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ESTIMATION OF CUMULATIVE DAMAGE OF BEAM-COLUMNS AND FRAMES SUBJECTED TO REPEATED LOADING

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ABSTRACT

A method of estimating the cumulative damage of structures and their members subjected to repeated loading is proposed. Progress in cumulative damage in structures subjected to repeated loading converges or diverges with the increase of the repetition cycle, depending on the magnitude of the constant vertical force. From this fact,, critical axial force for convergence-displacement relation is proposed as a basis for estimating the cumulative damage of structures. In this paper, 1) the critical force for convergence of the reinforced concrete beam-columns are given by tests and analyses, 2) that of deteriorating steel beam-columns subjected to sinusoidal dynamic loading by elasto-plastic analyses and 3) that of frames subjected to constant vertical forces and a repeated horizontal force by finite element method. Usefulness of critical force for convergence - displacement relation for estimating the cumulative damage of structures and their members subjected to repeated loading are verified by these tests and analyses.

KEYWORDS

Critical force; cumulative damage; dynamic excitation; repeated loading; vertical load; beam-column; frame.

INTRODUCTION

The instable behavior such as the local buckling or the crash of concrete often causes strength deterioration in beam-columns subjected to the constant axial force and the repeated horizontal force during an extremely strong earthquake (Fig. 1). Since the deteriorating behavior closely correlates with the accumulation of deformation, it is necessary to investigate the accumulation of deformation. The strength deterioration and the accumulation of deformation in frames and their members subjected to repeated loading are named "cumulative damage" in this study. Progress in cumulative damage in beam-columns subjected to repeated loading converges ("a" in Fig. 2) or diverges ("b" in Fig. 2) with the increase of the repetition cycle, depending on the magnitude of the axial force. From this fact, the convergence boundary of the accumulation of damage is expected to exist. The method of evaluating the cumulative damage of beam-columns and that of frames for controlling the cumulative damage are proposed based on the convergence boundary of the accumulation of deformation in beam-columns or structures subjected to repeated loading. The validity of the proposed method for evaluating the cumulative damage is verified by tests and analyses of reinforced concrete beam-columns, dynamic analyses of steel beam-columns and finite element analyses of steel frames.

EVALUATION OF CUMULATIVE DAMAGE BY CRITICAL FORCE FOR CONVERGENCE

Critical axial force for convergence is a limit value of the axial force when the accumulation of deformation converges, and the relation between critical axial force for convergence and converged axial displacement is proposed for evaluating the cumulative damage of the beam-column such as strength deterioration. The value of the critical axial force of the beam-column is given by the minimum value of the axial force varying while a repeated horizontal force and an axial deformation in the converged state are applied (Uchida *et al.*, 1992, 1995). Since "critical axial force for convergence" is very long, it is termed "critical axial force" hereafter. The relation between the critical axial force and the maximum value of displacement varying in the converged state is defined as critical axial force-axial displacement relation. The critical axial force p-axial displacement δ_v relation of a beam-column subjected to a repeated horizontal force (Fig. 1) is shown in Fig. 3(a). In general critical axial force-axial displacement relation forms the hysteresis loop. The accumulation of deformation and the progress in strength deterioration immediately converges under the constant axial force p_1 and p_1^*, but diverges under the constant axial force p_2 and p_2^* (Fig. 2, 3(a)). Point A in Fig. 3(a) represents a bifurcation point on the path of the accumulation of deformation (Uetani *et al.*, 1983). At the onset of bifurcation, the accumulation mode of deformation changes and the critical axial force deteriorates likewise the bifurcation in the usual load-displacement relation. Horizontal force h-horizontal displacement δ_h relation in Fig. 3(b) represents the resisting capacity for the horizontal force, while the shape of the critical axial force p -vertical displacement δ_v relation in Fig. 3(a) indicates the resisting capacity for the cumulative damage.

CRITICAL AXIAL FORCE OF REINFORCED CONCRETE BEAM-COLUMNS

Methods of Experiment

Reinforced concrete beam-columns under a axial force and a repeated horizontal force were tested for obtaining the critical axial force. Figure 4 shows configurations and dimensions of specimens. Specimens RC1~RC3 are bending failure type specimens(shear reinforcement ratio ρ=0.85%) and Specimens RC4~RC6 shear failure type ones(ρ=0.43%). Experimental dimensions, strength of concrete and material properties of reinforced bars are listed in Table 1., Table 2. and Table 3.,

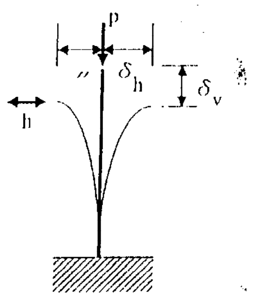


Fig. 1. Beam-column subjected to repeated loading.

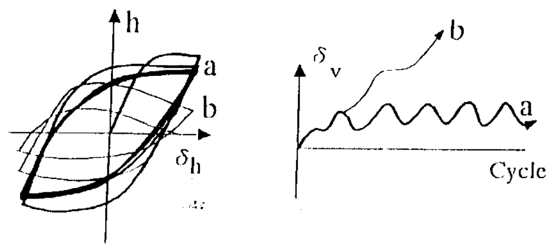
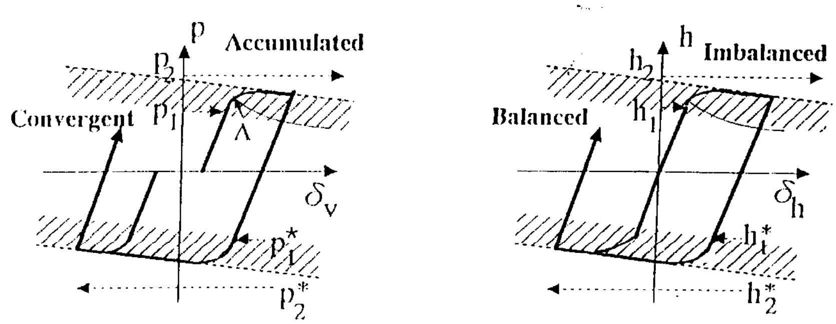


Fig. 2. Cumulative damage.



(a) p-δ_v curve (b) h-δ_h curve

Fig. 3. Critical axial force and horizontal force.

respectively. Figure 5 shows a test set-up in which a specimen is fixed at its end and subjected to vertical and horizontal loads applied by hydraulic jacks. A slider is attached to the top of the hydraulic jack for axial compressive and tensile loadings. The calculation method of critical axial force was presented in the theorem published in Refs. (Uchida *et al.*, 1992, 1995), and validity of the method have been assured by analyses.

Specimens RC2 and RC5 were tested for obtaining the critical axial force for convergence. The critical axial force is given by the minimum value of the axial force varying in the third cycle when a repeated horizontal force and a constant axial displacement are applied. Specimens RC1 and RC6 were tested in order to verify the validity of critical forces. A constant axial force varied in the stepwise manner in the vicinity of the critical axial force-displacement relation was loaded Specimens RC1 and RC6 under a repeated horizontal force with a constant displacement amplitude 10mm.

Test results are exhibited in Figs. 6.~8. Figure 6 shows axial force p-axial displacement δ_v relations in which the maximum value and the minimum value of varying axial force are indicated by ◇-◇ and○-○, respectively. Axial force p is normalized by the ultimate compressive strength of concrete cross section. The critical axial force-displacement relation is expressed by the inner curve formed by ◇-◇ and○-○curves. In Figs. 7(a), (b), p-δ_v relations derived from tests of Specimens RC1 and RC6 under a constant axial force are shown together with the critical axial force-displacement relation presented in Fig. 6. Steps of the constant axial force loading are designated by①~⑩ and

Table 1. Experimental dimensions.

Spec. No.	B (mm)	D (mm)	l (mm)	h (mm)
RC1	149.2	151.2	750.1	549.4
RC2	149.0	153.1	751.7	550.0
RC3	150.1	152.1	751.0	548.7
RC4	150.6	150.5	749.7	549.9
RC5	150.3	150.6	750.6	549.3
RC6	151.7	151.6	749.2	549.1

B=width, D=depth of cross section
l=length, h=length measured for δ_v

Table 2. Strength of concrete.

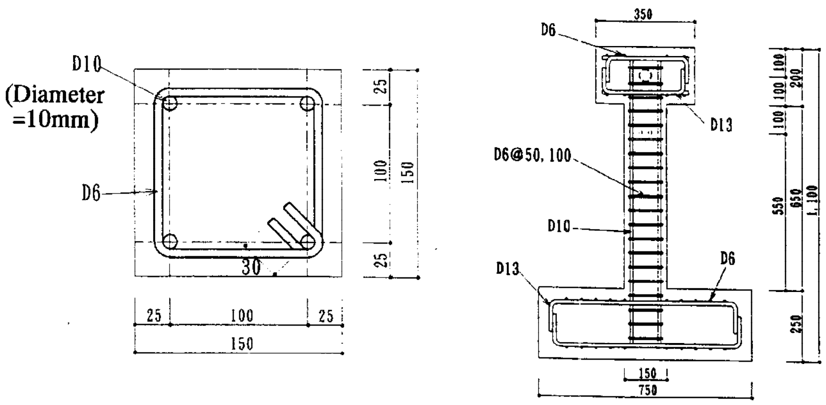
Spec. No.	Age (day)	F_c (kg/cm²)
RC1	25	412
RC2	20	339
RC3	29	407
RC4	56	446
RC5	43	401
RC6	47	420

F_c=compression strength

Table 3. Material properties of re-bars.

Re-Bars	σ_y (t/cm²)	σ_u (t/cm²)	δ (%)
D-6	3.96	6.11	18.4
D-10	3.56	5.22	20.2
D-13	3.44	5.02	19.4

σ_y=yield stress,, σ_u=tensile strength, δ=elongation



(a) Cross section (b) Elevation

Fig. 4. Specimen. (Unit=mm)

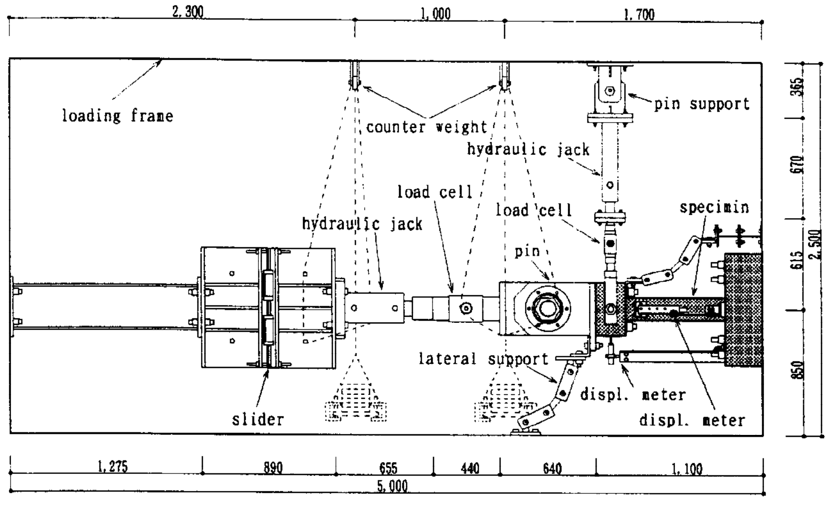


Fig. 5. Test set-up.

①-⑧. Symbols ○ and ● indicate the convergence and the divergence of the accumulation of displacement, respectively. Figures 8(a) and (b) show the horizontal force h-horizontal displacement δ_h and h-axial displacement δ_v relations in loading steps 3 and 4 of Specimen RC1, respectively. Horizontal force h is normalized by the theoretical ultimate strength $\{a_t\, \sigma_y\, (D\text{-}2d_c)+0.12BD^2F_c\}/\ell$, where a_t=total area of cross section of tension reinforced bars, d_c= depth of cover concrete, ℓ=length of specimen. These figures express that the accumulation of displacement converges in the beam-column under the axial force less than the maximum critical force but diverges under the axial force greater than that. Consequently, it is noted from these results that the convergent and divergent behavior of the accumulation of axial displacement can be predicted by the critical axial force for convergence-displacement curve very well.

CRITICAL AXIAL FORCE OF STEEL BEAM-COLUMN SUBJECTED TO DYNAMIC EXCITATION

Calculation of Critical Axial Force

The axial force-axial displacement relation of a steel beam-column subjected to dynamic excitation is presented. Figure 9 shows one lumped mass model with a beam-column composed of a elastic-plastic body and a rigid body analyzed. The cross section of a elastic-plastic body has three elements(Fig.10). The stress-strain relation was assumed to be the degrading type in consideration

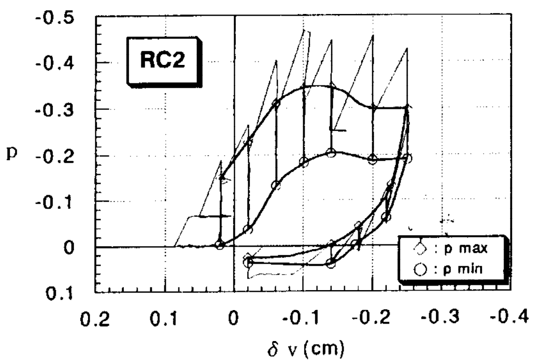
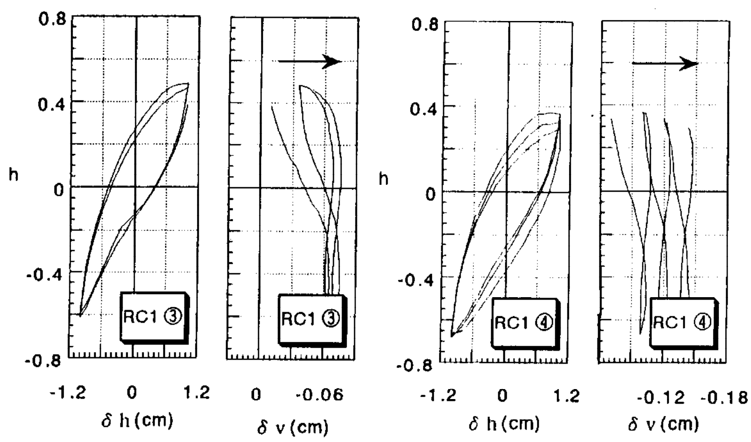
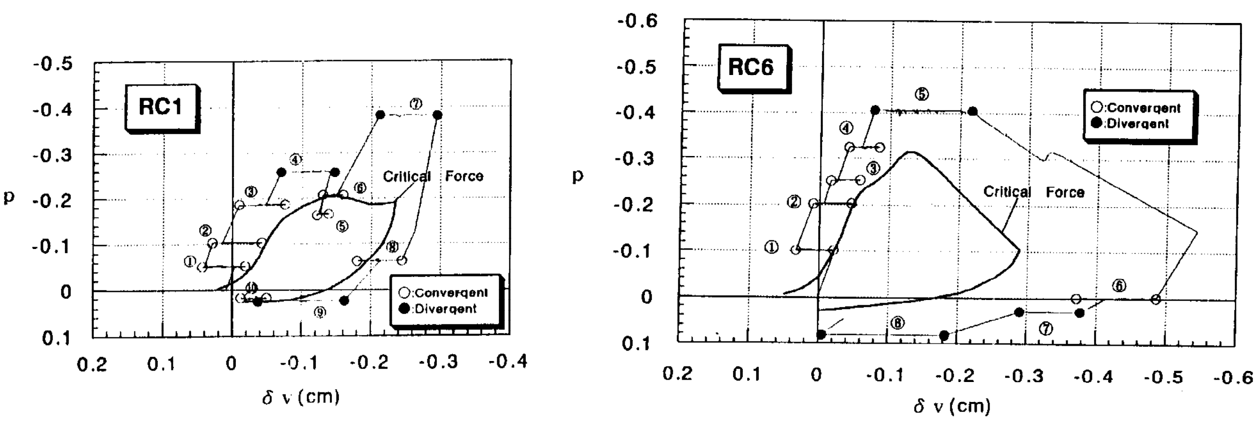


Fig. 6. Axial force-axial displacement relation.



(a) p = -0.18 (b) p = -0.26

Fig. 8. Hysteresis loop of Specimen RC1.



(a) Bending failure (b) Shear failure

Fig. 7. Critical axial force-axial displacement relation.

of the local buckling. The stress s and the strain e are quantities normalized by yield ones.

The critical axial force for convergence of beam-columns subjected to dynamic excitation are given by the following calculation procedure derived from extended application of the theorem in Refs.(Uchida et al., 1992, 1995). The critical axial force for convergence, however, is defined to be the critical one when the dynamic response remains stable but not stationary in case of static loading after the dynamic excitation terminates.

1. Get time-history responses of horizontal displacement d_h and the axial displacement d_v by the dynamic analysis of a beam-column subjected to a dynamic excitation (Fig. 12. (a)).
2. Assume displacements in the converged state of accumulation to be $d_h+\Delta d_h$ and $d_v+\Delta d_v$, where Δd_h and Δd_v are certain amount of displacement increments. The critical axial force for convergence can be derived from a minimum value in the time-history of axial force p of the beam-column subjected to assumed displacements d_{ha} and d_{va} (Fig. 12. (b)).
3. Go to Step 1 and get the time-history of displacement response of a beam-column under a critical axial force obtained in Step 2. The maximum value of displacement response and the critical axial force becomes the critical axial force-axial displacement curve.

The numerical computation method to solve the equation of motion is Newmark's β method, where β is taken equal to 1/4. Incremental method combined with initial stress method is taken for iterative calculation procedure.

Results and Discussions

Analytical parameters are as follows:
$a_1=a_3=0.3$, $a_2=0.4$: Dimensionless area of elements of the cross section shown in Fig. 10.
$d_1=d_3=1$: Dimensionless distance between the center of elements and the centroid of the cross section.
$e_B=-3$, $e_C=-10$: Strain at which degrading of a stress starts as shown in Fig. 11.
$\mu_{c1}=0.02$, $\mu_{c2}=-0.08$, $\mu_{c3}=-0.01$, $\mu=0.02$: Slope of the stress-strain relation shown in Fig. 11.
$\sigma_y = 2.4t/cm^2$: Yield stress. $h=0.01$: Damping ratio.
$\lambda=20, 40, 60, 80$: Slenderness ratio of beam-columns.
$A_r=1.0, 1.5$: Ratio of the intensity of excitation to the initial yield strength of the beam-column under a static horizontal force.
Ground acceleration is sinusoidal with the initial value equal to zero, of which frequency is 0.8 times the natural frequency of the beam-column. Critical axial force-horizontal displacement d_h and axial displacement d_v relations in case of $\lambda=80$ and $A_r=1$ are shown in Figs. 13. (a), (b) . The axial force p with the compression being taken negative is normalized by yield one.

Results of dynamic response analyses of beam-column under the axial force in the vicinity of the maximum critical axial force p_{max} are shown in Fig. 13. Symbols ◎ and ● represent convergence

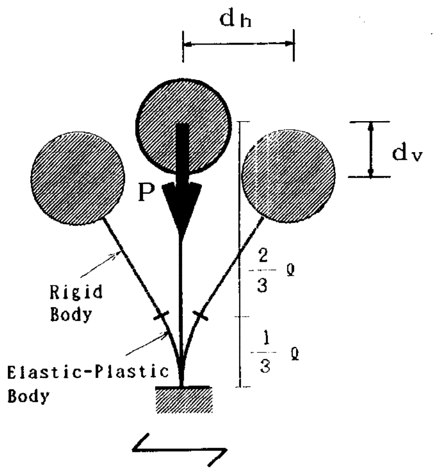


Fig. 9. One lumped mass model.

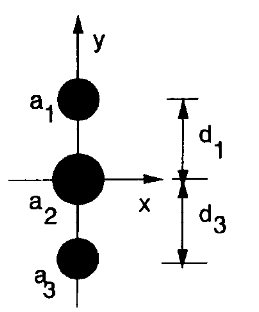


Fig. 10. Cross section of elastic-plastic body.

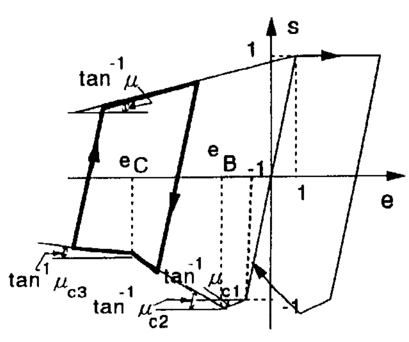


Fig. 11. Stress-strain relation.

and divergence of accumulation of displacements, respectively. The symbol ■ expresses the point that the strength of the beam-column became zero, i.e. the beam-column collapsed. It is noted from Fig. 13 that the accumulation of deformation of the beam-column under the axial force greater than the maximum critical force diverges but converges under that less than the maximum one. Therefore, the validity of obtained critical forces can be assured by these results.

Figure 14. shows the maximum critical axial force p-slenderness ratio λ relation indicated by solid lines and the limitation of axial force prescribed in Ref. (Architectural Institute of Japan, 1990) by a dashed line. The figure expresses that the maximum critical axial forces are remarkably less than the limitation and the value of critical axial force decreases as the value of A_r increases.

CRITICAL VERTICAL FORCE OF STEEL FRAME SUBJECTED TO REPEATED LOADING

Maximum Critical Vertical Force for Convergence of Frame

Steel frames subjected to a constant vertical force and a repeated horizontal force were analyzed by the finite element method in order to investigate the critical vertical force, and a formula for calculating the maximum

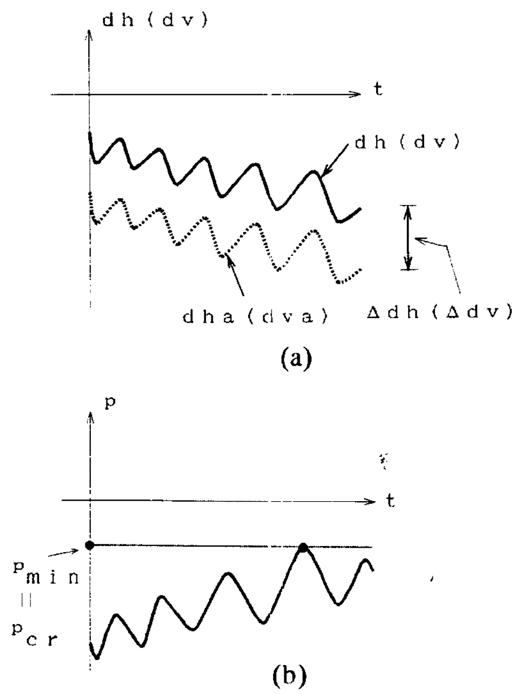


Fig. 12. Assumption of δ_h, δ_v and critical axial force.

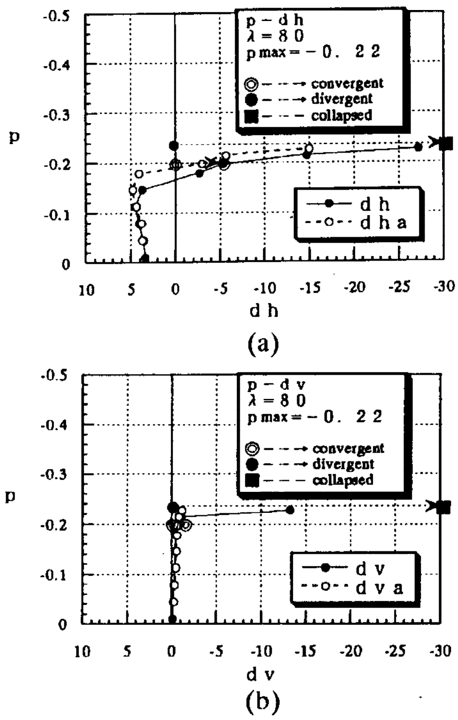


Fig. 13. Critical axial force for convergence.

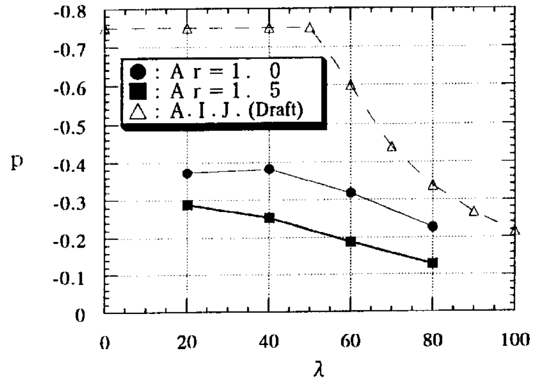


Fig. 14 Limitation of axial force.

critical vertical force is proposed. Figure15. shows an analytical model of the frame subjected to a constant vertical load P_b in the middle of the beam and the repeated horizontal force with a constant displacement at the top of the beam-column in the left-hand side. Vertical forces P_c loaded on the top of beam-columns were taken equal to zero. Members of the frame and the cross section (Fig. 16.) were divided into elements. Figure 17. shows the history of displacement with an amplitude of δ_{ba} given in the finite element analysis using the displacement incremental method. The degrading type of stress s-strain e relation shown in Fig. 18. was adopted for the analysis in consideration of local buckling.

The critical vertical force for convergence was defined as the critical force under the accumulation of deformation converges to a given value. Equation (1) was proposed for the maximum critical vertical force $p_{bcr,max}$ of the model shown in Fig. 15.

$$P_{bcr,max} = 4M_{pb}/\ell_b \qquad (1)$$

where M_{pb} = full plastic moment of the beam, ℓ_b = length of beam.

Analytical parameters are as follows:
CASE1 (Beam-Yield Type Frame, $P_{bcr,max}$ = 10.7t)
b_b x d_b = 10 x 10 cm, b_c x d_c = 10 x 15 cm
CASE 2 (Column-Yield Type Frame, $P_{bcr,max}$ = 24t, 16t (modified in consideration of averaging the distributed moment in plastic elements))
b_b x d_b = 10 x 15 cm, b_c x d_c = 10 x 10 cm
where b, d = width and depth of the cross section , respectively. Subscripts b and c indicate quantities with respect to the beam and the beam-column, respectively.
Followings are other parameters chosen in analyses :
δ_{Ha}=1.5cm, μ_t=0.005, μ_{c1}=-0.005, μ_{c2}=-0.01, μ_{c3}=-0.005, length of beam-column ℓ=100cm, ratio of area of a element to that of gross section =1/3, distance between center of elements of the cross section = d/3, yield stress σ_y =2.4t/cm^2, Young's Modulus E=2100t/cm^2

Results and Discussions

Curves in Fig. 19. are the maximum critical vertical force-horizontal displacement amplitude δ_{Ha} relations obtained from numerical analyses. Upper curves are for column-yield type frames, and lower ones for beam-yield type frames. Solid symbols ●, ■,▲indicate the divergence of displacement accumulation and hollow symbols ○,□,△ the convergence of that. Results of non-deteriorat-

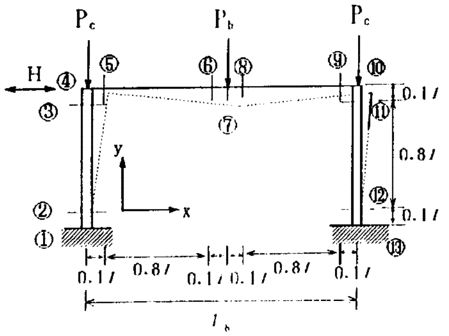


Fig. 15. Analytical model of frame.

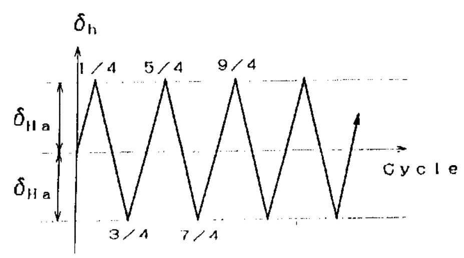


Fig. 17. History of displacement.

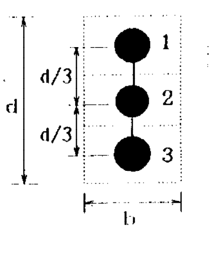


Fig. 16. Cross section.

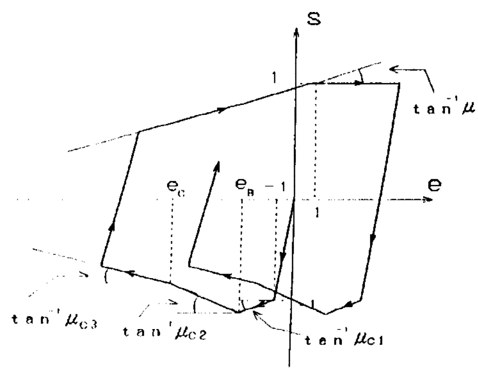


Fig. 18. Stress-strain relation.

ing beam-yield type of frames (μ_c=0.001) are presented in addition to those of deteriorating type in Fig. 19. Thick solid lines indicate the maximum critical vertical forces $P_{bcr,max}$ derived from Eq. (1). $P_{bcr,max}$(C.Y., Modified) in Fig. 19. was calculated with the full plastic moment modified in consideration of averaging the distributed moment of an element in the analysis. Figures 20(a), (b) show the horizontal force H-horizontal displacement at node number 4 δ_h relation, H-vertical displacement at node number 7 δ_v relation of the beam-yield type frames in case of P_b = 9t, 10t, respectively. These results exhibit that the cumulative damage of frames can be evaluated by the maximum critical vertical force obtained from Eq. (1).

CONCLUSIONS

1. Cumulative damage of beam-columns and frames subjected to repeated horizontal loading or dynamic excitation can be predicted fairly well by the proposed critical force for convergence-displacement curves. Therefore, critical force - displacement curve is useful for estimating the cumulative damage.
2. Constant axial force and constant vertical force affect the cumulative damage of beam-columns and frames remarkably.
3. A gap between the applied constant force and the critical force represents the magnitude of cumulative damage.

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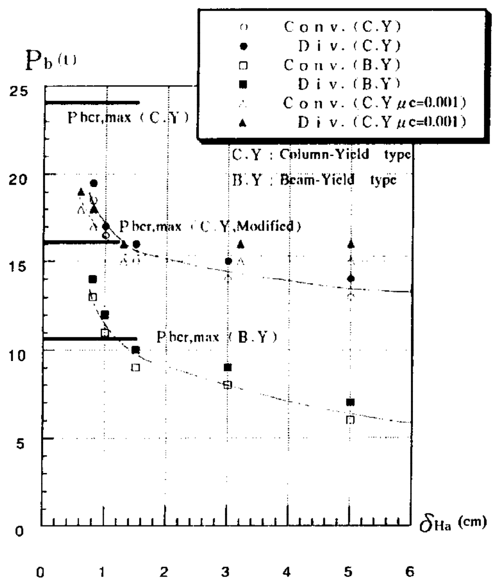
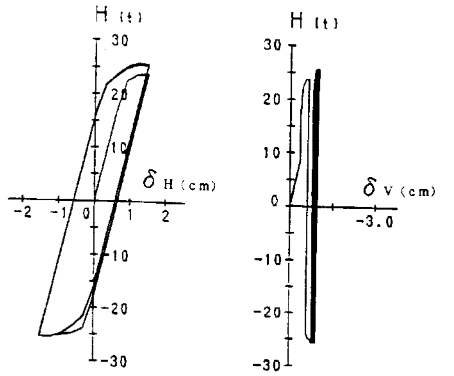
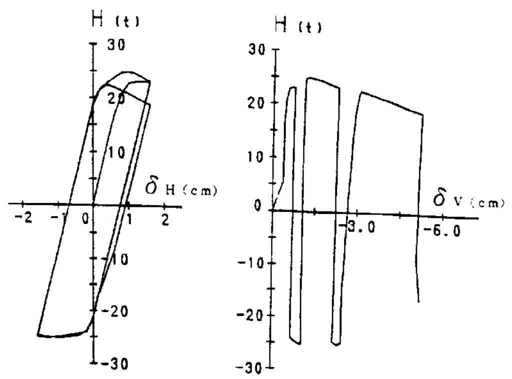


Fig. 19. Maximum critical vertical force-displacement amplitude relation.



(a) p_b = 9t



(b) p_b = 10t

Fig. 20. Cumulative damage of beam-yield type frame.