

## SEISMIC EVALUATION OF REINFORCED CONCRETE STRUCTURES USING MODAL DATA

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### ABSTRACT

This paper describes an investigation into the use of changes in vibrational characteristics to detect and quantify earthquake damage in reinforced concrete structures. Such changes may be quantified by in-situ measurement or by analytical techniques. Several recently developed damage measures based on dynamic behaviour are evaluated, and some new indices are proposed. In particular, attempts are made to make use of changes in the higher modes of vibration, rather than relying on the fundamental alone. The indices are assessed using the results of a series of tests on reinforced concrete beams, in which the changes in vibration characteristics caused by controlled levels of damage were measured.

### KEYWORDS

Seismic damage; structural assessment; damage index; vibrations; modal testing; reinforced concrete structures; bending; shear.

### INTRODUCTION

The seismic evaluation of a structure normally involves making an estimate of the damage caused by a given earthquake (either by analytical prediction or post-earthquake observation), followed by an assessment of the likely consequences of that damage, in terms of structural safety, repair cost etc. One of the major difficulties in this process is the quantification of damage. Various damage indices have been proposed (see Williams and Sexsmith, 1995 for a full review), but most are at an early stage of development, and validation remains limited, particularly for damage modes other than ductile flexure. The objectives of the work described in this paper were therefore:

- (i) To develop damage indices based on dynamic characteristics, which can be readily computed from the results of a dynamic analysis, or from the measured seismic response of an instrumented structure.
- (ii) To contribute to the validation of these indices for a wide variety of damage modes, including ductile and brittle flexure, shear and combined shear-flexure.

Damage detection techniques based on vibration measurements can be divided into two broad categories: global damage indices, giving an overall measure of the structural deterioration, based on changes in the

natural period; and parameters based on changes in the mode shape, which can yield information on both the magnitude of damage and its distribution across the structure. In this paper, several recently developed measures of both types are evaluated and some new indices are proposed. In particular, attempts are made to make use of changes in modes of vibration other than the fundamental, in order to yield more detailed information on damage location.

## DAMAGE INDICES BASED ON VIBRATIONAL CHARACTERISTICS

### Global damage indices

Damage to a structure will generally result in a loss of stiffness and hence in a lengthening of the natural period. DiPasquale and Cakmak (1989) have suggested that changes in the natural period during and after earthquake loading can be used as measures of the seismic damage caused. Such changes can be measured directly in the case of instrumented buildings, or estimated from a non-linear dynamic analysis. They proposed several softening indices based on the initial (undamaged) fundamental period,  $T_1^u$ , the longest period achieved during the earthquake,  $T_1^m$ , and the final period of the damaged structure,  $T_1^f$  (usually rather less than  $T_1^m$ ). In this study, two types of index based on the initial and final periods are examined:

$$D_{s1} = 1 - T_n^u / T_n^f \quad (1)$$

$$D_{s2} = 1 - (T_n^u / T_n^f)^2 \quad (2)$$

where the subscript  $n$  refers to the mode number. The possible values of these indices range from zero for an undamaged structure up to a theoretical maximum of 1. Attempts were made to correlate these indices with damage state using the first three modes of vibration.

A second possible approach to damage quantification is the use of the modal assurance criterion (Ewins, 1984; Salawu and Williams, 1994). The MAC can be defined as:

$$\text{MAC}_n = \frac{\left[ \sum_j (\phi_{nj})_d (\phi_{nj})_u \right]^2}{\left[ \sum_j (\phi_{nj})_d^2 \sum_j (\phi_{nj})_u^2 \right]} \quad (3)$$

Normally the MAC is used for comparing experimental mode shapes with theoretical predictions, but in this case it is used to compare damaged mode shapes with the original undamaged ones. Thus, for our application,  $\phi_{nj}$  is the mode shape coordinate for mode  $n$  at location  $j$ , the subscript  $d$  refers to the particular damage level and  $u$  refers to the undamaged structure. A value of the MAC close to 1 implies little or no damage, while a value close to zero implies severe damage. As with Equations (1) and (2), the MAC can be computed independently for different modes.

A related measure, the normalised area difference (NAD) is proposed here. This is defined as the sum of the differences in value between normalised mode shapes of the damaged and undamaged structure, that is:

$$\text{NAD}_n = \int |\phi_d - \phi_u| dx \quad (4)$$

## Damage location techniques

The most obvious method of locating damage using vibration characteristics is simply to examine the changes in the mode shape,  $\delta\phi$ , caused by the damage:

$$\delta\phi_{nj} = (\phi_{nj})_d - (\phi_{nj})_u \quad (5)$$

Salawu and Williams (1994) suggested that, since the mode shape changes tend to be greatest around the nodes, a clearer indication can be obtained by dividing by the original mode shape, giving the relative difference (RD):

$$RD_{nj} = \frac{(\phi_{nj})_d - (\phi_{nj})_u}{(\phi_{nj})_u} \quad (6)$$

The above indicators can be computed for each of several measured modes of vibration. Many researchers have recognized that a better approach is to combine data for the various modes to give a single parameter. One way of doing this is by using the coordinate modal assurance criterion (COMAC):

$$COMAC_j = \frac{\left[ \sum_n (\phi_{nj})_d (\phi_{nj})_u \right]^2}{\left[ \sum_n (\phi_{nj})_d^2 \sum_n (\phi_{nj})_u^2 \right]} \quad (7)$$

Another approach to combining data for the various modes is to compute the flexibility,  $H$ , defined as (Ragavendrachar and Aktan, 1992):

$$H_j = \sum_n \left( \frac{\phi_{nj}}{f_n} \right)^2 \quad (8)$$

The usefulness of Equations (7) and (8) is likely to increase if a large number of modes of vibration can be accurately determined. However, in this study it was possible to make use of only the first three modes.

## REINFORCED CONCRETE BEAM TESTS

The indices outlined above are at a relatively early stage of development and validation. It was therefore felt to be appropriate to assess them using tests on very simple elements rather than on complex structures. For this reason, testing was restricted to simply supported reinforced concrete beams. In order to test the ability of the indices to cover a broad range of failure modes, beams were designed to fail in ductile flexure (under-reinforced), brittle flexure (over-reinforced) or shear. The dimensions of the beams were: length 2.2m; overall depth 140 mm; and breadth 100 mm. Nominal bending capacities were 10.9 kNm for the under-reinforced beams and 13.9 kNm for the over-reinforced, while calculated shear strengths for those beams intended to fail in shear varied from 16.8 to 20.2 kN.

Two types of loading were used, a central point load and four-point bending, the latter giving a region of constant bending moment over the middle third of the beam (see Fig. 1). The aim of this was to inflict both highly localised and more widely distributed damage on the beams.

The testing procedure was the same in all cases. Modal characteristics (natural frequencies, mode shapes, damping ratios) were first determined by applying the instrumented hammer procedure (Maguire and Severn, 1987) at eight equally spaced points along the length of the beam. The beam was then loaded at gradually

increasing amplitude until a certain damage level was reached. The load was then removed, the residual midspan deflection of the beam measured and the hammer test repeated. This procedure was repeated at increasing damage levels up to complete collapse. The damage states used in the tests were as follows:

- 0 Undamaged.
- 1 Light damage, defined as either the first yield of the flexural reinforcement or the appearance of the first shear crack.
- 2 Severe damage, likely to be irreparable, defined as the development of very large cracks and/or crushing of concrete in the compression zone.
- 3 Collapse, defined as gross deformation of the beam, extensive spalling of concrete and debonding of reinforcing bars.

Typical damage patterns corresponding to levels 1, 2 and 3 are depicted in Fig. 2, which shows the development of damage in a shear-deficient beam subjected to four-point loading.

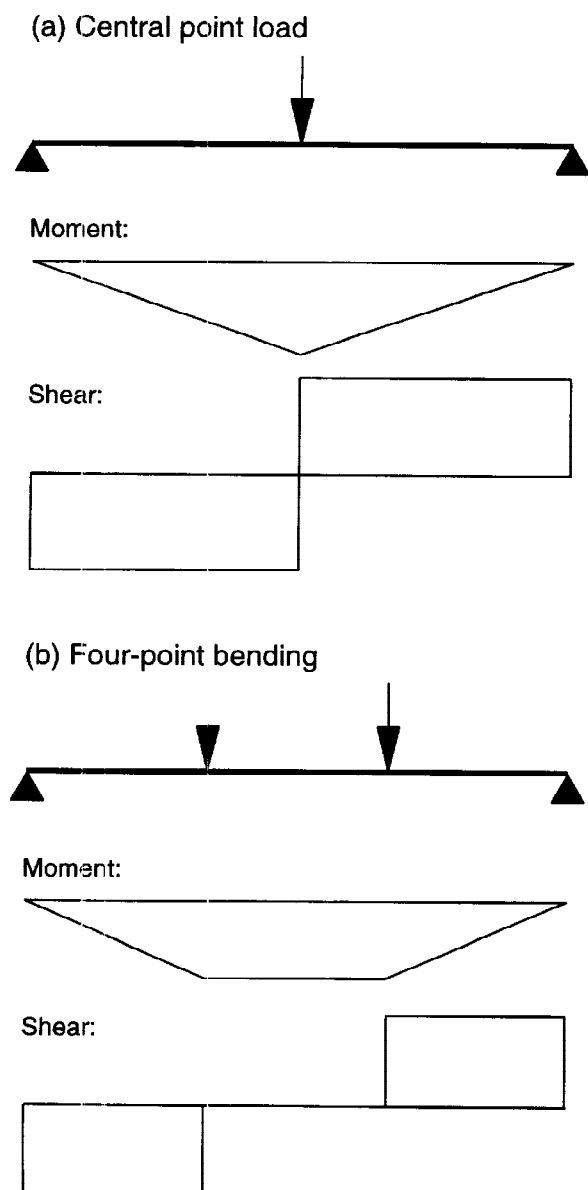


Fig. 1 Loading configurations

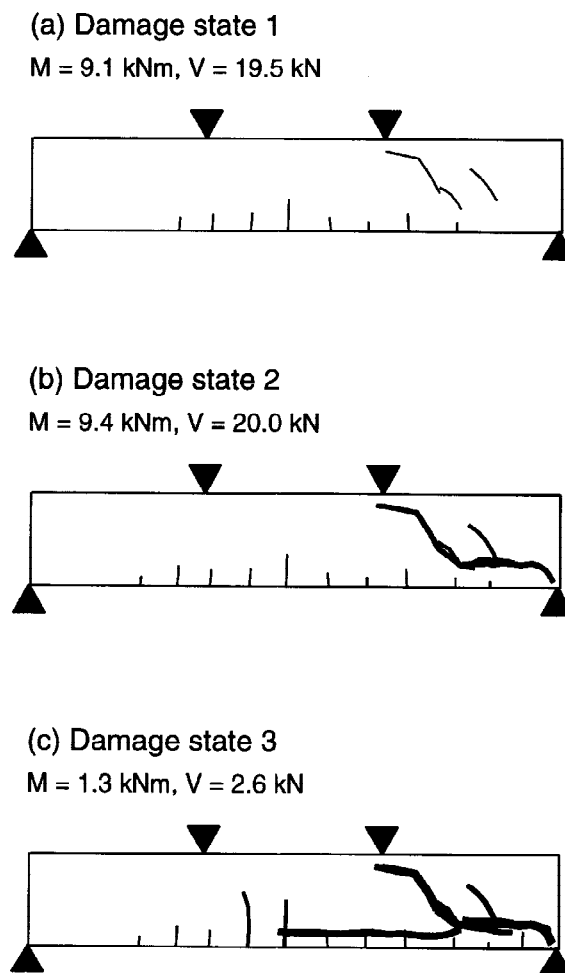


Fig. 2 Damage states 1, 2 and 3 for Test 2b (Shear-deficient beam with 4-point loading)

## RESULTS

### Natural frequencies and mode shapes

In general, increases in damage level resulted in reductions in the natural frequencies of the beams in each of the first three modes of vibration. For example, Fig. 3 shows the variation of the natural frequencies with damage state for four tests on beams failing in flexure. There is a reasonably consistent downward trend in both the first and third modes, with the second mode giving somewhat less clear results. This is presumably because the majority of the damage in these tests was located near the centre of the beam, which is a node point for the second mode.

It is also possible to plot the variation in mode shape through the tests, though these data are generally quite difficult to interpret. Fig. 4 shows results for the first two modes of vibration of the shear-deficient beam 2b, whose failure mode is illustrated in Fig. 2. It can be seen that the extensive shear damage to the right half of the beam caused a significant change in the first mode shape only in the collapse damage state, while the changes in the second mode shape are difficult to relate to the observed damage.

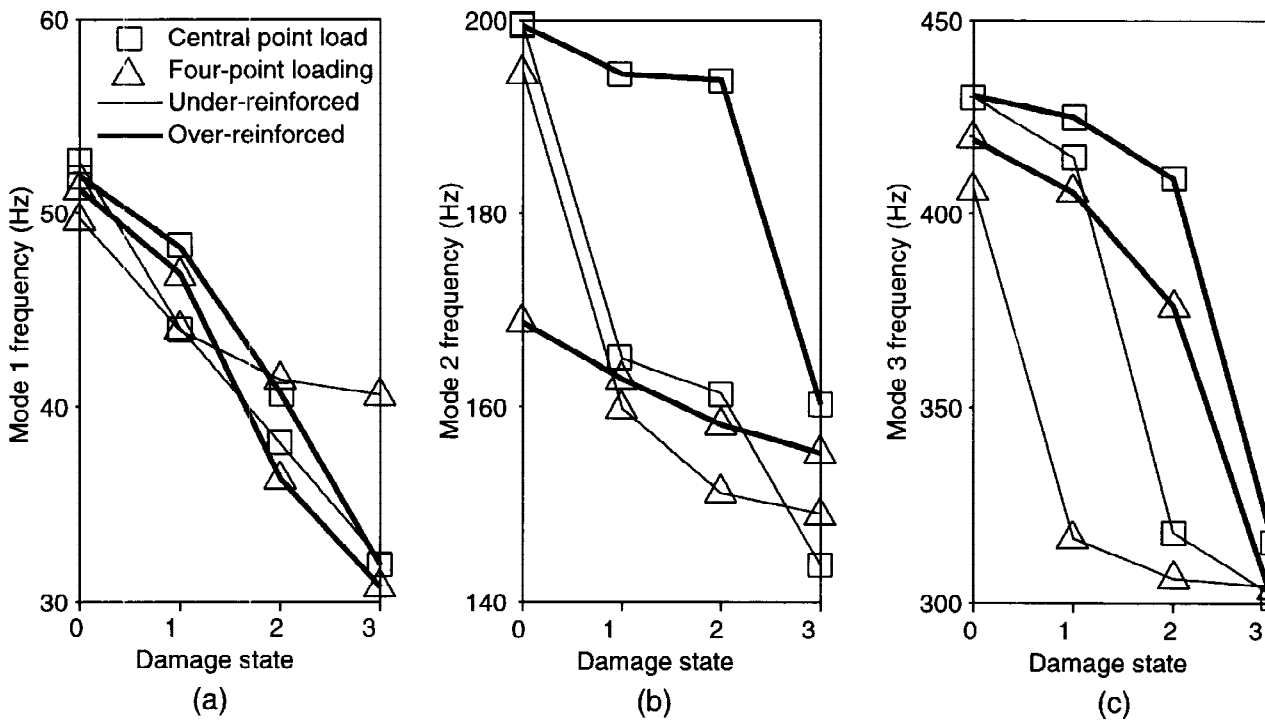


Fig. 3 Variation of frequency with damage state in flexural tests, (a) mode 1, (b) mode 2, and (c) mode 3

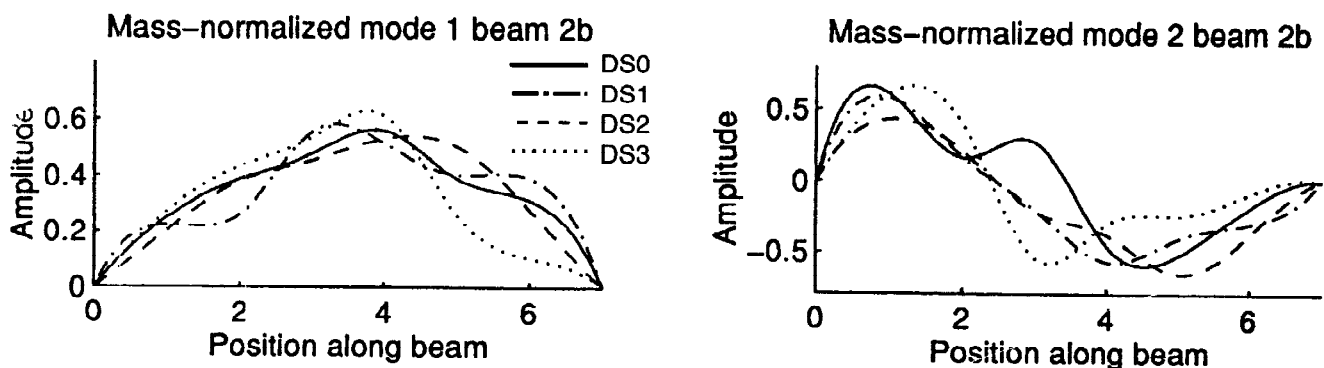


Fig. 4 Variation of mode shape with damage state for Test 2b (shear-deficient beam with 4-point loading)

## Global damage indices

Being closely related to frequency, both of the softening indices, Equations (1) and (2), showed a strong upward trend with damage level, the higher exponent used in Equation (2) giving a slightly clearer distinction between damage levels. Values of  $D_{s2}$  based on the first two modes of vibration are shown in Fig. 5, from which it is clear that the first mode yields much more consistent results. The difference between beams suffering shear and flexural failure modes is also clearly visible in these graphs, with the more brittle shear failures generally yielding very low values at damage state 1, but considerably higher values than the flexural tests at damage state 3. It is therefore difficult to assign reliable values of the index to the different damage states, making it of limited use as an assessment tool.

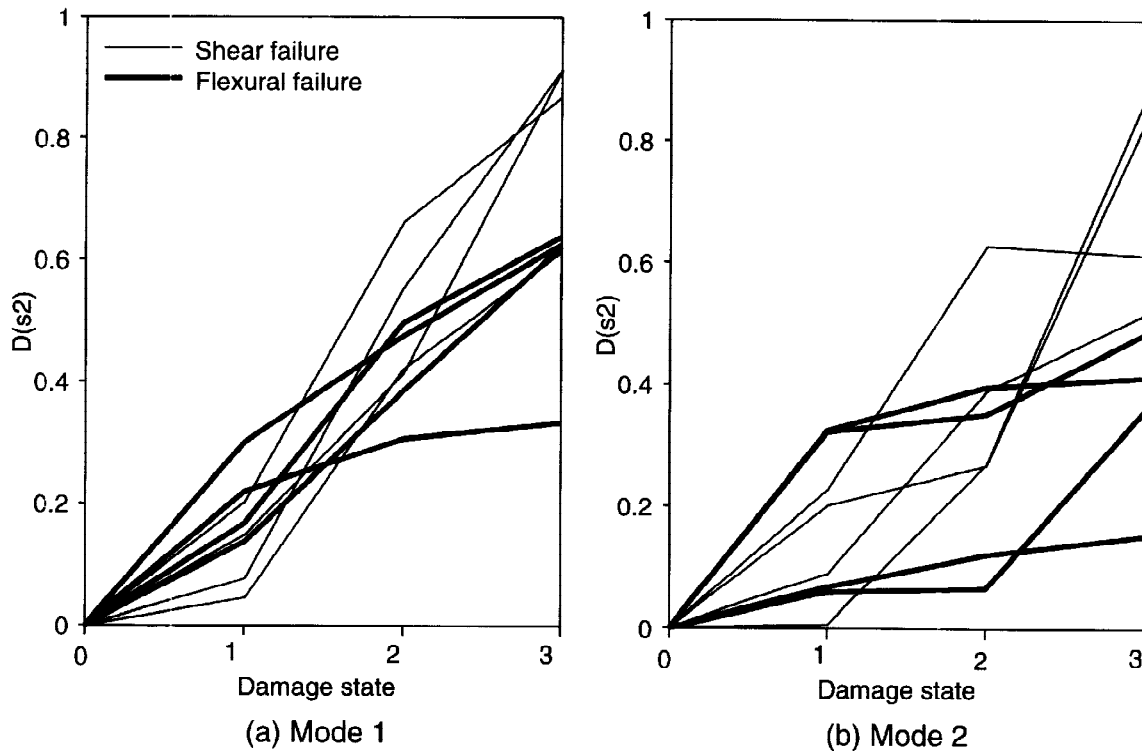


Fig. 5 Values of softening index  $D_{s2}$  (Equation 2) for flexural and shear tests

Results for the modal assurance criterion (MAC) and normalised area difference (NAD) are presented for the first mode only, in Figs. 6 and 7 respectively. The MAC takes a value of 1.0 for no damage and shows only quite small changes through the tests, staying above 0.9 in all cases. As with the softening indices, it yields greater changes of value for severe shear damage than for flexural failure. Rather more substantial reductions in value were evident in the MAC for the higher modes (not shown here), with the third mode appearing to show considerable sensitivity to low levels of damage, but then little further change as the damage level increased. Nevertheless, the MAC generally does not appear to be sufficiently sensitive to be useful for damage assessment of concrete structures.

The performance of the NAD, a somewhat simpler parameter than the MAC, is more promising, as can be seen in Fig. 7. The upward trend is far more marked, the sensitivity to low levels of damage is greater, and the discrepancy between the shear and flexural test results is far less obvious than for the other measures. However, the spread of values is quite wide and, unlike the other indices, there is no theoretical limiting value which denotes complete collapse. The second mode results were broadly similar to those for the fundamental mode, while the third mode again exhibited a relatively high sensitivity to the lower levels of damage.

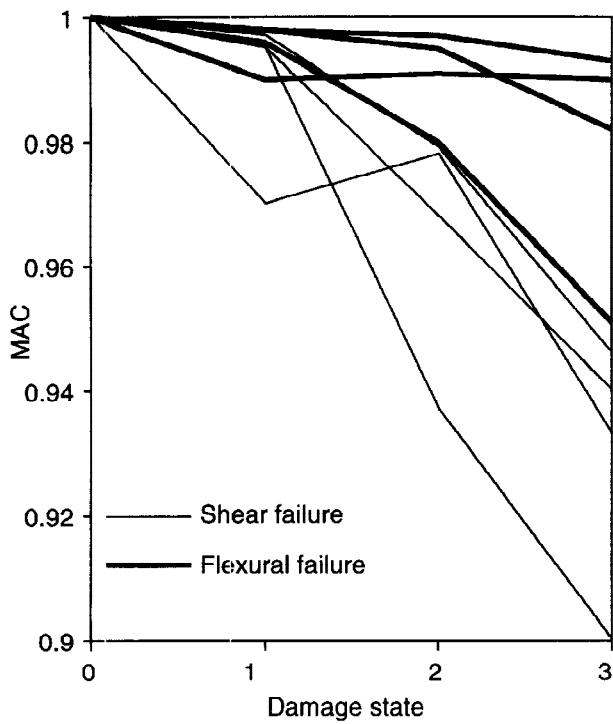


Fig. 6 Modal assurance criterion for mode 1

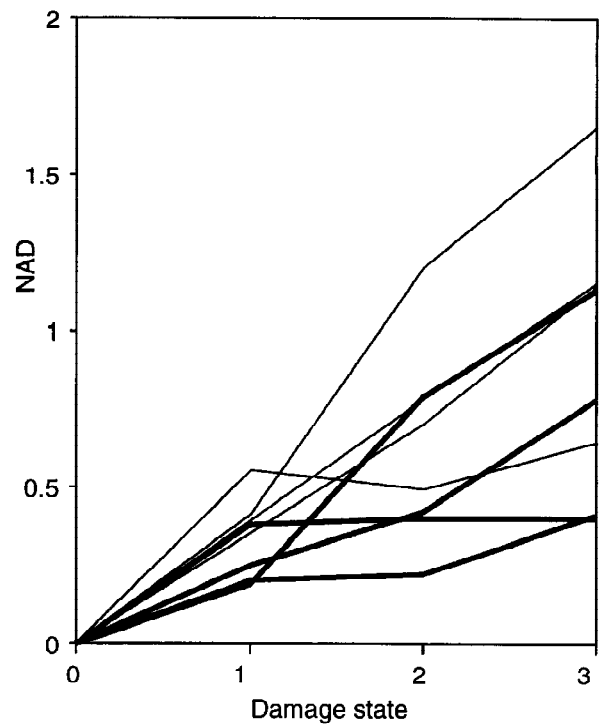


Fig. 7 Normalised area difference for mode 1

### Damage location techniques

In assessing the damage location techniques, it is more instructive to study the shear-dominated tests, as these resulted in varying damage positions, whereas the major damage in the flexural tests always occurred at midspan. Both of the mode shape difference methods, Equations (5) and (6), gave similar results. Fig. 8 shows the difference in normalised first mode shape,  $\delta\phi$ , for four shear tests, with the area of maximum observed damage indicated by the horizontal bar. It can be seen that the method succeeds in locating the general area of damage in each case, but only at the collapse damage state. The higher modes gave similar, though slightly less consistent, results. A method is needed which can identify lower levels of damage.

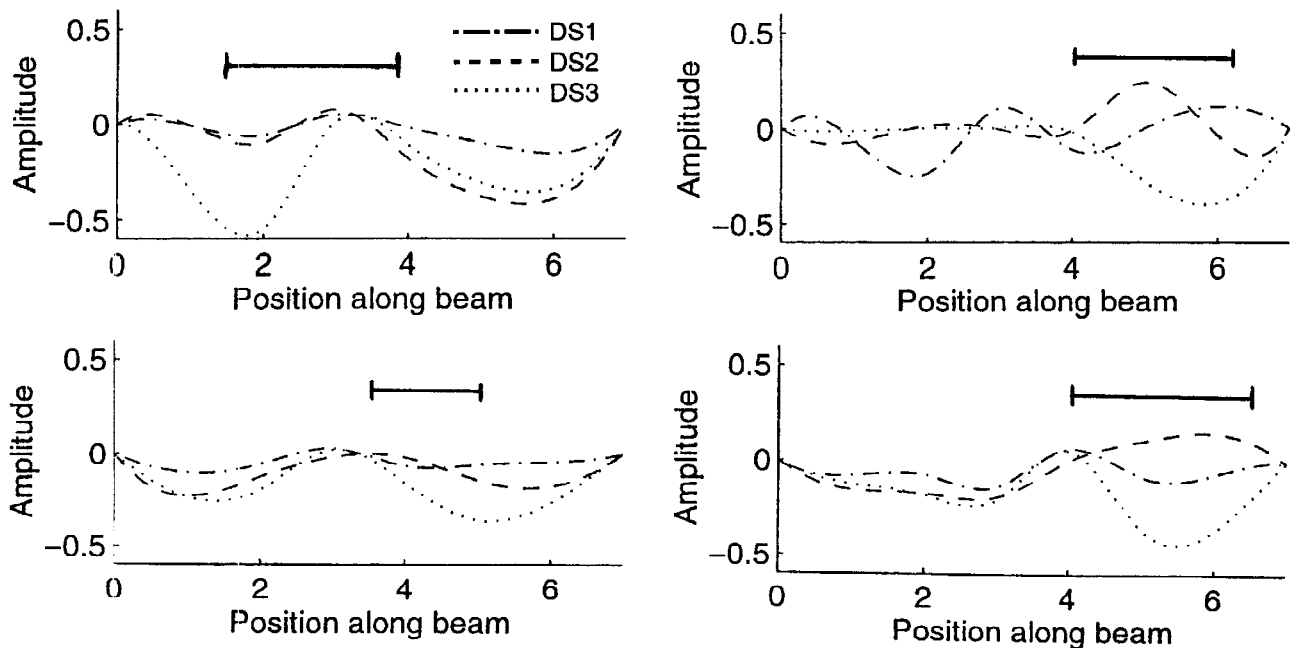


Fig. 8 Mode shape difference plots (Equation 5) for four beams failing in shear

Of the two methods of combining data from the various measured modes the COMAC, Equation (7), yielded very poor results, with a clear and correct indication of the damage location obtained in only one of the shear tests. Rather more success was achieved with the flexibility, Equation (8), perhaps because the  $1/f^2$  term gives a high weighting to the fundamental mode. Fig. 9 shows the changes in flexibility from the undamaged state measured during four shear tests (all normalised to a maximum of 1.0), with the areas of observed damage again indicate by horizontal bars. These results are an improvement on the mode shape difference methods as it is possible to approximate the damage location in both damage states 2 and 3.

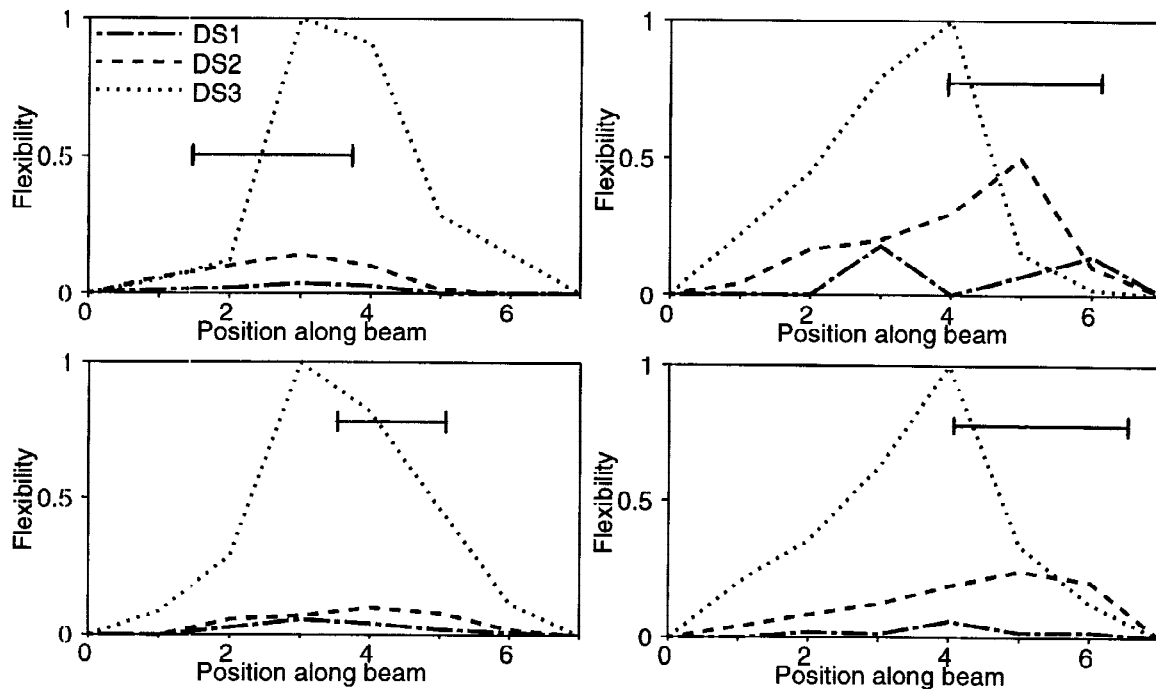


Fig. 9 Flexibility plots (Equation 8) for four beams failing in shear

## CONCLUSIONS

These results show that relatively high levels of damage cause significant changes in the dynamic characteristics of reinforced concrete elements. Given sufficiently high quality data, it should be possible to use these changes to estimate both the magnitude and location of the damage. However, it is clear that reinforced concrete structures present significant difficulties in obtaining top quality data, and that the interpretation of results is complicated by the existence of more than one possible failure mode. Nevertheless, several of the measures presented above (softening indices, normalised area difference, flexibility) have the potential to become powerful assessment tools in the near future.

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