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OPTIMIZED PARAMETERS OF DYNAMIC PENDULUM ABSORBERS FOR SIMPLE ENGINEERING STRUCTURES

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ABSTRACT

In this paper, a simple and new system for the pasive control of structural vibration is presented, in which pendulums are attatched to structure to dissipate some excitation energy. First of all, such a systematic scheme and its equation of motion are given, and then the optimized tuning frequency ratio for the composite system is derived in the case of steady-state response. Furthermore, the effects of parameters on the optimum values are investigated based on the numerical calculations. The results show that such a system presented herein is effective to reduce the amplitude of structural vibration.

KEYWORDS

Engineering structure; dynamic pendulum absorber; composite system; equation of motion; steady-state response; numerical calculation; optimized parameter.

INTRODUCTION

Engineering Structures have traditionally relied on their ability to dissipate energy through engineering design to resist the dynamic loads. In the last decade or two, however, an inereasing amount of attention has been given by scientists and engineers all over the world to the mitigation of damage caused by the various dynamic loads. A significant portion of this effort has been devoted to the study and use of passive and active systems to control and reduce structural damage. Down to date, there have been a number of successful applications of control concepts where the objectives, limits and criteria for the controllor are essentially determined by specific design requirements. Hence, these are promising methods.

This paper is concerned about the investigation of a simple approach for the pasive control of structural vibration, namely the pendulum are used in structure to reduce the response to dynamic load excitation. First of all, such methed is presented, and then the optimum tuning frequency ratio for the composite system is given. Furthermore, the effects of parameters on the optimum value are carried out. Finally, some conclusions with practical significance are drawn referring to the numerical calculations.

The study of engineering application on this control system under earthquake will be given in our further work.

METHOD AND FORMULATION

For the sake of simplicity and convenience, let us consider a typical single - degree - of - freedon structure which can be simplified from a civil engineering structure with a n mass pendulum subjected to the horizontal excitation, illustrated in Fig. 1. The gravitational stiffness matrix of a n mass pendulum can be referred to Reference (Wang et al. ,1981). By some derivations, the equation of motion for such systen can be gotten as follows

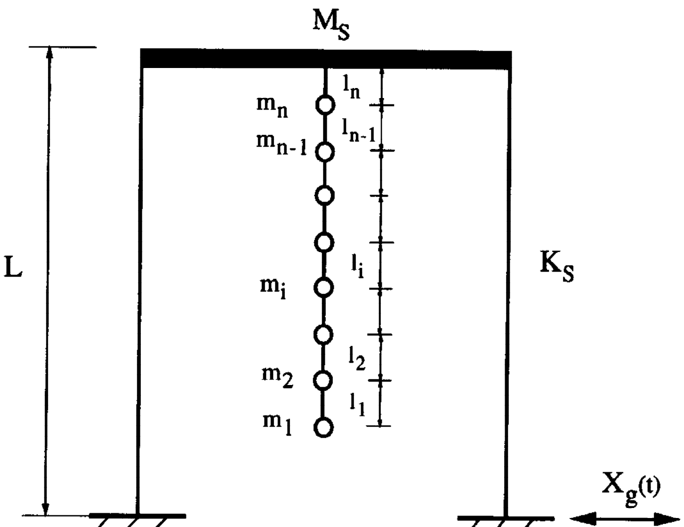

Fig. 1 Structural System with Pendulum

$$[M]\{\ddot{X}(t)\} + [C]\{\dot{X}(t)\} + [K]\{X(t)\} = -[M]\{E\}\ddot{X}_g(t) \qquad (1)$$

where

$\{X(t)\}, \{\ddot{X}(t)\}, \{\dot{X}(t)\}$ = displacement, velocity and acceleration vectors, respectively;
$\{E\}$ = unit vector ;
$[D]$ = damping matrix;
$\ddot{X}_g(t)$ = ground horizontal acceleration history;
$[M] = \mathrm{diag}[m_1, m_2, \cdots, m_m, M_s]$ = mass matrix;
$[K]$ = stiffness matrix given by

$$[K] = g \begin{bmatrix} \frac{m_1}{l_1} & -\frac{m_1}{l_1} & & & & & \\ -\frac{m_1}{l_1} & \frac{m_1}{l_1} + \frac{m_1+m_2}{l_2} & -\frac{m_1+m_2}{l_2} & & & & \\ & \ddots & \ddots & \ddots & & & \\ & & -\frac{\sum_{j=1}^{i-1} m_j}{l_{i-1}} & \frac{\sum_{j=1}^{i-1} m_j}{l_{i-1}} + \frac{\sum_{j=1}^{i} m_j}{l_i} & -\frac{\sum_{j=1}^{i} m_j}{l_i} & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & -\frac{\sum_{j=1}^{n-1} m_j}{l_{n-1}} & \frac{\sum_{j=1}^{n-1} m_j}{l_{n-1}} + \frac{\sum_{j=1}^{n} m_j}{l_n} & -\frac{\sum_{j=1}^{n} m_j}{l_n} \\ & & & & & \frac{\sum_{j=1}^{n} m_j}{l_n} & \frac{K_s}{g} + \frac{\sum_{j=1}^{n} m_j}{l_n} \end{bmatrix} \qquad (2)$$

in which

g = gravitational acceleration;

M_s = mass of single-degree-of-freedom structure;

K_s = stiffness of single-degree-of-freedom structure.

The other notations in the stiffness matrix [K] can be selfexplanatory with the help of Fig. 1, in which the stiffness cefficients are

$$\left.\begin{aligned}
k_{i-2,i} &= 0\\
k_{i-1,i} &= -\frac{g\sum_{j=1}^{i-1} m_j}{l_{i-1}}\\
k_{i,i} &= \frac{g\sum_{j=1}^{i-1} m_j}{l_{i-1}} + \frac{g\sum_{j=1}^{i} m_j}{l_i}\\
k_{i+1,i} &= -\frac{g\sum_{j=1}^{i} m_j}{l_i}\\
k_{i+2,i} &= 0\\
k_{n+1,n+1} &= K_s + \frac{g\sum_{j=1}^{n} m_j}{l_n}
\end{aligned}\right\} \quad (i = 1, 2, \cdots, n)$$

OPTIMUM SOLUTION OF STEADY-STATE RESPONSE FOR SIMPLE SYSTEM

Numerical calculations have shown that the effectiveness of vibrational reducing of a mass pendulum for structure is better than that of two mass peudulum (LI, 1995). In order to study the effects of parameters on the response of system, the system with a mass pendulum is here used as a example shown in Fig. 2. The structure is assumed to be subjected to the action of a harmonic ground motion $Xg(t) = a_0 \exp(i\omega t)$ and only the steady-state response is considered.

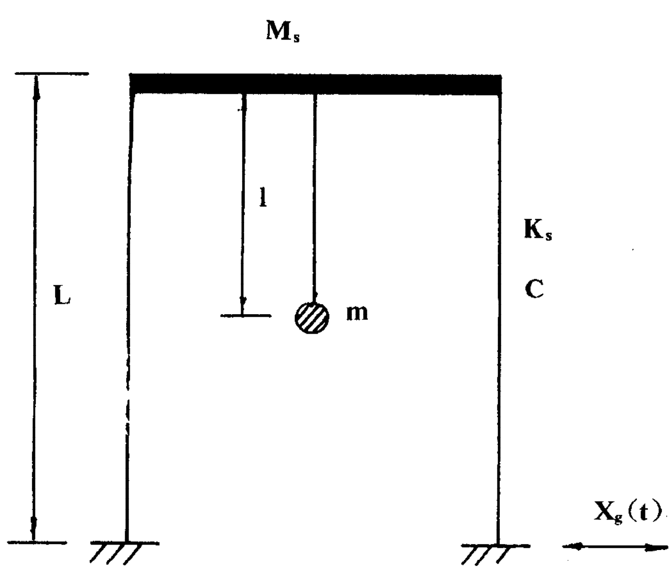

Fig. 2 Structural System with a Mass Pendulum

Since the damping of pendulum is small, it is omitted here. According to the Fig. 2, the equation of motion can be readily gotten as

$$\left.\begin{aligned} m\ddot{x}_1 + \frac{mg}{l}(x_1 - x_2) &= ma_0\omega^2\exp(i\omega t) \\ M_s\ddot{x}_2 + C\dot{x}_2 + K_s x_2 + \frac{mg}{l}(x_2 - x_1) &= ma_0\omega^2\exp(i\omega t) \end{aligned}\right\} \quad (3)$$

For the steady - state response, the solutions of the above equation may be supposed as follows

$$\left.\begin{aligned} x_1(t) &= \hat{x}_1\exp(i\omega t) \\ x_2(t) &= \hat{x}_2\exp(i\omega t) \end{aligned}\right\} \quad (4)$$

Substituting the equation (4) in (3), the dimmensionless displacement response of structure can be derived as following form

$$\eta = \left|\frac{\hat{x}_2}{x_0}\right| = \left\{\frac{[(1-\alpha^2)\gamma\beta^2-\alpha^2+\alpha^4]^2+[2\xi\alpha(\gamma\beta^2-\alpha^2)]^2}{[(1-\gamma\alpha^2)\beta^2-\alpha^2+\alpha^4]^2+[2\xi\alpha(\beta^2-\alpha^2)]^2}\right\}^{\frac{1}{2}} \quad (5)$$

where

$\alpha = \omega/\omega_s =$ forced frequency ratio;

$\beta = \omega_p/\omega_s =$ tuning frequency ratio;

$\omega_p^2 = g/l =$ natural frequency of the pendulum;

$\omega_s^2 = K_s/M_s =$ natural frequency of the structure;

$\xi = C/2M_s\omega_s =$ damping ratio of the structure;

$\gamma = 1 + \mu$;

$\mu = m/M_s =$ mass ratio;

$x_0 =$ steady - state displacement response of the system without pendulum.

The equation (5) may be used as the objective function for the optimum tuning frequency ratio β_{opt}. For the simplicity of derivations, let

$$\eta = \left|\frac{\hat{x}_2}{x_0}\right| = \left(\frac{H_1^2 + H_2^2}{H_3^2 + H_4^2}\right)^{\frac{1}{2}} \quad (6)$$

where

$H_1 = (1 - \alpha^2)\gamma\beta^2 - \alpha^2 + \alpha^4$

$H_2 = 2\xi\alpha(\gamma\beta^2 - \alpha^2)$

$H_3 = (1 - \gamma\alpha^2)\beta^2 - \alpha^2 + \alpha^4$

$H_4 = 2\xi\alpha(\beta^2 - \alpha^2)$

In order to get the miminum of response, one may apply the following fumula

$$\frac{\partial}{\partial\beta^2}\left(\frac{H_1^2 + H_2^2}{H_3^2 + H_4^2}\right) = 0$$

i. e.

$$\left.\frac{(H_1^2 + H_2^2)'(H_3^2 + H_4^2) - (H_1^2 + H_2^2)(H_3^2 + H_4^2)'}{(H_3^2 + H_4^2)^2}\right|_{\beta=\beta_{opt}} = 0 \quad (7)$$

By some derivations, the following equation can be gotten for β_{opt}

$$A\beta^4 + B\beta^2 + C = 0 \quad (8)$$

where

$$A = (a^2+d^2\gamma^2)(bc-de)+(c^2+d^2)(de\gamma-ab)$$
$$B = (b^2+e^2)[a^2-c^2+d^2(\gamma^2-1)]$$
$$C = \mu(b^2+e^2)(b-de)$$

in which

$$a = (1-\alpha^2)\gamma$$
$$b = -\alpha^2+\alpha^4$$
$$c = 1-\gamma\alpha^2$$
$$d = 2\xi\alpha$$
$$e = 2\xi\alpha^3$$

By analysis, one may know that there are no closed-form solutions in the expression (8). Therefore, some numerical results are given by using computer. Fig. 3 shows the variations of optimized tuning frequency ratio β_{opt} in terms of α for the specified values of mass ratio and damping ratio, namely $\mu=0.02$, 0.06, 0.1, 0.4 and 0.8 and $\xi=0.02$ and 0.05, in which the structural damping ratios are in the common used range of civil engineering structures (0.02 for the steel structure, 0.05 for the concrete structure). As the forced frequency ratio α increases, there is a linear increase in the optimized tuning frequency ratio β_{opt}, and the curved slope is less when the mass ratio μ is large.

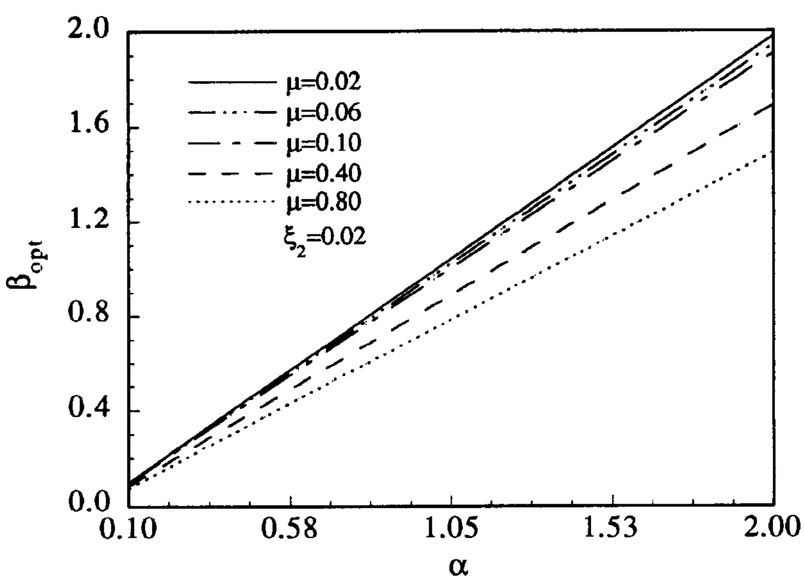

Fig. 3(a) Variations of Optimum β_{opt} with α for $\xi=0.02$

Table 1 and 2 give the optimum values of the dimmensionless displacement response η and tuning frequency ratio β with the variations of the specified values of the mass ratio μ and forced frequency ratio α. From the two tables it can be seen that both β_{opt} and η_{opt} deorease with increase of mass ratio μ. The structural damping ratio ξ in the common used range of civil engineering has only a small effect on the values of β_{opt}. In the vicinity of $\alpha\approx1.0$, the effectiveness of vibrational redcing on structural resonse is best for the range. Therefore, for large μ and $\alpha\approx1$, the amplitude reduction of structural vibration is most noticeable.

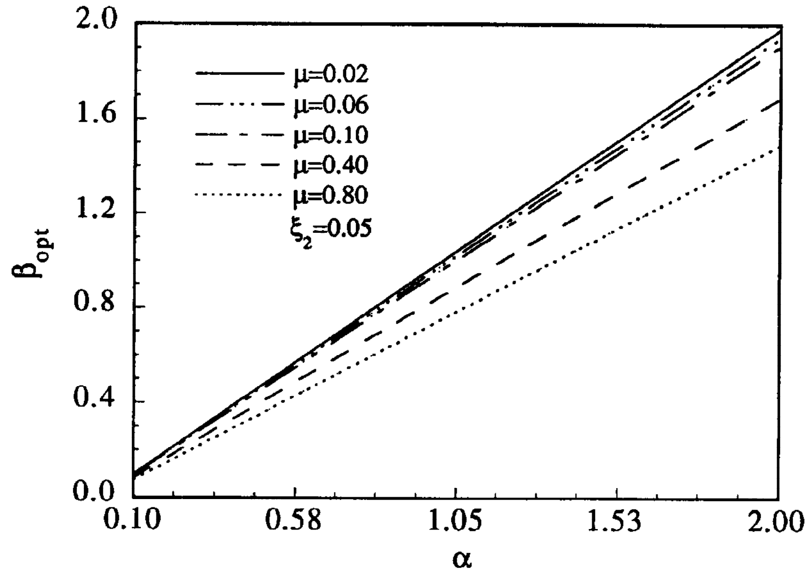

Fig. 3(b) Variations of Optimum β_{opt} with α for ξ=0.05

Table 1. Optimum Value of Dynamic Pendulum for Structure with Damping ξ_2=0.02

Mass Ratio μ	Forcing Frequency Ratio α	Optimum Values β_{opt}	η_{opt}
	.1	.9901768E−01	.2987192E−02
	.5	.4950705E+00	.4981576E−03
.02	1.0	.9901477E+00	.7078713E−06
	1.5	.1485126E+01	.8264982E−02
	2.0	.1979973E+01	.5216876E−01
	.1	.9713045E−01	.6685917E−03
	.5	.4856487E+00	.3122956E−03
.06	1.0	.9712859E+00	.5288670E−08
	1.5	.1456917E+01	.3527541E−03
	2.0	.1942459E+01	.6196382E−02
	.1	.9534670E−01	.9897197E−04
	.5	.4767309E+00	.1172019E−05
.10	1.0	.9534626E+00	.2263166E−07
	1.5	.1430201E+01	.1278254E−03
	2.0	.1906834E+01	.3146432E−02
	.1	.8451549E−01	.4702678E−05
	.5	.4225773E+00	.1476990E−05
.40	1.0	.8451543E+00	.1262609E−07
	1.5	.1267730E+01	.1213861E−04
	2.0	.1690321E+01	.1570149E−03
	.1	.7453562E−01	.1095460E−05
	.5	.3726780E+00	.3234645E−06
.80	1.0	.7453560E+00	.1905825E−08
	1.5	.1118034E+01	.1608465E−05
	2.0	.1490716E+01	.3595152E−04

Table 2. Optimum Value of Dynamic Pendulum for Structure with Damping $\xi_2=0.05$

Mass Ratio μ	Forcing Frequency Ratio α	Optimum Values β_{opt}	η_{opt}
	.1	.9901763E−01	.2934129E−02
	.5	.4950708E+00	.4520511E−03
.02	1.0	.9901487E+00	.1132656E−04
	1.5	.1485129E+01	.7942396E−02
	2.0	.1980008E+01	.4556459E−01
	.1	.9713049E−01	.6841516E−03
	.5	.4856482E+00	.2899123E−03
.06	1.0	.9712861E+00	.6532279E−06
	1.5	.1456916E+01	.3837186E−03
	2.0	.1942468E+01	.5619169E−02
	.1	.9534672E−01	.1045513E−03
	.5	.4767308E+00	.1606223E−04
.10	1.0	.9534625E+00	.8718399E−07
	1.5	.1430199E+01	.9548501E−04
	2.0	.1906838E+01	.2974326E−02
	.1	.8451549E−01	.4702700E−05
	.5	.4225773E+00	.1478476E−05
.40	1.0	.8451543E+00	.3143368E−07
	1.5	.1267730E+01	.1209913E−04
	2.0	.1690321E+01	.1470004E−03
	.1	.7453562E−01	.1095466E−05
	.5	.3726780E+00	.3240364E−06
.80	1.0	.7453560E+00	.4744706E−08
	1.5	.1118034E+01	.5354386E−06
	2.0	.1490715E+01	.2689314E−04

CONCLUDING REMARKS

A new system for the passive control of structural vibration has been presented, and the equation of motion for this system subjected to ground motion excitation has been given. It is evident that it is simple, applicable and the cost is less.

The investigation has clearly denoted the fact that when a pendulum is added to a engineering structure the amplitude of structural respouse can largely be reduced. The optimized pendulum perameters have been determined, which are focused on the case of the steady−state response and their numerical results have been given.

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