate further discussion, the following additional notations are introduced:

ω: Frequency of a harmonic excitation.

Ω: Natural frequency of main mass, $\Omega = \sqrt{\frac{K}{M}}$.

ω_a: Natural frequency of damper mass, $\omega_a = \sqrt{\frac{k}{m}}$.

g_1: Ratio of excitation frequency to main mass natural frequency, $g_1 = \frac{\omega}{\Omega}$. For MDOF structures, $g_1 = \frac{\omega}{\Omega_1}$, where Ω_1 is the first modal frequency of the structure.

f: Frequency ratio, $f = \frac{\omega_a}{\Omega}$.

ζ_d: Damping ratio of TMD.

ζ: Damping ratio of main mass.

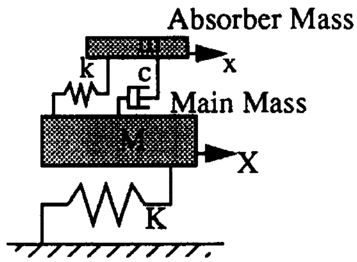

Fig. 1. Damped Vibration Absorber Suggested by Den Hartog (1928)

For undamped main structure, optimum values of TMD parameters are given as:

TMD parameter	Harmonic main mass excitation	Harmonic base excitation
f_{opt}	$\frac{1}{1+\mu}$	$\frac{1}{1+\mu}\left(\sqrt{\frac{2-\mu}{2}}\right)$
$\zeta_{d_{opt}}$	$\sqrt{\frac{3\mu}{8(1+\mu)}}$	$\sqrt{\frac{3\mu}{8(1+\mu)}}\left(\sqrt{\frac{2}{2-\mu}}\right)$

TMD Design for Damped Structures

In the presence of damping in the main mass, the equation of motion of the main mass, Eq. (1) is modified by adding the term $C\dot{X}(t)$ to the left-hand side. No closed-form expressions can be derived for the optimum damper parameters, however, they may be obtained by numerical trials. In a numerical trial approach, several combinations of damper parameters ζ_d and f are investigated in a systematic manner until the combination which minimizes the higher response peak is reached. This approach was used by Randall *et al.* (1981) to develop design graphs for obtaining optimum damper parameters for damped SDOF structures under main mass excitation. Warburton and Ayorinde (1980) tabulated numerically searched optimum values of absorber parameters for certain values of structure parameters. Rana (1995) developed tables of numerically searched optimum damper paramerters covering a practical range of structural parameters for harmonic main mass and base excitations. For a general N DOF damped structure-TMD system with a TMD placed at the jth floor of the structure, the equations of motion, for a typical rth floor mass, can be written as:

$$M_r\ddot{X}_r(t) + C_r\dot{X}_r(t) + K_rX_r(t) - \delta_{rj}[c\{\dot{x}(t) - \dot{X}_r(t)\} + k\{x(t) - X_r(t)\}] = P_r(t) \tag{3}$$

where

$$\delta_{rj} = \begin{cases} 0 & r \neq j \\ 1 & r = j \end{cases}$$

and for the damper mass:

$$m\ddot{x}(t) + c\{\dot{x}(t) - \dot{X}_r(t)\} + k\{x(t) - X_r(t)\} = p(t) \tag{4}$$

where

$$p(t) = \begin{cases} \frac{m}{M_j}P_j(t) & \text{for base excitation} \\ 0 & \text{for main structure excitation} \end{cases}$$

If TMD is to be designed to control ith structural mode with modal properties M_I, K_I and C_I, the design problem is essentially similar as that of designing a TMD to control a SDOF structure. For the ith structural mode-TMD system, equations of motion can be written as:

$$M_I\ddot{y}(t) + C_I\dot{y}(t) + K_Iy(t) - \phi_{ij}[c\{(\dot{x}(t) - \dot{X}_j(t)\} + k\{x(t) - X_j(t)\}] = P_i(t) \tag{5}$$

$$m\ddot{x}(t) + c\{\dot{x}(t) - \dot{X}_j(t)\} + k\{x(t) - X_j(t)\} = \Delta p(t) \tag{6}$$

where Δ is zero for main mass excitation and one for base excitation and

$P_i(t) =$ $\underline{\phi}_i^T \underline{P}(t)$, $\underline{P}(t)$ being the load vector acting on structure.
$y(t)$ Generalized displacement of ith mode.

Comparing Eqs. (5) and (6) to equations of motion for SDOF structure-TMD system, Eqs. (1) and (2), one can observe that these two pairs of equations differ on two accounts, namely, the presence of the term ϕ_{ij} in Eq. (5) and the presence of term $X_j(t)$ in stead of $y(t)$ in both Eqs. (5) and (6). However, if the structure's ith mode shape vector is normalized with respect to its jth element which corresponds to the TMD location (jth floor), ϕ_{ij} becomes unity and $X_j(t) = \phi_{ij} y(t) = y(t)$, and Eqs. (5) and (6) reduce to the same form as Eqs. (1) and (2). Thus if ϕ_{ij} is unity, expressions for calculating steady-state ith modal response and damper response in a MDOF structure-TMD system will be exactly the same as those for main mass and damper mass responses respectively in a SDOF structure-TMD system. In which case, design aids (e.g., Randall *et al.*, 1981; Warburton and Ayorinde, 1980; Rana, 1995), developed for designing a TMD for a SDOF structure can be directly used to design a TMD for a certain structural mode of a MDOF structure. Design procedure using this approach is illustrated by designing a TMD to control the first mode of a flexible and lightly damped 3 DOF structure with properties given in Table 1:

Table 1. Properties of the 3 DOF Structure Considered

$$\mathbf{M}(Kg) = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \qquad \mathbf{K}(N/m) = \begin{bmatrix} 1000 & -1000 & 0 \\ -1000 & 2000 & -1000 \\ 0 & -1000 & 2000 \end{bmatrix}$$

$$\mathbf{C}(N-s/m) = \begin{bmatrix} 1.519 & 0.272 & 0.097 \\ 0.272 & 1.343 & 0.176 \\ 0.097 & 0.176 & 1.246 \end{bmatrix} \qquad \mathbf{\Phi} = \begin{bmatrix} 0.737 & -0.591 & -0.328 \\ 0.591 & 0.328 & 0.737 \\ 0.328 & 0.737 & -0.591 \end{bmatrix}$$

$\Omega_1 = 0.708306$ Hz, $\Omega_2 = 1.98463$ Hz, $\Omega_3 = 2.86787$ Hz
$\zeta_1 = 2.0\%, \zeta_2 = 0.5\%, \zeta_3 = 0.3\%$

Observing the structural modal matrix Φ in Table 1, it can be said that in the first mode, the top floor will undergo largest steady-state deflection under a harmonic excitation. Therefore the TMD should be placed at the top floor for best control of the first mode. Since the TMD will be placed at the top floor, mode-shape vector $\underline{\phi}_1$ should be normalized with respect to its first element to calculate the structure's first modal mass. Therefore, the normalized $\underline{\phi}_1$ is given as:

$$\underline{\phi}_{1n} = [1.000 \;\; 0.802 \;\; 0.445]^T$$

The first-mode modal mass is given as: $M_1 = \underline{\phi}_{1n}^T \mathbf{M} \underline{\phi}_{1n} = 18.41$ Kg. If the damper mass is taken to be 2% of the entire building mass, then $m = 0.6$ Kg. Therefore, the damper mass to the structure's first-mode modal mass ratio is: $\mu_1 = \frac{0.6}{18.41} = 0.03259$. The first-mode modal damping ratio is known to be $\zeta_1 = 2\%$. Using these known values of μ_1 and ζ_1, the optimum damper parameters, f_{opt} and $\zeta_{d_{opt}}$, can be found from a numerical search. One obtains

$$f_{opt} = 0.952, \;\; \zeta_{d_{opt}} = 0.11.$$

This completes the design of the TMD tuned to the first mode of a 3 DOF structure.

MULTI-TUNED MASS DAMPERS

A single TMD can only control the mode for which it is designed. For controlling an additional mode a seperate TMD can be used. Therefore, the concept of multi-tuned mass dampers (MTMD), i.e., having a seperate TMD for every structural mode appears to be worth investigating. Much of the research in the area of multi-tuned mass dampers (e.g., Xu and Igusa, 1992; Yamaguchi and Harnpornchai, 1993; Kareem and Kline, 1995) has been done with the aim of controlling a single mode only. In this paper, analyses are carried out with the purpose of controlling multiple modes. The design method illustrated earlier is used to tune each damper.

Assuming that 2% of the building mass is the total available mass for all the dampers, an appropriate mass distribution among the various TMDs must first be determined. A response analysis of the structure with harmonic base excitation of frequencies varying over a range which covers all three natural frequencies was done. Obtained peak response ratios are, in first mode - 50.0 (at the top floor), in second mode - 7.5 (at the first floor) and in third mode - 2.5 (at the middle floor). These response values indicate the relative importance of various modes in determining the overall structural response and TMD masses are distributed in the ratio of 50.0 : 7.5 : 2.5 for first-, second- and third-mode TMD, respectively.

Harmonic Analysis of Modal Interaction

When a TMD is installed in the structure to control a particular mode, properties of the finally obtained system differ from those of the original structure. Now, if an additional TMD tuned to another mode is also to be installed, it may not perform as expected because of this effective change in structural parameters. Also, the addition of a TMD may affect the performance of TMD(s) already present. This problem of modal interaction is discussed with the help of a harmonic base excitation analysis. The parameters of the various TMDs used are given in Table 2. Top-floor responses of the structure are shown in Fig. 2 and the following observations are taken:

- To effectively control any particular mode, a seperate TMD, specifically tuned to that mode, must be provided.
- The structural response of the first controlled mode is marginally *increased* due to the presence of TMDs tuned to other modes while the structural response of second and third controlled mode is marginally reduced due to the presence of TMDs tuned to other modes.

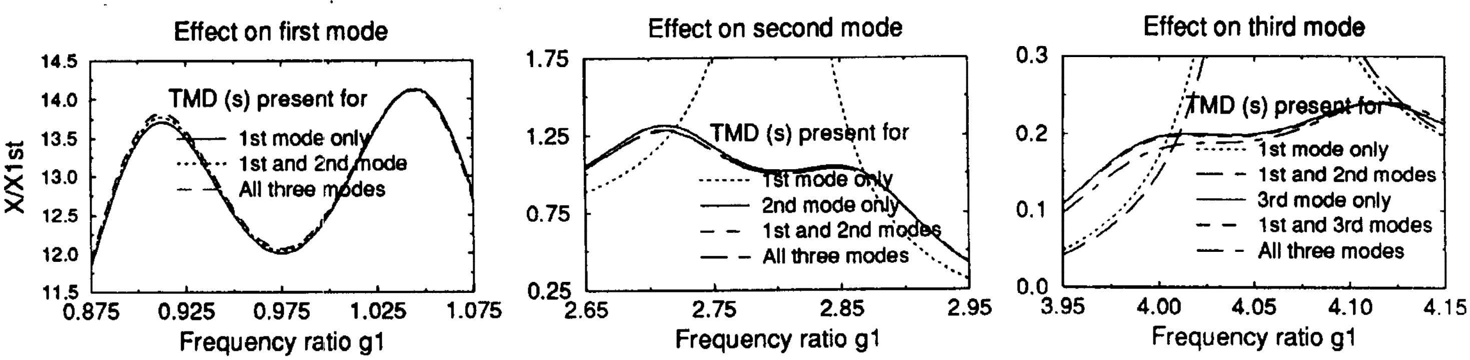

Fig. 2. Top Floor Response Showing Effect of TMD(s) on Various Modes

Time-History Analysis

It was observed in the harmonic analysis that, the presence of higher-mode TMDs causes some deterioration in the first-mode response. At the same time, to control higher modes, a seperate TMD must be provided. Therefore, a MTMD is designed and investigated with the help of time-history analyses under El Centro and Mexico earthquakes using one, two and three TMDs, respectively. Results of these analyses are summarized in Table 2.

It is clear that the designed MTMD does not result in appreciable response reduction in addition to what is already possible by a first-mode TMD. Based on the above harmonic and time-history analyses, it can be concluded that, effect of controlling the higher modes is getting nullified by a marginal increase in the first mode response.

Table 2. 3 DOF Structure-MTMD System Time-History Analyses Results

TMD(s) description		Floor	El Centro		Mexico	
Tuned to (location)	Parameters	no.	Peak(cm)	RMS(cm)	Peak(cm)	RMS(cm)
		3rd	13.29	4.58	-22.33	5.60
1st mode:	m=0.5 Kg	2nd	9.87	3.61	-17.76	4.52
(3rd floor)	k=9.11 N/m	1st	5.26	2.12	-10.20	2.59
	c=0.43 N-s/m	TMD-1	-43.00	16.06	75.51	19.02
1st mode:	as above	3rd	13.62	4.57	-22.22	5.62
(3rd floor)		2nd	9.66	3.61	-17.89	4.53
2nd mode:	m=0.075 Kg	1st	5.14	2.06	-10.20	2.59
(1st floor)	k=11.53 N/m	TMD-1	-43.08	16.09	75.70	19.06
	c=0.07 N-s/m	TMD-2	18.24	6.45	-12.82	3.25
1st mode:	as above	3rd	13.63	4.57	-22.26	5.63
(3rd floor)		2nd	9.69	3.61	-17.91	4.54
2nd mode:	as above	1st	5.15	2.07	-10.21	2.60
(1st floor)		TMD-1	-43.13	16.11	75.84	19.09
3rd mode:	m=0.025 Kg	TMD-2	18.21	6.45	-12.84	3.25
(2nd floor)	k=8.09 N/m	TMD-3	11.41	4.28	-19.23	4.83
	c=0.02 N-s/m					
		3rd	15.65	6.45	-23.47	5.66
w/o TMD		2nd	11.02	5.13	-19.07	4.61
		1st	6.18	2.94	-10.92	2.64

ACTIVE TUNED MASS DAMPERS

The concept of active TMD or ATMD, has been an area of interest to researhers for some time (Morison and Karnopp, 1973; Chang and Soong, 1980). An ATMD offers benefits over a TMD like controlling multiple structural modes and reduced damper mass and stroke requirement. At present there exist several examples of real-life implementations of active mass dampers (Soong *et al.*, 1994). Considering the difficulties involved in the accurate measurment of floor displacements due to the lack of absolute reference during a seismic event, algorithms have recently been proposed (e.g., Dyke *et al.*, 1993; Nishimura *et al.*, 1992a, b) which require only acceleration or acceleration and velocity measurements but not displacement measurements. The algorithm proposed by Nishimura *et al.* (1992a, b) essentially utilizes the same methodology of optimization, as originally proposed by Den Hartog (1956). This algorithm is further studied in this paper. The equations of motion of a SDOF structure-active mass damper system can be written as follows:

$$m\ddot{x} + k(x - X) + c(\dot{x} - \dot{X}) = \Delta\mu P(t) + u(t) \tag{7}$$

$$M\ddot{X} + KX - k(x - X) + C\dot{X} - c(\dot{x} - \dot{X}) = P(t) - u(t) \tag{8}$$

where $u(t)$ is the control force. Assuming an undamped SDOF structure, Nishimura *et al.* (1992a, b) proposed the following expression for control force calculation:

$$u(t) = -MG_a\ddot{X} - c_{opt}G_v(\dot{x} - \dot{X}) \tag{9}$$

Where G_a and G_v are acceleration- and velocity-feedback gains respectively. G_a should always be less than unity since a $G_a \geq 1.0$ results in a complete annihilation of inertial resistance causing instability. In the presence of inherent damping in the main structure, the optimum damper parameters can be found by a numerical search. However for an undamped SDOF structure, closed-form expressions for optimal parameters have been derived:

	Harmonic main mass excitation	Harmonic base excitation
f_{opt}	$\frac{1}{1+\mu}\sqrt{1 - G_a}$	$\frac{1}{1+\mu}\sqrt{\frac{2(1-G_a)-\mu}{2}}$
$\zeta_{d_{opt}}$	$\approx \sqrt{\frac{3(\mu+G_a)}{8(1+\mu)}}$	$\approx \sqrt{\frac{3(\mu+G_a)}{8(1+\mu)}}\sqrt{\frac{2}{2-\mu}}$

Following above discussion, one can obtain optimum damper parameters when the damper is activated using acceleration- and velocity-feedback control force. However it is preferrable to activate the TMD only selectively while using it in passive mode for most of the times. Therefore there is a need to study the performance of an active damper whose parameters (frequency ratio f and damping ratio ζ_d) are maintained at their optimum values of the passive mode. In this practical situation, G_a and G_v are the parameters which should be optimized.

Harmonic Analysis

Figure 3 is obtained by performing steady-state harmonic base excitation analyses on a SDOF structure-active damper system. This figure shows the plots of the peak response ratio values for different G_a-G_v combinations with changing structural parameters. It is seen that at higher μ and/or ζ, the benefit of activating the TMD becomes less significant. From this figure it is also clear that

- Increasing G_a and accordingly adjusting G_v results in significant reduction of peak response ratios. For parameters $\mu = 0.02$ and $\zeta = 0.02$, for example, the peak response ratio reduces from 7.6 (passive TMD) to 2.4 (at $G_a = 0.8$ and $G_v = 4.0$). The effect of increasing G_a diminishes at higher G_a values.
- For each G_a value, there exists an optimum G_v, denoted here as $G_{v_{opt}}$. $G_{v_{opt}}$ increases with increasing G_a. Adjusting G_v to its optimum value is important in response reduction.

Performing a similar harmonic analysis on the 3 DOF structure considered, Fig. 4 is obtained. This figure indicates that a significant response reduction is possible in all three modes. Although the value of $G_{v_{opt}}$ is different for different modes, the reduction in the response of higher structural modes by using a G_v which is optimum for the first mode would still be significant.

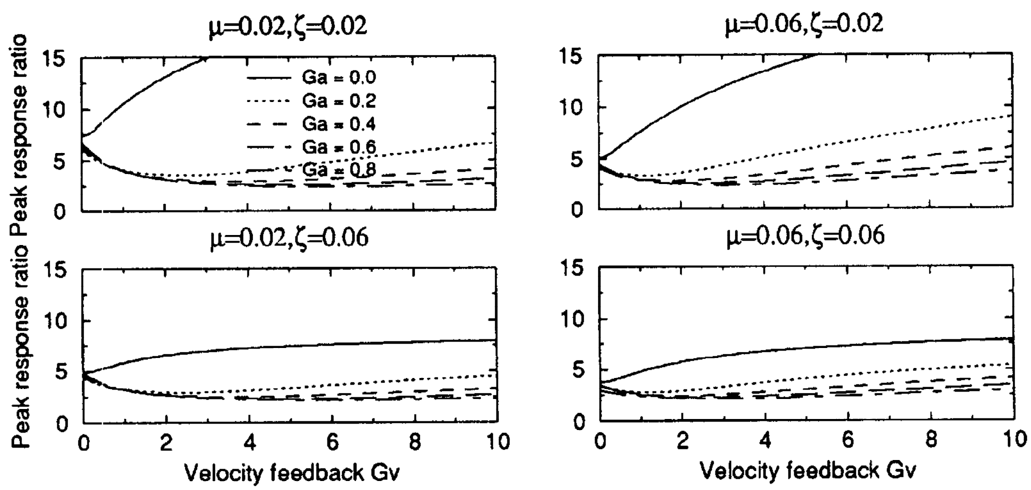

Fig. 3. Effect of Various Ga-Gv Combinations on Response: SDOF Structure

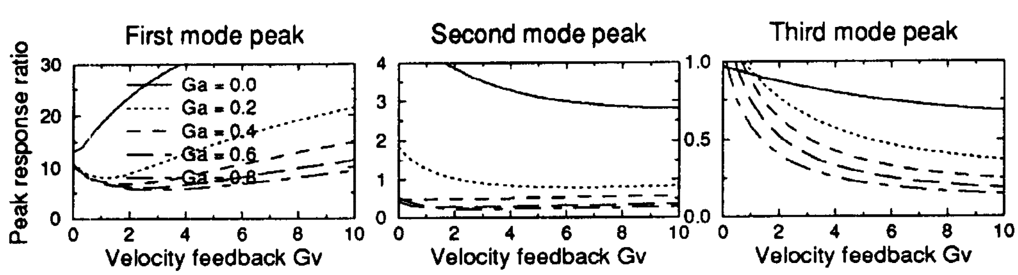

Fig. 4. Effect of Various Ga-Gv Combinations on Response: 3 DOF Structure

Time-History Analysis

Figures 5 and 6 summarize the results of the time-history analyses performed on the 3 DOF structure under El Centro and Mexico earthquakes respectively. Under El Centro earthquake, significant additional reduction is observed. Under Mexico earthquake, the structure derives a relatively smaller benefit from active control. Regarding the selection of suitable values of G_a and G_v, it is noted from above analyses that increasing G_a in general results in a further response reduction. However, this effect is smaller at higher G_a values. For the structure analyzed here, a G_a of 0.6 seems to be reasonably good. Also, it is to be remembered that, a $G_a \geq 1.0$ will result in an unstable system. The value of $G_{v_{opt}}$ depends on the value of G_a. Although $G_{v_{opt}}$, as observed in the time-history results, usually differs from harmonic excitation analysis results; providing a $G_{v_{opt}}$ as suggested by harmonic excitation analysis usually proves to be a good, if not the best, choice.

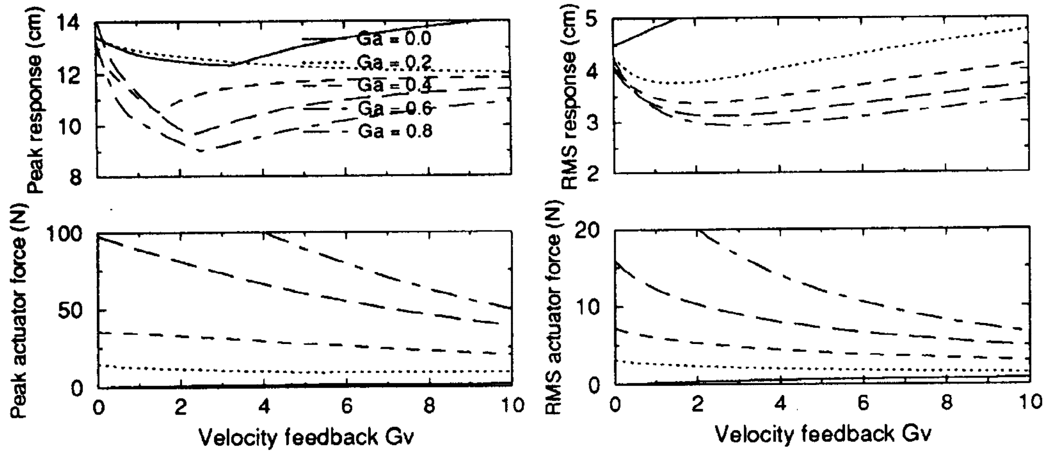

Fig. 5. Top Floor Response and Actuator Force for Various Ga-Gv Combinations: El Centro

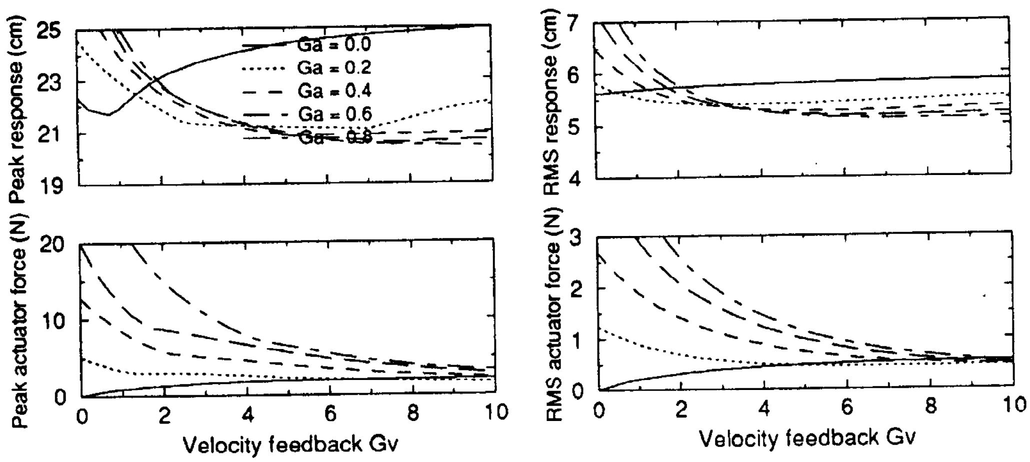

Fig. 6. Top Floor Response and Actuator Force for Various Ga-Gv Combinations: Mexico

Harmonic Detuning Analysis

To determine the additional benefit that can possibly be obtained by adjusting the damper parameters to their optimum active mode values rather than keeping them fixed at their optimum passive mode values, detuning analysis using steady-state harmonic base excitation analysis is done on a SDOF structure-active damper system. The peak response ratio values so obtained are plotted in Fig. 7. In this analysis, optimum G_v which is obtained by harmonic analysis (Fig. 3) has been used.

In addition to noting the significant additional response reduction offered by ATMD, it is observed that the benefit of adjusting parameter f is not significant. Since damper parameter ζ_d is already adjusted to its active state optimum value by using the appropriate G_v, a change in ζ_d causes performance deterioration. From Fig. 7, it can be concluded that it is permissible to let the damper parameters remain fixed at their optimum values of passive state even when the damper is switched to the active mode. This is significant in simplifying the implementation of such a control system without compromising control efficiency.

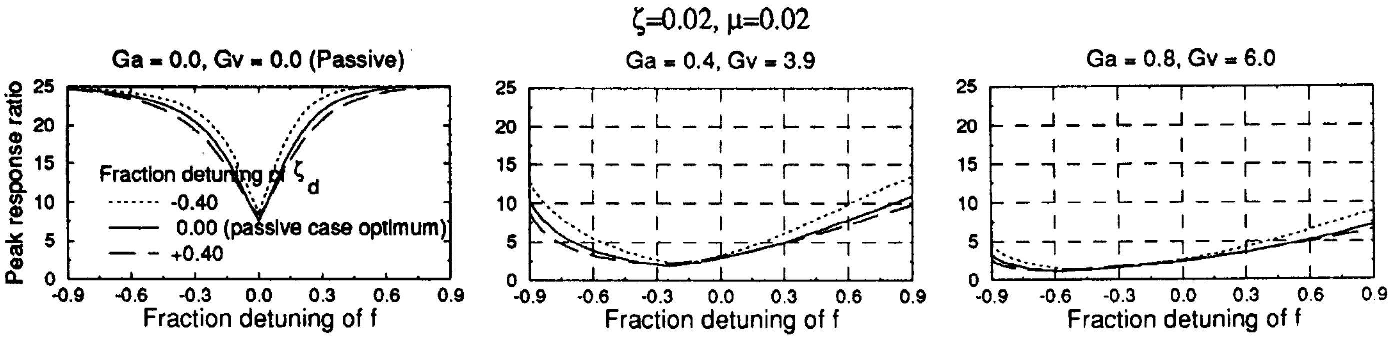

Fig. 7. Detuning Analysis to Find Optimum Damper Parameters in ATMD

CONCLUSIONS

A study performed on TMD and their generalizations is presented. A simplified procedure for TMD design is illustrated. Control of multiple structural modes using a MTMD is not found to be effective due to modal contamination in the 3 DOF structure considered. A recently proposed active TMD algorithm is simplified and it is shown to be effective in response control. Selection of appropriate values of feedback gains is also explained.

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