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DISCONTINUOUS STRUCTURAL ANALYSIS

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ABSTRACT

A discrete finite element method is proposed to model the discontinuous behavior of masonry and bridge structures under severe earthquake conditions. Masonry structures are represented as systems of multiple masonry units interacting through contact stresses at their boundaries. Bridge structures are represented as multiple girders and piers interacting through deformable bearings. The method allows analysis to be extended to failure conditions.

KEYWORDS

Bridge; masonry; discontinuous; discrete element; finite element; failure analysis; collapse

INTRODUCTION

The assumption of continuity is universally used as the basis for the idealization of most structural systems, including bridges and buildings. Numerical techniques such as the Finite Element Method, in which these systems are represented as single framed or continuous structures, are widely used in earthquake engineering. However, these methods, founded on continuity assumptions, are not capable of capturing the discrete behavior that these structures may exhibit under severe earthquake conditions.

Bridges for instance, are generally composed of individual elements, that may move independently from one another, and interact only at some bearing points. Such a discontinuous behavior is difficult to model using conventional frame analysis. Masonry structures on the other hand, exhibit very complex seismic behavior due to the discrete nature of the material, which is difficult to model using continuum constitutive relations.

In this paper a discrete finite element approach is proposed to model the discontinuous behavior of bridges and masonry structures. In this method, the structure is modeled as a system of multiple interacting bodies. Each individual body may be a single rigid element, a single beam element, a single quadrilateral plane stress finite element or a general finite element mesh. The individual structural bodies may undergo arbitrarily large rotations and displacements, and interact through contact stresses, that are continually updated as the bodies move and deform. Analysis can be extended to failure conditions.

DISCRETE ELEMENT METHOD AND DISCRETE FINITE ELEMENT

The Discrete Element Method is a numerical technique specifically designed to solve problems where continuity cannot be ensured throughout the analysis. The method is capable of analyzing systems of multiple bodies undergoing large dynamic or pseudo static absolute or relative motions.

Discrete element analysis has advanced considerably since the introduction of the method in early 70's (Cundall, 1971). Three-dimensional discrete element analysis was performed in mid 80's, (Ghaboussi and Barbosa 1986 and 1990). The Discrete Finite Element Method (DFEM), in which each deformable body is discretized as a finite element mesh was developed in late 80's (Barbosa and Ghaboussi 1989, 1990 and 1992). Modeling of fracturing of individual bodies is possible although not done routinely. Additional research and development is needed to produce practical methods for modeling the fracturing of individual bodies. Coupling of the discrete element analysis with other processes such as ground water flow through fractured rock or temperature effect and thermal diffusion have also been accomplished, (Ghaboussi and Barbosa, 1988, Barbosa 1990).

Discrete element analysis has been applied almost exclusively to geotechnical engineering problems. The methodology has been used mostly in studies of deformation of granular materials, granular flow problems and modeling of jointed rock masses. The Discrete Finite Element Method is however completely general in its ability to handle a wide range of material constitutive behavior, inter-body interaction force laws, impact, arbitrary geometries and therefore is a valuable tool for solving a large variety of discontinuum structural engineering problems, which are intractable by conventional techniques.

Application of the Discrete Finite Element Method in structural engineering, particularly in earthquake engineering problems, is the focus of this paper. After a brief description of the general method, the major issues in modeling masonry structures and bridges using DFEM will be discussed. Some illustrative examples are presented.

METHOD OF ANALYSIS

In this method, structural systems are represented as a large number of individual interacting bodies. The method of analysis consist of a mechanical model to represent the bodies and an interaction model that specifies the contacts and bearings between bodies. Each body can be idealized either as a single finite element or as a finite element mesh. Individual bodies interact either through contact stresses, distributed over the actual area of contact, or through contact forces concentrated at some defined bearing points. Normal and shear contact stresses and/or bearing forces are computed incrementally from incremental relative displacements and the contact/ bearing constitutive properties. For frictional contacts a Coulomb type friction law is typically used to determine the initiation of sliding and to limit the magnitude of the shear contact stresses. Bodies may slide over each other, separate, re-establish contact, etc.

Thus, a bridge can be represented as a collection of individual bodies, the girders and the piers, interacting through deformable bearings and contacts. Piers and girders can be modeled as either single rigid elements, single beam elements or rigid frames (body model). Bearings could be elastic, hysteretic, frictional, etc. (interaction model). Masonry walls can be idealized as large numbers of masonry units, interacting through the mortar material at their boundaries. Each masonry unit can be modeled as a single rigid element or as a single quadrilateral plane stress finite element (body model). Interactions between units can be cohesive and frictional (interaction model).

The method of analysis is a dynamic time stepping incremental procedure. For each step of dynamic analysis three major computational operations are performed: (a) displacements and deformations induced in each deformable body are computed, based on the mechanical model used to represent the bodies; (b) new contacts between pairs of deformable bodies are identified; and (c) interaction stresses between bodies are updated using the contact model.

This cycle of dynamic calculations is repeated several times to trace the movements and deformations of each body during and after the earthquake excitation. The analysis can be continued, until a final motionless condition is reached. In cases of severe shaking that final motionless condition could be the configuration after total collapse of the structure. At every stage of the analysis, both external and internal equilibrium conditions are satisfied for all deformable bodies. At the final motionless condition, applied loads and self weight on each deformable body are balanced by the contact stresses developed at the body's surface. Furthermore, for this motionless condition, the internal stresses on every body are those due to the contact stresses acting at the body's surface, as computed from standard finite element analysis.

BODY MODEL

In the DFEM each interacting body is idealized through finite elements. The equations of motion of the system of finite elements representing each body can be expressed as

$$\mathbf{M}\ddot{\mathbf{U}}_t + \mathbf{C}\acute{\mathbf{U}}_t = \mathbf{P}_t - \mathbf{I}_t \qquad (1)$$

where **M** is the nodal mass matrix, **C** is the damping matrix, and $\ddot{\mathbf{U}}_t$ and $\acute{\mathbf{U}}_t$ are the nodal acceleration and velocity vectors at time *t*, respectively. The vector $\mathbf{P}_t$ is the external load vector in the configuration at time *t*, and $\mathbf{I}_t$ is the internal resisting force vector that correspond to the element stresses in that configuration. Included in $\mathbf{P}_t$ are contributions from contact stresses and any other external surface tractions as well as self-weight and any other external body force. The right hand side term in this equation, $(\mathbf{P}_t - \mathbf{I}_t)$, is the unbalanced force vector. All components of this vector will be zero when the final motionless configuration is reached.

The internal resisting force vector can be written in terms of element stresses as

$$\mathbf{I}_t = \Sigma_k \int_{kvt} ({}_k\mathbf{B}^T{}_t)\,({}_k\sigma_t)\,dV \qquad (2)$$

where the subscript k is an element label, $\mathbf{B}_t$ is the element strain-nodal displacement matrix for the configuration at time *t*, V_t is the volume of the element at this deformed configuration, and σ_t is the element stress tensor (written in form vector), at this configuration. The summation is over all the elements k, representing the deformable body. The symbolism pertaining to element assembly is omitted for clarity.

Equation (3) is based on the principle of virtual work and provides a consistent relation between nodal forces and element stresses (Zienkiewicz, 1977). For plane stress elements, the internal resisting force vector is evaluated by Gaussian integration. For frame elements, the internal resisting force vector represents simply the internal forces at the ends of the member, induced by the deformations of the element. For rigid elements the internal resisting force vector is zero.

Equations (1) and (2) represent the equilibrium conditions of the system of finite elements defining any body and must be satisfied throughout the complete time history of the motion of the body. These relations are general and valid for any type of material (elastic, elasto-plastic, etc.). Since bodies are allowed to undergo large displacements and rotations, Updated Lagrangian Method is used. The details of the formulation can be found in (Barbosa and Ghaboussi 1989 and 1990).

INTERACTION MODEL

Individual bodies may interact through deformable bearings and actual contacts. Processing of deformable bearings is fairly simple. They are pre-defined as input at the start of the analysis. Contacts on the other hand, must be detected automatically as the calculation progresses, since their status may change considerably throughout the analysis. Bodies that initially were separated may collide during the motion, then separate, and re-establish contact later.

Bearings can be represented as simple multiaxial springs or links connecting two points, on different bodies. Points attached to a link may be located inside the interacting body, on its surface or even outside the body's surface. The point of interaction can be assumed to be rigidly attached either to the surface of the body, or to one of the nodes of the deformable body. Bearings may have a finite length or be zero-length links. In a three-dimensional model of a bridge structure, each bearing is then represented as a multiaxial spring: a vertical, a longitudinal and a transverse spring. The interaction spring forces are computed incrementally based on the relative displacements of the points attached. Springs can be linear or non-linear. For each direction a limit relative displacement can be specified, after which the bearing is eliminated. An explicit integration scheme is used to compute the history of the interacting forces, which makes modeling nonlinear spring behavior a very simple process.

Deformable bodies may interact through different types of contacts. In the most general case of three-dimensional bodies, each body is represented as a polyhedron. The surface of a polyhedron is composed of three different elements: corners, edges and faces. In general six different types of contacts are possible between two polyhedrons: corner to corner, corner to edge, corner to face, edge to edge, edge to face and face to face. Of these six contact types, two are very unlikely to occur, and hence are usually not considered; corner to corner contacts and corner to edge contacts. Corner to face and edge to edge contact types are basically point contacts, while the remaining two are distributed contacts. Edge to face contact is a line contact and face to face is a surface contact. Schemes for detecting and processing all these types of contacts are described in (Barbosa 1990, and Ghaboussi and Barbosa 1990)

In the two-dimensional examples presented in this paper only one general type of contact is considered: face to face contact. Although some contacts may be full face to face contacts and others partial face to face contacts, all contacts are treated similarly. The actual area of contact is determined at each time step. As shown in Fig. 1, a corner to face contact is thus a special case of a face to face contact with a small contact area. Contact stresses are computed from the contact properties, assuming a very simple constitutive law.

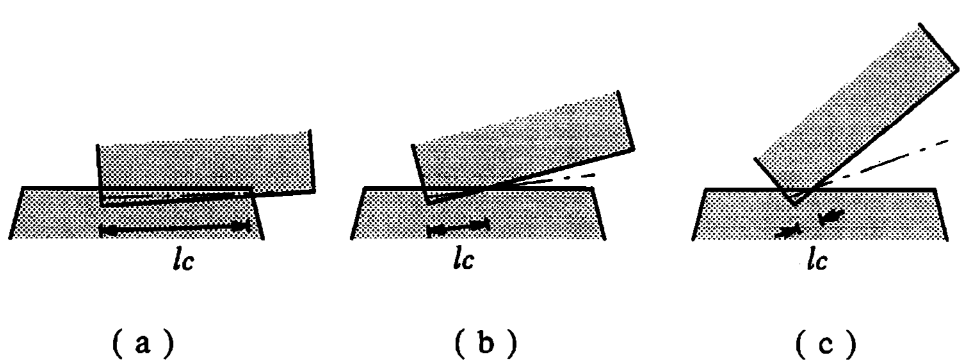

Fig. 1. Contact area between bodies

To define the normal stress mobilized by contact between two bodies, a distributed overlap δn, representing the local deformation at the contact, is assumed to develop at the body boundaries, as shown in Fig. 2(a). The normal stress distributed along the contact area is then computed as a function of such a deformation. A simplified linear relation between contact stress and contact deformation is usually assumed. Hence, the normal stress is computed as

$$\sigma = Kn\ \delta n \tag{3}$$

where Kn is the contact normal stiffness (stress/displacement). For this simplified linear relation the normal contact stress between bodies is determined uniquely by the relative spatial positions of the bodies. However, normal contact stresses are computed incrementally, hence nonlinear contact models can be implemented without major difficulty.

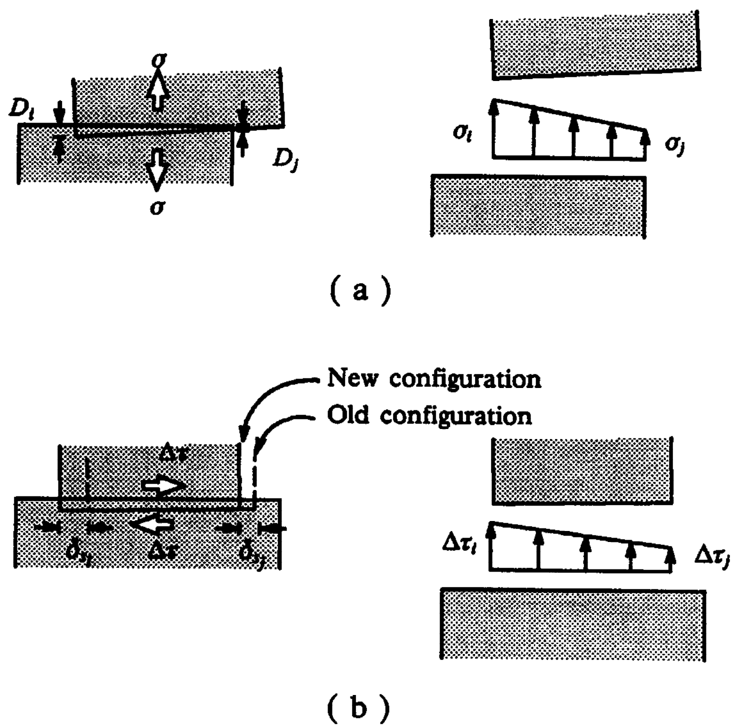

Fig. 2. Normal and shear contact stresses between bodies.

For a pure frictional model, contacts are deactivated when the two bodies separate. That is when there is no inter-body overlap. For cohesive contacts in bonded bodies, such as masonry units bonded by mortar, a very small separation between the bodies is allowed. That limit small separation is determined by the normal stiffness and the tensile strength of the bonding material. Once that limit separation is exceeded, the bond is lost and if contact is re-established it is treated as a purely frictional contact.

The shear contact stress is computed incrementally as it depends on the deformation path to which the contact has been subjected. The increments of shear stress are computed from the progressive shear displacements between the bodies. For an increment of shear displacement δs, as shown in Fig. 2(b), the increment of shear stress is given by

$$\Delta\tau = Ks\ \delta s \tag{4}$$

where *Ks* is the contact shear stiffness. This distributed increment of shear stress is then added, nothing the sense of movement, to the shear stress already existing between the two bodies, i.e.

$$\tau = \tau^{old} + \Delta\tau \tag{5}$$

In the case of cohesive contacts, if at some stage the computed shear stress at any point along the contact area exceeds the maximum shear resistance ($c + \sigma \tan \phi$, where c is the cohesion and ϕ is the friction angle), the bond breaks and the contact becomes purely frictional.

For a frictional contact, if the computed shear stress exceeds the maximum frictional resistance ($\sigma \tan \phi$), slip occurs and the shear stress assumes the limiting value $\sigma \tan \phi$. Consequently, after each increment of normal and shear displacement, the total shear stress must be evaluated. If the shear stress is less than the limiting frictional resistance, elastic deformability conditions are re-established at the contact.

In addition to static contact stresses, contact damping stresses can be included in the discrete finite element method. Stiffness proportional viscous damping is used in the proposed model. The normal damping contact stress is computed as β Kn δn, where β is the viscous damping proportionality factor and δn is the normal contact velocity. Similarly the shear damping contact stress is computed as β Ks δs, where δs is the shear contact velocity.

COMPUTATIONAL SCHEME

The computational scheme proceeds by following the motion of the individual bodies and tracing their interactions through the repeated application of three major computational operations: (a) body motion calculations, (b) interaction processing and (c) contact detection.

In the body motion operation the displacements and deformations induced in each structural body by out-of-balance forces are computed by direct integration of the equations of motion. The total forces acting at the degress of freedom, resulting from contact stresses, bearing forces, applied loads and self weight, are combined with the internal body stresses into a conventional integration scheme to compute body displacements. These body displacements are then used to compute body deformations and the new internal body's stresses. The body displacements are also used to update the geometry of the assemblage of interacting bodies.

In earlier applications explicit integration was used (Barbosa and Ghaboussi, 1990). However, implicit integration as described in (Barbosa 1994), allows greater time steps, specially when bodies include small frame elements, resulting in a much more efficient integration method.

In interaction processing, the stresses developed between pairs of bodies either in direct contact or attached by deformable bearings, are calculated based on their relative movement and the interaction model. In addition, those cases in which contact between bodies is lost and not likely to be re-established, are identified and deleted from the list of contacts. The first step in the processing of each contact is to compute total and incremental values of both normal contact deformation and shear contact displacement, as well as the current area of contact, using the updated geometry of the bodies. Next, based on these quantities and the contact model, the normal and shear contact stresses are updated. Then, the computed contact stresses are transformed to energy equivalent nodal loads at the element degrees of freedom. Finally, these energy equivalent nodal loads are added to the vectors of external loads $P_{t+\Delta t}$ of both bodies.

The purpose of the contact detection operation is to identify new pairs of bodies that have established contact. In order to find these contacts, each body must be checked with respect to all other bodies. Usually the space of the problem is divided into several regions, which allows to limit the checks to neighbor bodies.

This computational operation is the most time-consuming part of this method of analysis, therefore efficiency becomes a major requirement in the computer implementation of contact detection procedures.

The main scheme for contact detection consists of a multilevel check, aimed at disregarding cases of no contact as early in the contact detection process as possible and to identify cases of high likelihood of contact with as little computational effort as possible. In a first level check a quick test, based on simple comparisons of body coordinates, is performed to determine if contact between any two neighbor bodies is possible. If contact is possible, another test is made to find possible combinations of faces in contact, based on the relative orientation of the two bodies. Finally, these candidate combinations are examined in a more detailed test to determine whether contact has occurred.

EXAMPLES

Different analytical and experimental validations of discrete element methods have been carried out in the past (Barbosa 1990, Barbosa and Ghaboussi 1990 and 1992). In this section only two demonstrative examples are presented to illustrate the potential of the proposed method of analysis as a valuable analysis and design tool for bridge and masonry structures.

Seismic analysis of a bridge structure

In this example a seven-span, slab-on-stringer, concrete bridge is analyzed for a severe earthquake loading. The example illustrates the capability of the proposed method of analysis to capture aspects of the seismic response of bridge structures, that are either untractable or extremely difficult to model with conventional techniques. First, the method allows to consider the limited deformation capacity of bearings, in the overall seismic analysis of the structure. This is a significant aspect as failure of bearings has resulted in collapse of several bridges during recent earthquakes. Second, the analysis allows to specify different ground motions at the base of the structure, which is not easy to do in conventional structural finite element analysis. Whereas for small structures such as buildings, it is reasonable to assume a unique ground motion at the base of the structure, for long structures, such as bridges, actual base excitations may change considerably along the length of the structure, due to changes in ground conditions. Those differential motions result in additional distress to the structure. Finally, and most importantly, is the capability of the method to extend the analysis to failure conditions. There are no limitations to infinitesimal displacements or small changes in geometry. The analysis produces the time history of motion, internal forces and interactions for all elements and bearings, up to the final post-collapse configuration of the structure.

The structure analyzed consists of seven single span units, with alternate fixed and expansion bearings, i.e. the left abutment has a fixed bearing, the adjacent pier has two expansion bearings, one for each adjacent span, the next pier has two fixed bearings, the next pier has two expansion bearings, etc. The three interior spans are 80 ft, the intermediate spans 60 ft, and the two exterior spans 30 feet long. The heights of piers, from the interior piers towards the stub abutments are 60, 50, 40, and 20 ft, respectively.

Fixed bearings were modeled as two elastic springs: an axial spring (vertical) and a shear spring (horizontal). No rotational restraint was considered. A limit bearing shear displacement of 10 inches was specified, after which, the bearing was assumed to fail, and considered to behave as an expansion bearing. Expansion bearings were assumed to be frictional and were treated as contacts between the beam elements and the pier elements. The initial length of contact for all bearings was 15 inches. All elements were assume to be rigid. The structure rested on two huge rigid ground blocks. A different base motion was specified for each ground block.

In a first step of the analysis, the assemblage of elements was allowed to consolidate under its self weight and the superimposed dead load. The ground blocks were fixed at this stage of the analysis. Next, after a final static equilibrium condition was reached, the seismic analysis was initiated. The motions of the right ground block, were specified to be the earthquake motions measured at El Centro, California in 1940. Both horizontal (NS component) and vertical components were included in the analysis. The same horizontal and vertical records were used for the ground block on the left abutment, however, the time scale was compressed by a factor of 0.8, so that the frequency content of the input motion, corresponded to a slightly stronger foundation material. The acceleration scale of the records was expanded so that the horizontal peak acceleration was 0.8 g. The resulting relative horizontal velocity between the two base blocks, for the specified ground motions, is shown in Fig. 4. It is noted that the relative peak velocity resulted approximately 1.5 times greater than the absolute peak velocity of the two ground blocks. This relative motions naturally, impose considerable straining on the structure, additional to the inertial effects of the absolute motion.

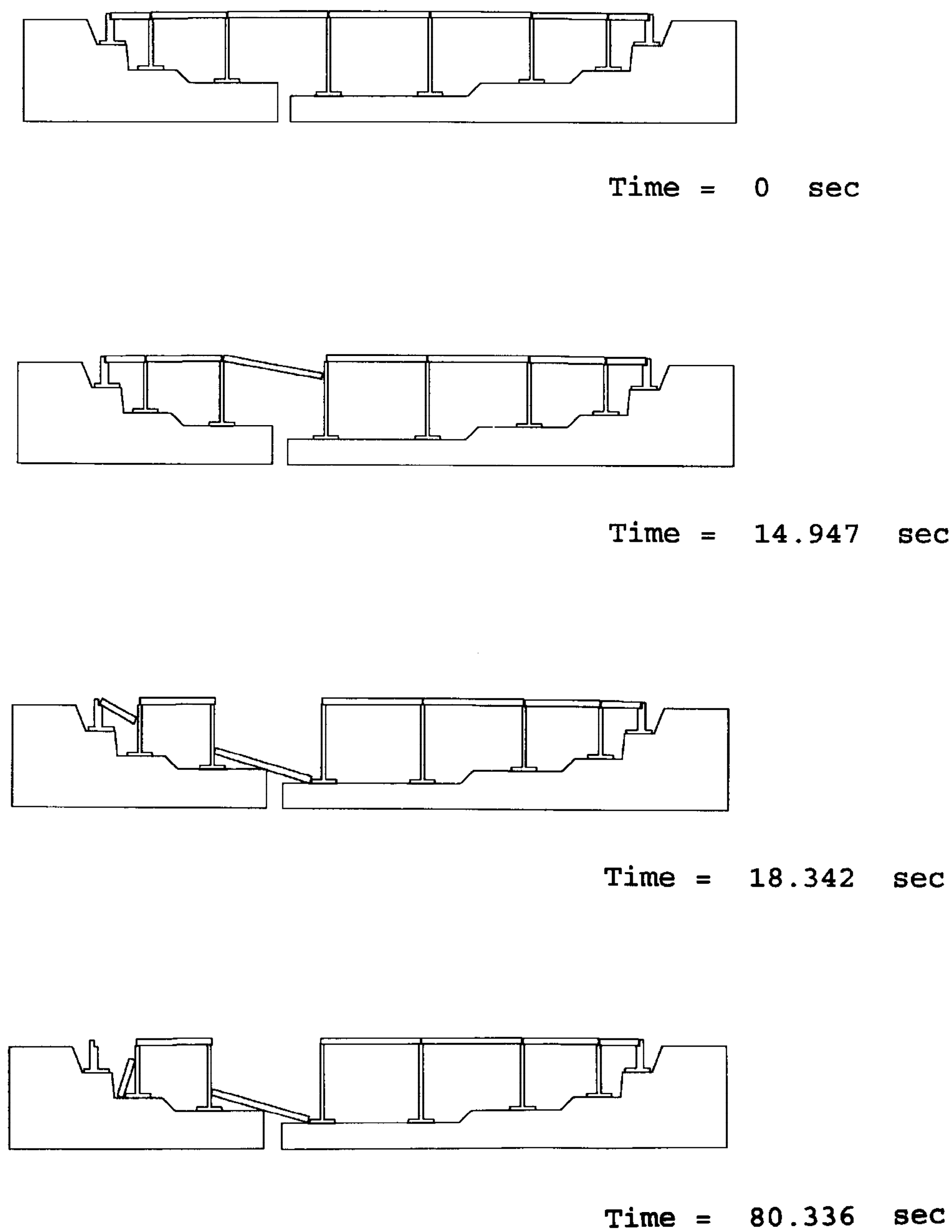

Fig. 3. Seismic analysis of a concrete bridge

Four configurations of the structure at different stages of the analysis are shown in Fig 3. These are actual computer outputs of the simulation. Displacements are not magnified in those figures. Time zero corresponds to the final static configuration after application of dead load. The configuration at time 80.3 sec, which is 26.6 seconds after the earthquake ended, is essentially the final motionless configuration. All element velocities at that configuration are less that 0.00001 in/sec.

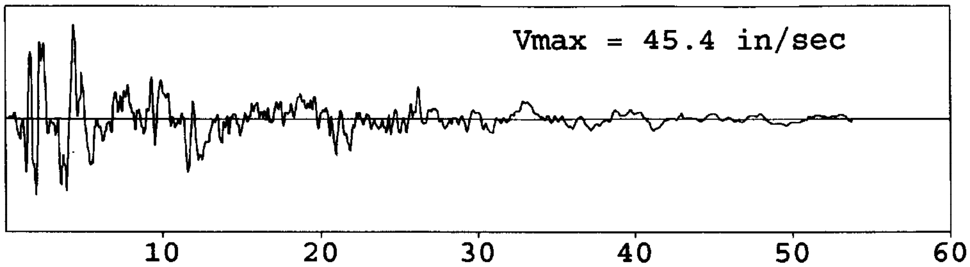

Fig. 4. Relative velocity, between left abutment and right ground blocks

Failure of the bridge followed a chain of events that developed during the first 25 seconds of shaking. First, the left fixed-bearing on the third left span failed, reaching its limit shear displacement, 11 seconds after the shaking started. Next, at time 14.4 sec, the expansion bearing on the same span lost its support on the pier, which made the span fall. The left end of the falling deck hit the right ground block at time 15.9 sec. The other end hit the adjacent pier, 0.4 seconds after that. The impact, of the left end of the falling deck on the third left pier produced the failure of the fixed bearings of two leftmost spans. At time 17.7 sec the support length of the first left span became zero, which initiated its fall. At about 23 seconds after earthquake shaking, the configuration of the bridge was already essentially the same failed final configuration.

Seismic analysis of a masonry wall

This example is the seismic analysis of an unreinforced masonry wall. The wall is three stories height with an asymmetrical layout of window and door openings. The story height is eight feet, and the total width of the wall is 16 feet. The masonry units are assumed to be elastic. Each unit is modeled as a single, free standing, quadrilateral isoparametric finite element. The floor slabs are modeled as rigid blocks. The structure rest on a rigid base block. The structure was first consolidated under its own weight and then subjected to the seismic excitation. The horizontal motion of the base block was specified to be that of the El Centro (NS component) with the acceleration scaled to a peak value of 1.0 g. The base block was fixed in the vertical direction.

Configurations for different stages of a seismic analysis are shown in Fig. 5. The results presented are those for a case of a very low strength mortar, in which the shearing resistance is mostly frictional. The friction angle of the mortar was assumed to be 35 degrees, and the cohesion intercept 204 psf, which corresponds to a tensile strength of 400 psf.

An interesting feature of the proposed method is that it allows to perform numerical experiments inexpensively under totally controlled conditions. To study for instance the effect of the tensile strength of the mortar on the mode of failure of the wall, several analysis to failure can be done, varying only the input parameter under study, i.e. the tensile strength of mortar. To obtain similar information experimentally, failing numerous prototypes or even small scale models would be much more expensive. Furthermore, in a real test series, it is difficult, if not impossible, to produce two identical prototypes to perform two tests under identical conditions, varying only the desired parameter. Experimental settings can be controlled only to a limited extent. With the proposed method one can easily obtain behavioral information by studying the influence of each significant parameter.

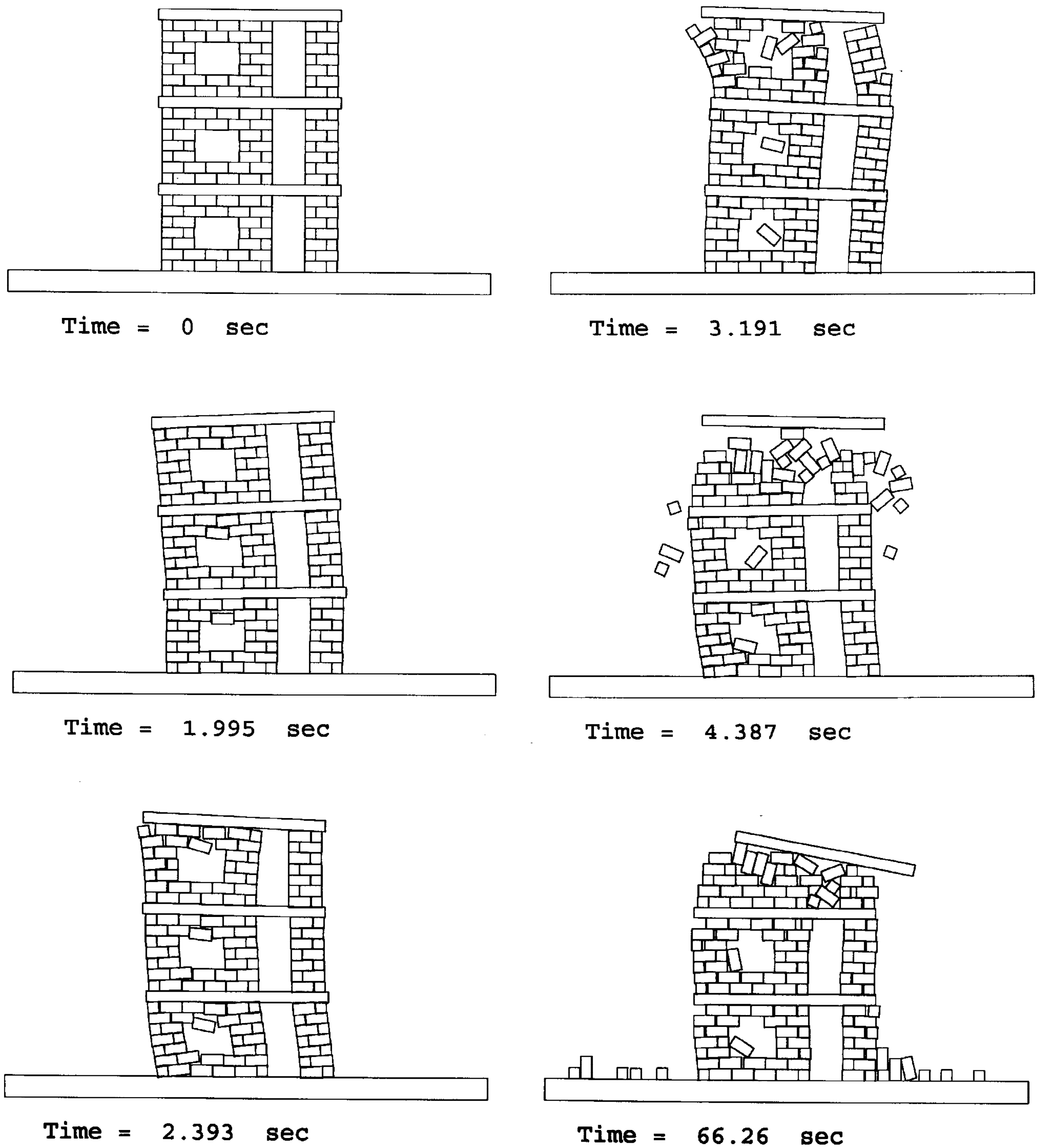

Fig. 5. Seismic failure analysis of a masonry wall

For the low strength mortar case shown in Fig. 5, most damage occurs at the upper floor where the normal stresses are lower and the frictional resistance of the mortar is smaller. Failure occurs early in the seismic shaking, which is when the strongest ground motions occur.

CONCLUSIONS

A computer procedure has been presented, that allows to model realistically the discontinuous behavior of structural systems. The method allows analyses for a very broad range of conditions from small displacement static situations to the very large and rapid displacement conditions, associated with strong base excitation. Analyses can be extended to failure conditions, including the total collapse of the assembly of structural elements. The formulation of the method as well as computational aspects of its implementation are discussed in the paper. Examples illustrate the potential of the method as a valuable analysis and design tool for bridge and building engineering.

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