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SEMI-ACTIVE TUNED MASS DAMPERS WITH PULSE CONTROL

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ABSTRACT

A semi-active tuned mass damper (TMD) with pulse generators is proposed. First, desirable TMD motion to maximize the exponential decay rate of the free response of the structure is derived using the analytical perturbation solutions of vibration modes of TMD/single degree of freedom (SDOF) structure system. Then, the pulse force to achieve this TMD motion is expressed analytically with the assumption that the duration of pulse is much shorter than the natural period of the structure. The proposed control scheme is simple and transparent, because the specification of control effect is given in terms of the exponential decay rate of the structure. To make comparison between the proposed method and the conventional direct pulse control where the structure is directly controlled by pulse force, an algorithm for direct pulse control whose performance can be specified by the exponential decay rate is also developed. Numerical studies show that the proposed method needs much less control effort than the direct pulse control. The proposed algorithm is generalized to control of structures with widely spaced natural frequencies using the single mode approximation, and its efficiency is confirmed by numerical simulations.

KEYWORDS

Dynamic vibration absorbers; perturbation solutions; pulse control; seismic protection; semi-active control; structural control; tuned mass dampers; vibration control.

INTRODUCTION

Pulse control of civil structures (Figure 1a) has been shown to be effective in suppressing structural response subject to non-stationary disturbances (Masri, et. al., 1981, Udwadia and Tabaie, 1981a,b), and the algorithm is extended to the seismic control problem of nonlinear structures (Masri, et. al., 1982, Reinhorn, et. al., 1987). Although pulse generators can be assembled for experimental scale (Safford and Masri, 1974, Masri and Safford, 1976, Miller, et. al., 1989), pulse control has not yet been applied to real civil structures where larger pulse forces are required. Development of new control algorithms which require limited amount of pulse force is necessary to make pulse control feasible.

A tuned mass damper (TMD) is a commonly used device to control civil structures. Although TMDs are very effective in suppressing vibrations caused by stationary disturbances, its performance to control seismic response is relatively limited (Kaynia, et. al., 1981, Sinha and Igusa, 1995). The reason is that TMDs usually need some time interval before they become fully operational. Active TMDs are one of the solutions to overcome this drawback (Chang and Soong, 1980). Several control algorithms for active TMDs are proposed to improve efficiency, while preserving the full advantage of passive TMDs (Mita and Kaneko, 1992, Nishimura, et. al., 1993, Watanabe and Yoshida, 1994). Semi-active TMDs, which require only limited amount of control force, are also extensively investigated and various control strategies, such as control with initial TMD displacement (Ohrui, et. al., 1992, Abé and Igusa, 1995) and control with variable damping components (Hvorat, et. al., 1983, Abé and Igusa, 1995), have been proven to be effective. The current paper proposes another semi-active strategy for TMDs which uses pulse generators. Pulse generators are employed to control motion of TMD and make TMD effective (see Figure 1b) rather than to control structures directly (see Figure 1a), in order to reduce necessary control forces. Using the perturbation solutions of transient response of the TMD/single degree of freedom (SDOF) structure system (Abé 1995, Abé and Igusa 1995), pulse force to maximize the damping ratio of the structural response is derived.

The paper starts with development of control strategies. First, a simple direct pulse control algorithm for SDOF structures is developed. This algorithm is basically an extension of the control law of Masri et. al. (1982). Then the perturbation solutions for the TMD/SDOF structure system are briefly reviewed and a control algorithm for semi-active TMDs is derived analytically. Fundamental characteristics of the proposed algorithm are explained using impulse responses of SDOF structures. Since the performance of control is given by system damping ratio in both algorithms, design of control algorithms is straightforward once the specification of seismic performance of structures is given in terms of damping ratio, which can easily be specified by the response spectra of the design earthquake. The design procedure and the seismic performance of the proposed method for multiple degrees of freedom systems are also demonstrated with numerical simulations of a cantilever beam subject to the El Centro earthquake. The proposed strategy is shown to be much more effective than conventional passive TMDs and requires less control effort than the conventional pulse control.

CONTROL ALGORITHMS

In this section, two pulse control algorithms are developed: (i) direct pulse control of SDOF structures (Figure 1a) where pulse generators are used to control structures directly; and (ii) indirect pulse control via semi-active TMDs (Figure 1b) where TMDs are driven by pulse to enhance performance.

Direct pulse control of SDOF structures

Consider a linear SDOF structure with mass m_s, natural frequency ω_s and no damping. Displacement of the structure x is taken relative to the ground. Pulse is applied at the zero-crossings of displacement x, where velocity $\dot{x}$ is maximum, to make energy dissipation most efficient (Masri, et. al. 1982). A direct pulse control algorithm to give similar free response of the same SDOF system with damping ratio ζ_e is derived as follows. It is well known that with damping ratio ζ_e, the amplitude of free response decays by the factor of $\exp(-\pi\zeta_e)$ at each zero-crossing. Assuming that the duration of pulse Δt is much shorter than the natural period of the structure, the necessary pulse force u to give the equivalent damping ratio of ζ_e yields;

$$u = m_s\left(e^{-\pi\zeta_e} - 1\right)\dot{x} / \Delta t \text{, at each zero-crossing of } x. \tag{1}$$

Impulse response of the SDOF structure with $\zeta_e = 0.1$ is shown in Figure 2a. The structural response is given by square root of structural energy normalized by initial structural energy, which is basically the envelop process of the response. Time is normalized by the natural period of the structure. Pulse duration Δt is taken to be one fiftieth of the natural period. The simulation results show that the response by proposed pulse control algorithm closely approximates the response of an SDOF structure with damping ratio ζ_e. The associated pulse force normalized by the structural mass m_s and initial velocity is also shown in Figure 2b.

Semi-active tuned mass dampers with pulse control

Consider the TMD/SDOF structure system which is shown in Figure 1b. Pulse generators are attached to the TMD. The TMD has the mass of m_t, the natural frequency of ω_t, and the damping ratio of ζ. The TMD displacement y is taken relative to the structure. The TMD natural frequency is set to $\omega_t = \omega_s / (1 + \mu)$ by the optimal tuning ratio given by Den Hartog (1956), where $\mu = m_t / m_s$. Pulse is applied to the TMD at zero-crossings of displacement y to maximize the control effect on TMD motion.

Using a perturbation analysis (Abé 1995, Abé and Igusa 1995) which assumes that ζ and μ are the small parameters, the two modal frequencies and damping ratios of the system can be derived as a function of ζ as,

$$\omega_{1,2} = \omega_a(1 \pm \mathrm{Im}\gamma/2),\ \ \zeta_{1,2} = (\zeta \pm \mathrm{Re}\gamma)/2 \tag{3}$$

where $\omega_a = (\omega_s+\omega_t)/2$ and $\gamma = \sqrt{\zeta^2 - \mu}$. Both modal damping ratios are monotonically increasing in terms of ζ until $\zeta = \sqrt{\mu}$. When $\zeta > \sqrt{\mu}$, ζ_1 still increases while ζ_2 approaches to zero. This is the reason why a TMD has the optimal damping ratio. Although there are several criteria to define optimality of TMDs (Fujino and Abé 1993), the value of ζ to maximize modal damping ratios is employed in this paper, i.e., $\zeta = \sqrt{\mu}$. By this value of ζ, the modal damping ratios will be $\zeta_1 = \zeta_2 = \sqrt{\mu}/2$.

Using these perturbation solutions, free response with the initial condition of $x(0) = x_0$, $\dot{x}(0) = \dot{x}_0$, $y(0) = y_0$, $\dot{y}(0) = \dot{y}_0$, can also be derived (Abé and Igusa 1995). The response at zero-crossings of y is considered as the initial condition of the following half cycle. Because the responses of the TMD and the structure is approximately 90 degrees out of phase (Abé 1995), $y_0 = 0$ implies $\dot{x}_0 \approx 0$ at each zero-crossing. Substituting the condition $\dot{x}_0 = y_0 = 0$, one obtains time domain solutions for the following three cases:

<u>Case I: $\zeta < \sqrt{\mu}$.</u> In this case, the both modal damping ratios are lower than the passive optimal values as noted previously, so this value of ζ will not be used in the semi-active control.

<u>Case II: $\zeta = \sqrt{\mu}$.</u> The solution for free response of the structure yields;

$$x(t) = \left[x_0 + \frac{\sqrt{\mu}\,\omega_a x_0 - \mu \dot{y}_0}{2}\, t\right] \exp\!\left(-\frac{\sqrt{\mu}\,\omega_a t}{2}\right) \cos(\omega_a t). \tag{4}$$

This solution corresponds to the passive optimal configuration as noted above. Although both modal damping ratios appear to be equal to $\sqrt{\mu}/2$, this modal interpretation is not entirely appropriate because of the factor t appearing in the second term of the amplitude.

<u>Case III: $\zeta > \sqrt{\mu}$.</u>

$$x(t) = \frac{\cos(\omega_a t)}{2\gamma}\left\{\left[(-\zeta+\gamma)x_0 + \frac{\mu \dot{y}_0}{\omega_a}\right] \exp\!\left(\frac{-\zeta-\gamma}{2}\omega_a t\right) + \left[(\zeta+\gamma)x_0 - \frac{\mu \dot{y}_0}{\omega_a}\right] \exp\!\left(\frac{-\zeta+\gamma}{2}\omega_a t\right)\right\} \tag{5}$$

Although the first term has damping ratio of $\zeta_1 = (\zeta + \mathrm{Re}\gamma)/2$, the second term is lowly damped with damping ratio of $\zeta_2 = (\zeta - \mathrm{Re}\gamma)/2$. Hence, the performance of TMD cannot be enhanced by increasing ζ.

Consider Eq. (5) of case III. Idealistically, $\dot{y}$ can be changed instantly by introducing pulse. If $\dot{y}$ is adjusted to

$$\dot{y}^* = \omega_a(\zeta + \gamma)x_0 / \mu \tag{6a}$$

at the zero-crossing of y, the lowly damped term can be eliminated, and the remaining response becomes,

$$x(t) = x_0 \exp\left[\frac{-\zeta-\gamma}{2}\,\omega_a t\right] \sin(\omega_a t) \tag{6b}$$

Here, the structural response decays by the highest modal damping ratio. Similar argument holds for Eq. (4) of case II. The adjusted velocity and response will be;

$$\dot{y}^* = \omega_a x_0 / \sqrt{\mu}\,, \quad x(t) = x_0 \exp\left[-\tfrac{1}{2}\sqrt{\mu}\,\omega_a t\right] \cos(\omega_a t) \tag{7a,b}$$

which can also be derived from Eq. (6) by taking the limit of $\zeta \rightarrow \sqrt{\mu}$. The TMD velocity can be adjusted to $\dot{y}^*$ by applying a pulse of

$$u = m_t(\dot{y}^* - \dot{y}_0) / \Delta t\,, \tag{8}$$

using balance of momentum. Here, pulse is used to adjust TMD with expectation to make TMD more efficient rather than controlling structure itself. If the specification for response is given by equivalent damping ratio ζ_e and $\zeta_e \geq \sqrt{\mu}/2$, the damping ratio in Eq. (6b) should satisfy the relationship of

$$\zeta_e = (\zeta + \gamma)/2. \tag{9}$$

Solving Eq. (9) for ζ gives,

$$\zeta = \left(4\zeta_e^2 + \mu\right)/\left(4\zeta_e\right) \tag{10}$$

Using these relationships, the design procedure for the semi-active TMD can be constructed as follows:

1. Decide the specifications for the equivalent damping ratio ζ_e , the mass ratio μ and the duration of pulse Δt.

2a. If $\zeta_e < \sqrt{\mu}/2$, it is possible to reduce μ up to $\mu = 4\zeta_e^2$. Go back to step 1.

2b. If $\zeta_e = \sqrt{\mu}/2$, use Eq. (7a) and Eq. (8) to construct control algorithm.

2c. If $\zeta_e > \sqrt{\mu}/2$, obtain damping ratio of TMD ζ by Eq. (10), and use Eq. (6a) and Eq. (8) to construct control algorithm.

The design of step 2b is conservative in the sense that TMD works as good as optimally designed passive one even the pulse generator fails.

Results of numerical simulations are given in Figure 3 for $\mu = 0.01$. Two sets of equivalent damping ratios are considered: i.e. $\zeta_e = 0.05$ and 0.1. Duration of pulse Δt is one fiftieth of the natural period. The values $\zeta_e = 0.05$ and 0.1 correspond to $\zeta = 0.1$ and 0.125 respectively, by the relation of Eq. (10). The impulse response subject to sudden base motion is given in Figure 3a. It is observed that the proposed method gives very similar response to the SDOF structure with the damping ratio of ζ_e , which verifies the validity of the design procedure. The structural response with the passive optimal TMD is also plotted in the same figure. It can be seen that the response is much larger than that of the SDOF structure with $\zeta_e = 0.05$, although both modal damping ratios of the structure/TMD system are 0.05. The term with t appearing in Eq. (4) causes this effect. The pulse force normalized by m_s and the initial velocity is given in Figure 3b for the case of $\zeta_e =$ 0.1, which is the same as the previous example of Figure 2. The required control effort for the semi-active TMD is much smaller than that of the direct pulse control, because all the energy need to be dissipated by pulse force in the direct pulse control, while most of the energy is dissipated by the TMD in the semi-active TMD.

To make precise comparison of control effort of both methods, total control cost which is defined by the average control force per unit mass is introduced:

$$\bar{u} = \frac{1}{m_s T}\int_0^T |u| dt \,, \tag{11}$$

where T is the time of interest, which is taken as first 10 cycles in this case. The value of $\bar{u}$ for unit impulse input by the direct pulse control is 0.102 and 0.0208 by the semi-active TMD, in the case of $\zeta_e = 0.1$. Control cost for the semi-active TMD is about one fifth of the direct pulse control.

EXAMPLE: A CANTILEVER BEAM SUBJECT TO EARTHQUAKE EXCITATION

In this section, how the results of this paper can be applied to the vibration control of continuous structures is shown. Let the natural frequencies, damping ratios, modal masses, and mode shapes of the structure be denoted by ω_i, ζ_i, m_i, and $\phi_i(z)$, respectively, where z is a spatial coordinate. Previous studies have shown that the TMD/continuous structure system can be reduced to a single-degree-of-freedom structures controlled by a TMD, when the structural natural frequencies are widely spaced (Abé and Igusa 1994). If the first structural mode is to be controlled by a TMD, the same values for the damping ratio and the natural frequency are used for the design of TMD provided the mass ratio is generalized to the following modal quantity (Warburton and Ayorinde, 1980):

$$\mu = \phi_1(z_t)^2\, m_t/m_1 \tag{12}$$

where z_t is the location of TMD. The control law of Eq.(6a) and (8) is applied by taking x coordinate as the response at the attachment of TMD and y coordinate as the response of the TMD. For the direct pulse control, the control force of Eq. (1) is replaced by

$$u = \frac{m_1\left(e^{-\pi\zeta_e}\right)}{\left\{\phi_1(z_p)\right\}^2 \Delta t}, \text{ at each zero-crossing of } x, \tag{13}$$

where z_p is the location of the pulse generator and $\dot{x}$ is the velocity response of the structure at z_p.

The example structure is a cantilever beam with constant mass density ρ, cross sectional area A, second moment of inertia I, Young's modulus E, and length L which are so chosen to give the fundamental natural period of 1 second. Lowest 5 modes are used in the simulation and higher modes are truncated. Structural viscous damping with damping ratio of 1% is assumed for each mode. First 20 seconds of N-S component of the El Centro earthquake is used as the input disturbance. The direct pulse control setting is given in Figure 4a and the semi-active TMD setting is given in Figure 4b. In both cases, the devices are attached at the end of the beam. The target equivalent damping ratio is taken as $\zeta_e = 0.1$. Mass of TMD is set to $m_t = 0.0025\rho AL$ which is equivalent to $\mu = 0.01$. Duration of pulse Δt is taken as 0.02 seconds. According to the standard response spectra diagram of El Centro earthquake, maximum displacement of the SDOF structure with natural period 1 second and damping ratio 0.1 is given as 8.71cm. The effective participation factor for the 1st mode of a cantilever beam at the top is 1.57. Hence target equivalent damping ratio can be converted to target maximum response of 8.71cm $\times$ 1.57 = 13.7cm. Response of the end of the beam with passive configuration is given in Figure 5a, where maximum response is 29.3cm without control and 26.2cm with the passive TMD. It can be seen that TMD is almost ineffective in reducing maximum response. Figure 5b gives the response with direct pulse control and with semi-active TMD. The maximum response by the direct pulse control is 13.3cm and 14.7 cm by the semi-active TMD. Response by the semi-active TMD is slightly larger but the control effort shown in Figure 6 is considerably smaller than that of the direct pulse control. The average control force $\bar{u}$ of Eq.(11) is 4.6cm/s^2 for Figure 6a and 1.8cm/s^2 for Figure 6b, which implies the proposed semi-active TMD needs less than half of the control effort of the conventional pulse control.

CONCLUSIONS

A semi-active tuned mass damper with pulse generator is proposed for seismic control of civil structures. The algorithms are developed in a simple closed form, based upon analytical perturbation solutions of free response. The main conclusions of the paper are as follows:

1. Algorithms for conventional direct pulse control and control with semi-active TMD are developed analytically for SDOF structures. Because both algorithms are given in terms of the target equivalent damping ratio ζ_e, the design of controller is straight-forward once the specification of performance is given in terms of the damping ratio.
2. The proposed semi-active TMD is found to be much more effective than the passive optimal TMD while requiring much smaller control effort than the conventional direct pulse control. This property of the proposed method makes implementation of the pulse control more feasible.
3. The proposed algorithms can be applied to structures with widely spaced natural frequencies using the single mode approximation of the structures. Numerical simulations using an example of a cantilever beam confirm the efficiency of the proposed semi-active TMD.

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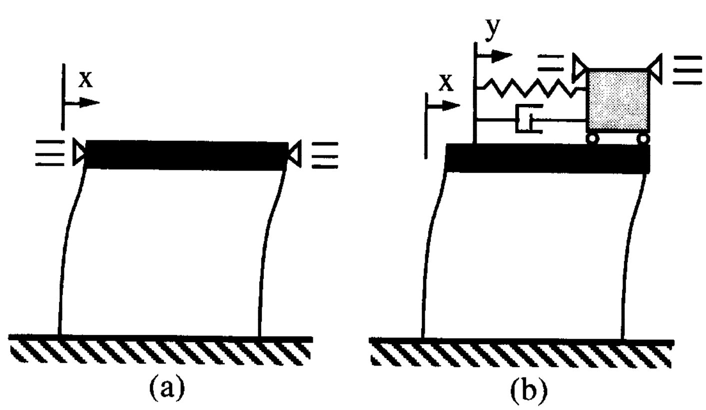

Figure 1. Controller configurations. (a) Direct pulse control, (b) semi-active TMD with pulse control.

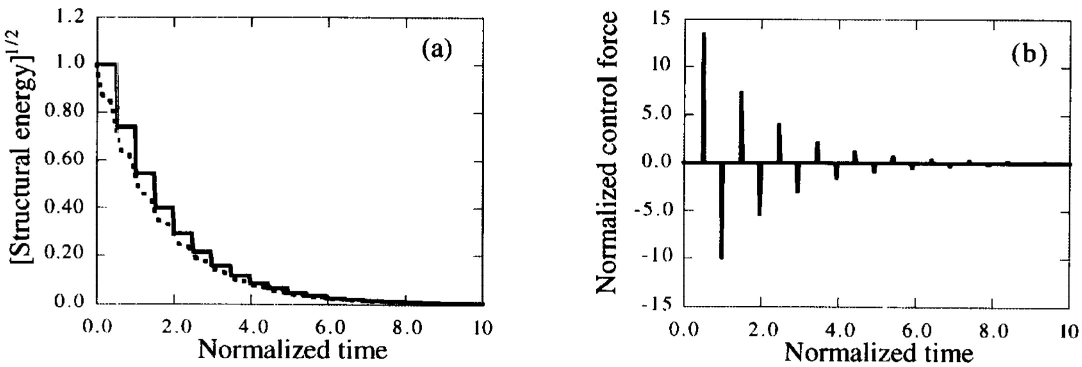

Figure 2. Impulse response of SDOF structure with direct pulse control ($\zeta_e = 0.1$).
(a) Structural response:———— with direct pulse control, - - - - - - SDOF structure with damping ratio ζ_e.
(b) Control force for direct pulse control.

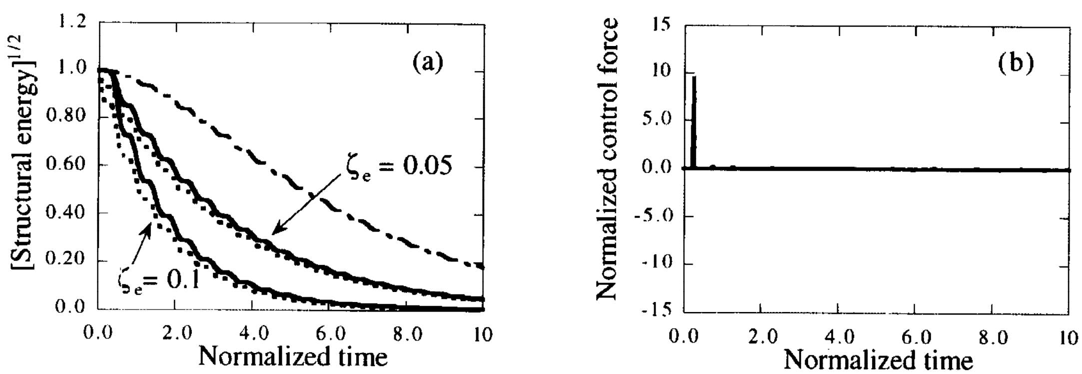

Figure 3. Impulse response of SDOF structure with semi-active TMD ($\zeta_e = 0.05, 0.1$).
(a) Structural response:———— with semi-active TMD, - - - - - - SDOF structure with damping ratio ζ_e,
— - — - — with passive TMD ($\mu = 0.01$, $\zeta_1 = \zeta_2 = 0.05$).
(b) Control force for semi-active TMD ($\zeta_e = 0.1$).

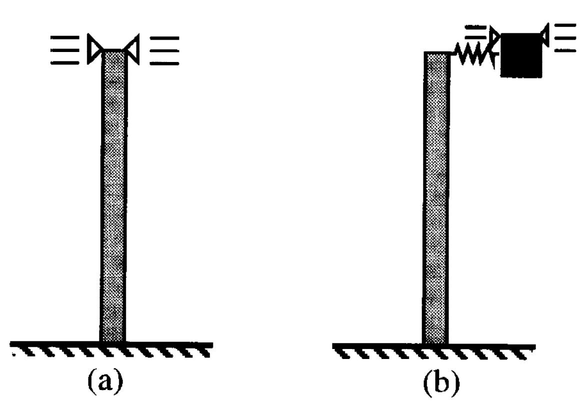

Figure 4. Controller configurations for the cantilever example. (a) Direct pulse control, (b) semi-active TMD with pulse control.

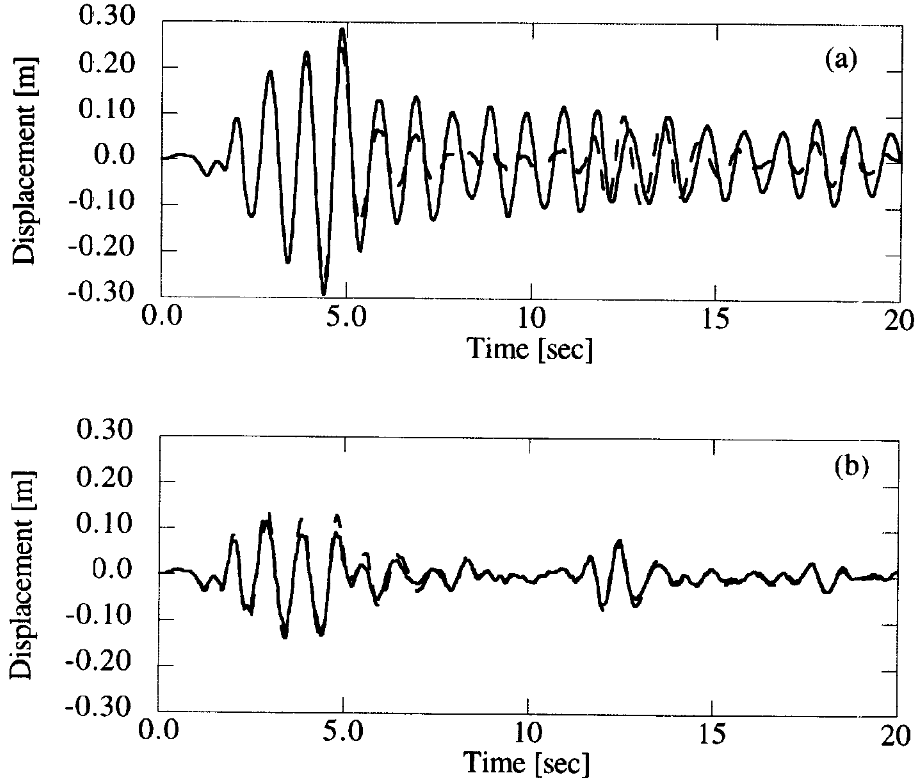

Figure 5. Response of the cantilever beam subject to El Centro earthquake.
(a) Passive configuration: ——— without control, - - - - - - with passive TMD.
(b) Pulse control: ——— with direct pulse control, - - - - - - with semi-active TMD.

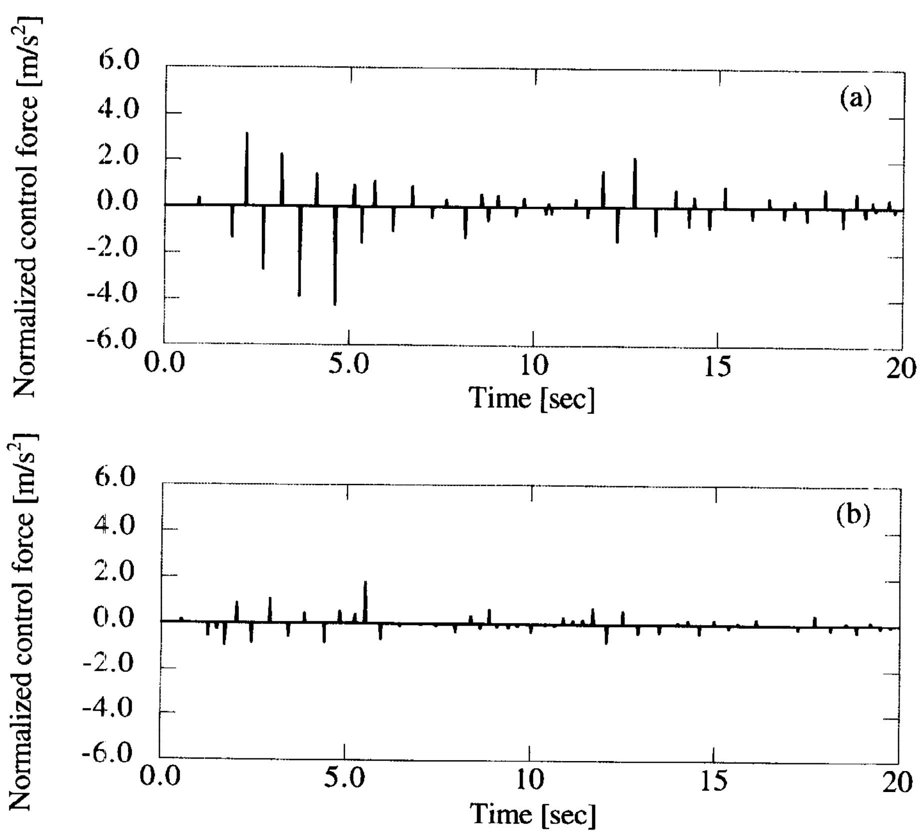

Figure 6. Control effort of pulse control algorithms. (a) Direct pulse control, (b) semi-active TMD.