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THE DESIGN OF FRAMED BUILDINGS ON EARTHQUAKE SPATIAL EFFECT

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ABSTRACT

The lessons from the past earthquakes show that the circular ground motions along with lateral ones are essential. In many cases the structure designed as a three-dimensional asymmetric model fails during the real earthquake. This fact leads to the decision that the earthquake circular components are acting on the structure during the earthquake. In most Design Codes earthquake ground motion circular components are not considered, and torque is calculated from the eccentrically loaded seismic force due to the asymmetry of the structure. It is should be noticed that during the earthquake symmetric structures are also subjected to torque.

Presented work gives the real values for torsional ground accelerations and shows how to use them in Design Codes during the analysis. As an example the real records of Georgian Earthquake (April 29, 1991) are used. Circular components are calculated from the recorded transitional ones. The normalized values are found which are compared with the values obtained by the proposed method. The proposed method uses wave theory to derive the analytical formulae for determination of accelerations for different seismic zones. It also shows how to link this method with acting Design Codes. The regional seismic characteristics are also found for several areas of Georgia. The design models of soil and structure to support the theory are presented. Each level of the building has three degrees of freedom: lateral, longitudinal and torsional. This structure is subjected to six-component earthquake effect. Finally, software programs are developed to design symmetric and asymmetric structures on earthquake spatial (six-component) effect.

KEYWORDS

Circular acceleration; torsional acceleration; normalized acceleration; records; seismic lateral load; seismic torsional load; center of mass; center of rigidity; shear wave velocity, predominant period.

THE ENGINEERING ANALYSIS OF GEORGIAN EARTHQUAKE

In April 29, 1991, western part of Georgia was struck by earthquake with magnitude of 6.9-7.2 in Richter Scale (Arefiev S. *et al.*, 1991). Shortly after main shock SMACH accelerographs were located in several areas by Swiss Seismological Service and over 400 records of aftershocks were recorded. These records are used to support presented theory. For the calculation of circular components from recorded translational ones, Newmark's (Newmark N.M. *et al.*, 1971) differentiation method is used.

In order to design the structure on earthquake spatial effect along with different coefficient the value of normalized acceleration is needed. The normalized values of circular accelerations are found using the following method (Korchinskij I.L. *et al.*, 1985): arithmetic mean value of the accelerations is found over the predominant period, which is calculated from the condition that the start of this period is at the point where the acceleration is greater or equal to its maximum value and the end is taken with the same condition. Approximately 400 records were analyzed and normalized values were found for different areas of Georgia, they are shown in Table 2 along with calculated values. The regional seismic characteristics are found for these areas (Marjanishvili Sh. ScC Thesis, 1993).

THE DETERMINATION OF TORSIONAL ACCELERATIONS

It is known that due to the earthquake the building experience horizontal, vertical and torsional effects. This process is very complicated and requires the study of spatial design models of the structure and corresponding seismic effects.

Elastic oscillation waves distributing in the earth's crust as a result of great earthquake energy release represents seismic waves and causes mainly three types of ground motions: longitudinal, transverse and surface. The laters occur as a result of medium limit change of longitudinal and transverse wave distribution and characterize ground motion in the base of the building foundation.

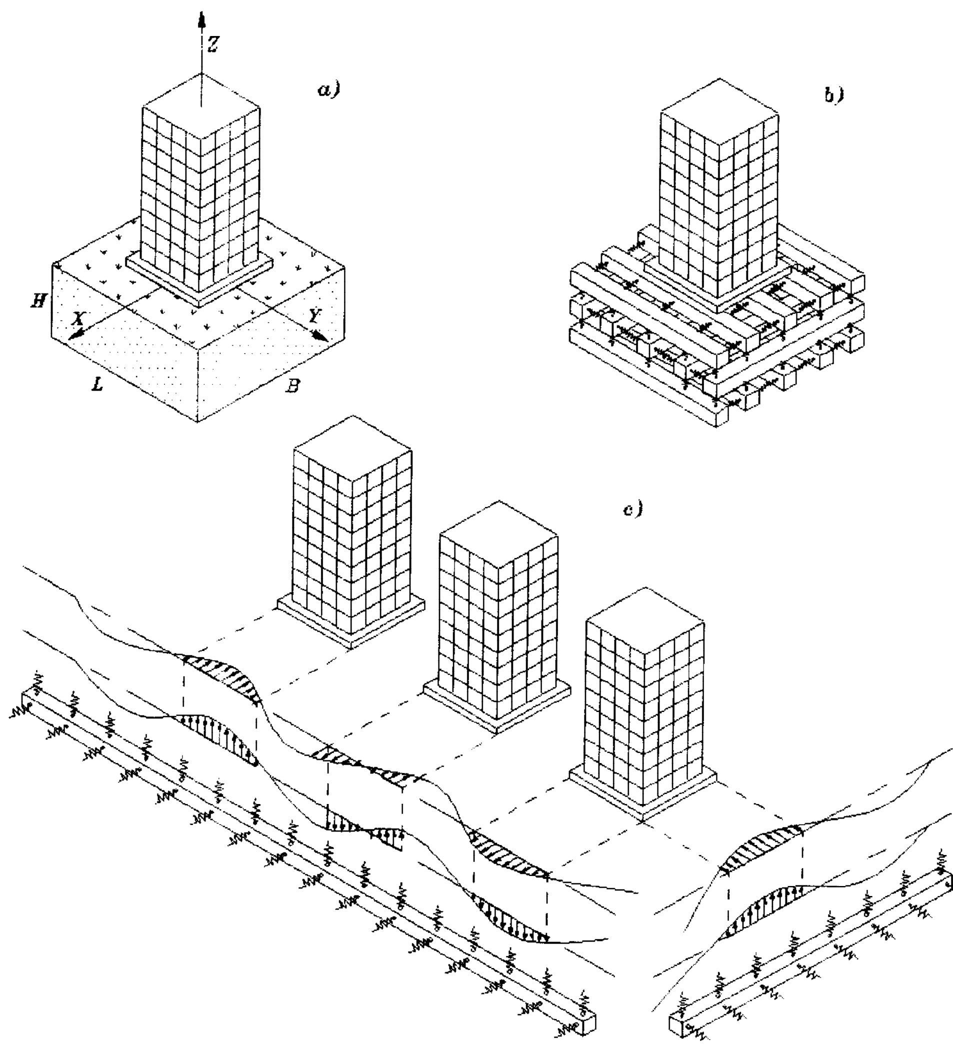

Fig. 1 Design soil model with structure
a) General view of the structure and soil; b) Design model of the soil;
c) Possible seismic waves.

To create soil spatial design model the ground should be isolated by the volume of *L, B and H* (See Fig.1 a). It is assumed that the building is mostly effected by the wave which has the period equal to the predominant period of the considered soil. If we assume that T_{pred} is predominant period of the soil and λ - is the length of seismic wave, the dimensions of isolated volume can be:

$$L = 0.5\lambda_x^{trans} \quad ; \qquad B = 0.5\lambda_y^{trans} \quad ; \qquad H = 0.25\lambda_z^{long} \tag{1}$$

Where $\lambda = C\ T_{pred}$; C - is the seismic wave velocity.

Seismic accelerations distributed on the ground surface along the axes *X, Y* and *Z* expanded according to the main oscillation forms will be:

$$u''^{*}_{g.x} = u''_{g.x}(\phi_y^{trans})_k = u''_{g.x} Cos\left[\frac{\pi(k-1)}{2} - \frac{2\pi y}{\lambda_y^{trans}}\right]$$

$$u''^{*}_{g.y} = u''_{g.y}(\phi_x^{trans})_k = u''_{g.y} Cos\left[\frac{\pi(k-1)}{2} - \frac{2\pi x}{\lambda_x^{trans}}\right] \tag{2}$$

$$u''^{*}_{g.z} = u''_{g.z}(\phi_{zx}^{trans})_k(\phi_{zy}^{trans})_r = u''_{g.z} Cos\left[\frac{\pi(k-1)}{2} - \frac{2\pi x}{\lambda_x^{trans}}\right] Cos\left[\frac{\pi(r-1)}{2} - \frac{2\pi y}{\lambda_y^{trans}}\right]$$

Where ϕ_K^{trans} - is the flat forms of soil natural oscillations.

Let us put the building on the isolated soil volume (see Fig.1 a). The solid ground can be changed with longitudinally and transverse located shear bars with elastic connections having C_1 and C_2 stiffness. The differential equation of the isolated volume of the soil could me identical to those of the shear bars with stiff connections. C_1 and C_2 stiffness characterize the soil. The distribution of seismic waves can be considered as the oscillations of these bars. Let us isolate one longitudinal and one transverse bar. For the different modes of the bars' oscillations there would be different seismic effects (see Fig.1 c). Figure 2 shows different possible cases of seismic effect. In many Design Codes only two cases are considered: a) or b) or e). When considering earthquake spatial effect it is required to account d) and, a) or b); c) and f); e) cases.

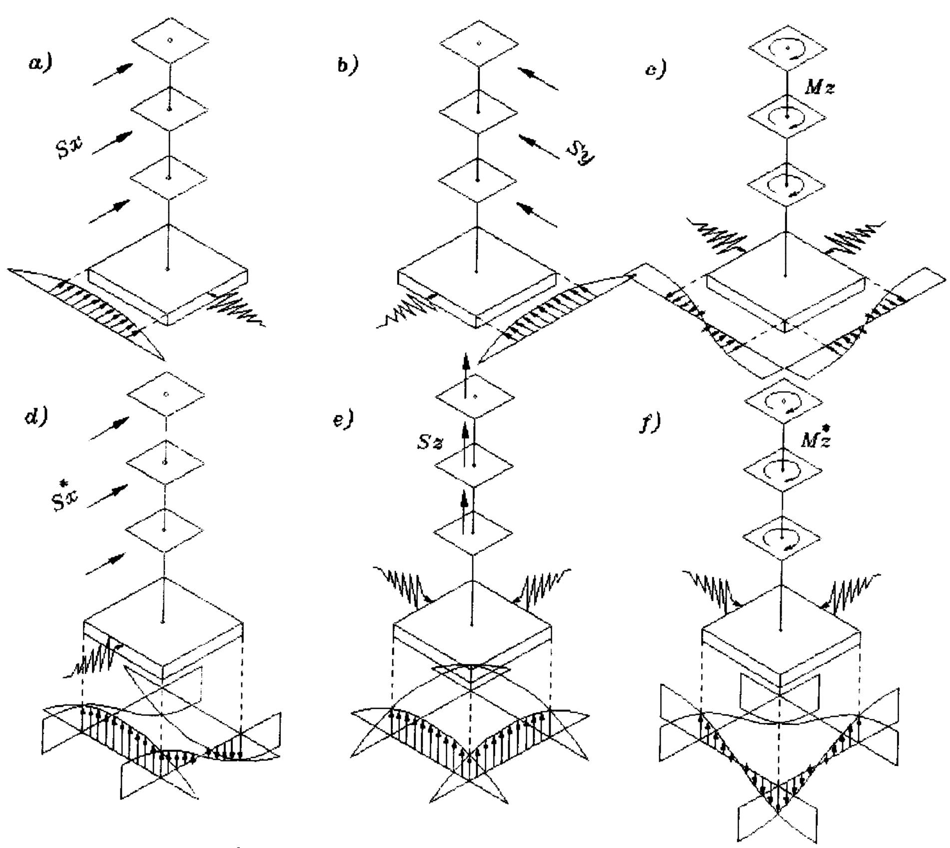

Fig.3 Possible cases of earthquake effects

In formulae (1) for odd value of k the earthquake lateral effect takes place, but for even value of k - circular one. The values of their accelerations will be calculated:

Linear:

$$u''^{*}_{g.x} = u''_{g.x} Cos(2\pi y / \lambda^{trans}_{y}) \qquad \text{(Fig.2.a);} \qquad (5.a)$$

$$u''^{*}_{g.y} = u''_{g.y} Cos(2\pi x / \lambda^{trans}_{x}) \qquad \text{(Fig.2.b);} \qquad (5.b)$$

$$u''^{*}_{g.z} = u''_{g.z} Cos(2\pi x / \lambda^{trans}_{x}) Cos(2\pi y / \lambda^{trans}_{y}) \qquad \text{(Fig.2.e);} \qquad (5.c)$$

Torsional:

$$u''^{*}_{g.x} = u''_{g.x} Sin(2\pi y / \lambda^{trans}_{y}) \qquad \text{(Fig.2.c);} \qquad (5.d)$$

$$u''^{*}_{g.y} = u''_{g.y} Sin(2\pi x / \lambda^{trans}_{x}) \qquad \text{(Fig.2.c);} \qquad (5.e)$$

$$u''^{*}_{g.z} = u''_{g.z} Sin(2\pi x / \lambda^{trans}_{x}) Sin(2\pi y / \lambda^{trans}_{y}) \qquad \text{(Fig.2.f);} \qquad (5.f)$$

Lateral-torsional:

$$u''^{*}_{g.z} = u''_{g.z} Cos(2\pi x / \lambda^{trans}_{x}) Sin(2\pi y / \lambda^{trans}_{y}) \qquad \text{(Fig.2.d);} \qquad (5.g)$$

$$u''^{*}_{g.z} = u''_{g.z} Sin(2\pi x / \lambda^{trans}_{x}) Cos(2\pi y / \lambda^{trans}_{y}) \qquad \text{(Fig.2.d);} \qquad (5.h)$$

In the soil design model it is assumed that the seismic wave has sine or cosine shape. In order to design the structure the normalized value of the acceleration is required. For this purpose the cosine shape should be changed into linear equal shape, since in the design model of the building the floor disks are not deformable. Equation 5 is integrated along the L length of the building and should be equal to those of linear shape:

For lateral ground motions (When $k=1$):

$$A^{*}_{x} = A^{\max}_{x} \Delta^{hor}_{1y} \quad ; \quad A^{*}_{y} = A^{\max}_{y} \Delta^{hor}_{1x} \quad ; \quad A^{*}_{z} = A^{\max}_{z} \Delta^{hor}_{1x} \Delta^{hor}_{1y} \qquad (6)$$

Where $A^{\max}_{K}$ - is the maximum acceleration according to corresponding axes.

$$\Delta^{hor}_{1x} = \int_{-Lx/2}^{Lx/2} Cos\frac{2\pi x}{\lambda^{tr}_{x}} dx \Bigg/ \int_{-Lx/2}^{Lx/2} dx = \frac{Sin \pi \chi_{x}}{\pi \chi_{x}} \qquad (7)$$

For torsional ground motions (when $k=2$):

In this case sine shape would be changed into linear. The acceleration at the end of the building is reverse and there will be moments, which give torsional acceleration according to Z axis, their integration gives:

$$\theta''^{*}_{z.x} = A^{\max}_{x} \frac{2}{L_{y}} \Delta^{hor}_{2y} \quad ; \quad \theta''^{*}_{z.y} = A^{\max}_{y} \frac{2}{L_{x}} \Delta^{hor}_{2x} \quad ; \quad \theta''^{*}_{z.z} = A^{\max}_{z} \frac{4}{L_{x} L_{y}} \Delta^{hor}_{2x} \Delta^{hor}_{2y} \qquad (8)$$

Where

$$\Delta^{hor}_{2x} = \frac{L_{x}}{2} \int_{-Lx/2}^{Lx/2} x Sin\frac{2\pi x}{\lambda^{tr}_{x}} dx \Bigg/ \int_{-Lx/2}^{Lx/2} x^{2} dx = 3\left[\frac{Sin \pi \chi_{x}}{\pi^{2} \chi^{2}_{x}} - \frac{Cos \pi \chi_{x}}{\pi \chi_{x}}\right] \qquad (9)$$

Where $\chi_{x} = L_{x} / \lambda^{tr}_{x}$ (10)

Table 1 shows normalized accelerations for three seismic zones by MSK-64 scale. The tortional accelerations are calculated with formula (8) and lateral ones are taken from Russian Design Codes (SNiP II-7-81, 1981). Torsional accelerations are calculated when the length of the building is $L=40m$, soil with predominant period of $T_{pred}=0.18$ sec. and the velocity of shear wave of $C=800$ m/sec. Using expressions (10) and (9) gives: $\chi = 0.28$; $\Delta^{hor}_{1} = 0.95$; $\Delta^{hor}_{2} = 0.88$

Table 1. Normalized lateral and torsional accelerations

MSK-64	Lateral acceleration	Torsional accelaration
7	1 m/sec2	0.044 rad/sec2
8	2 m/sec2	0.088 rad/sec2
9	3 m/sec2	0.176 rad/sec2

Table 2. shows the comparison of torsional accelerations obtained from the records and calculated ones (formula 8). Shear wave velocity is *800 m/sec* the length of the building is *40m.*

Table 2. Calculated normalized torsional accelerations

Station	Ax [g]	Ay [g]	Tz(Rec)	Tzx(Cal)	Tzy(Cal)	Tz(Cal)
Ambrolauri	0.154	0.305	0.235	0.091	0.18	0.274
Zemo Bari	0.138	0.083	0.292	0.088	0.052	0.14
Iri	0.047	0.041	0.069	0.03	0.027	0.057
Sachkhere	0.004	0.005	0.004	0.002	0.002	0.004

THE DESIGN OF THE BUILDINGS ON EARTHQUAKE SPATIAL EFFECT

Let us consider symmetric building having frames in longitudinal and transverse directions and floors as rigid disks (non-deformable). The differential equation of natural oscillations of this building due to torque may be expressed:

$$\tilde{K}\frac{\partial^2\theta_z}{\partial z^2} - M_0\frac{\partial^2\theta_z}{\partial t^2} = -M_0\theta''_{z,g}(t) \qquad (11)$$

Where $\tilde{K}$ - is building general stiffness matrix in torque;

M_0 - is the distributed inertial mass of the building along the height;

$\theta''_{z,g}(t)$ - is the torsional acceleration of the soil;

θ_z - is the angle of rotation of floor disk.

Solution of the equation (11) has the form:

$$\theta_{z,n} = \frac{\theta''^{\max}_{z,g}}{\omega_n^2}\theta_n(z)\,\eta_n^{vert}\omega_n\int_0^t f(\tau)e^{-\xi\omega(t-\tau)}Sin\omega_n(t-\tau)d\tau \qquad (12)$$

Where: ω_n - is natural oscillation on the building;

$\theta_n(z)\,\eta_n^{vert}$ - is seismic load vertical distribution function;

$f(\tau)$ - is seismic wave shape function;

ξ - is damping ratio.

Seismic load could be calculated

Lateral ground motion:

$$S_{xn} = k_1k_2k_\psi A_x M\beta_{xn}Z_{xn}(z)\,\eta_n^{vert} \qquad (13.a)$$

$$S_{yn} = k_1k_2k_\psi A_y M\beta_{yn}Z_{yn}(z)\,\eta_n^{vert} \qquad (13.b)$$

$$S_{zn} = k_1k_2k_\psi A_z M\beta_{zn}Z_{zn}(z)\,\eta_n^{vert} \qquad (13.c)$$

Torsional ground motion:

$$M_{zn} = k_1k_2k_\psi\theta''_z M_0\beta^{\theta}_{zn}\theta_n(z)\,\eta_n^{vert} \qquad (13.d)$$

Where θ''_{zn} acceleration is calculated using expression (9)

$k_1k_2k_\psi$ - are seismic coefficients (SNiP II-7-81, 1981)

β - is dynamic (spectral) coefficient

Considered buildings have three degrees of freedom on each level: longitudinal, transverse and torsional, so there is a system of three differential equations. In symmetric structure they are separated and can be solved separately. In asymmetric building they are linked and have the form:

$$\left\|\widetilde{K}\right\| \frac{\partial}{\partial z^2} \left\| \begin{matrix} u_x \\ u_y \\ \theta_z \end{matrix} \right\| - M \frac{\partial}{\partial z^2} \left\| \begin{matrix} u_x \\ u_y \\ r^2\theta_z \end{matrix} \right\| = -M \left\| \begin{matrix} u''_{x,g}(t) \\ u''_{y,g}(t) \\ r^2\theta''_{z,g}(t) \end{matrix} \right\| \qquad (14)$$

Where $\left\|\widetilde{K}\right\|$ - is building general stiffness matrix in X, Y directions and torque around Z axis;

r^2 - is radius of gyration.

During the oscillation of this building there is three centers of rotation and consequently three radiuses of rotation. In symmetric building two centers are located in the infinity, and one is coincided with mass and stiffness center.

Let us consider asymmetric building (see Fig. 3) and choose one of the centers. The shape of seismic wave is considered as cosine for the case when the mass center of the building is located on the crest of the seismic wave. Lateral seismic effect causes torque in the building. The torque distributed along the axes at the center of rotation will be:

$$(M_{nk}^{lat})_b = k_1 k_2 k_\psi A g \beta_{nk} \frac{M}{H}\left[(X_{nk}^0)^2 \Delta_{1x}^{hor} + (Y_{nk}^0)^2 \Delta_{1y}^{hor}\right]\frac{1}{R_{nk}} \mathcal{A}_n^{vert} Z_n(z) \qquad (15)$$

Where X_{nk}^0 and Y_{nk}^0 are coordinates of center of rotation from the center of rigidity.

During the seismic effect when the seismic wave is sine, the center on mass is located on the break point on the seismic wave, the building also experience torque. The value of the torsional moment in this case may be expressed:

$$(M_{nk}^{tor})_b = k_1 k_2 k_\psi A g \beta_{nk} \frac{M_0}{H}\left[\frac{\Delta_{2x}^{hor} X_{nk}^0}{0.5L_x} + \frac{\Delta_{2y}^{hor} Y_{nk}^0}{0.5L_y}\right]\frac{1}{R_{nk}} \mathcal{A}_n^{vert} Z_n(z) \qquad (16)$$

When these moments are transferred to the center of rigidity, there will be two perpendicular forces and one torsional moment on each level. Their values will be:

- Seismic lateral effect (see formula 15):

$$(S_{nk}^x)_b = \lambda_{nk}^{lat} (\omega_n^2)_b^x (Y_{nk}^0 - e_y) \mathcal{A}_n^{vert} Z_n(z) \qquad (17.a)$$

$$(S_{nk}^y)_b = \lambda_{nk}^{lat} (\omega_n^2)_b^y (X_{nk}^0 - e_x) \mathcal{A}_n^{vert} Z_n(z) \qquad (17.b)$$

$$(M_{nk}^z)_b = r^2 \lambda_{nk}^{lat} (\omega_n^2)_b^{tor} \mathcal{A}_n^{vert} Z_n(z) \qquad (17.c)$$

- Seismic torsional effect (see formula 16):

$$(S_{nk}^x)_b^* = \lambda_{nk}^{tor} (\omega_n^2)_b^x (Y_{nk}^0 - e_y) \mathcal{A}_n^{vert} Z_n(z) \qquad (17.d)$$

$$(S_{nk}^y)_b^* = \lambda_{nk}^{tor} (\omega_n^2)_b^y (X_{nk}^0 - e_x) \mathcal{A}_n^{vert} Z_n(z) \qquad (17.e)$$

$$(M_{nk}^z)_b^* = r^2 \lambda_{nk}^{tor} (\omega_n^2)_b^{tor} \mathcal{A}_n^{vert} Z_n(z) \qquad (17.f)$$

Where e_x and e_y are coordinates of center of mass from the center of rigidity

λ_{nk}^{tor} and λ_{nk}^{tor} are coefficients of seismic lateral and torsional effects and are calculated as:

$$\lambda_{nk}^{lat} = M \frac{k_1 k_2 k_\psi A g \beta_{nk}[(X_{nk}^0)^2 \Delta_{1x}^{hor} + (Y_{nk}^0)^2 \Delta_{1y}^{hor}]}{H(\omega_{nk}^2)_b R_{nk}(r^2 + R_{nk}^2)} \qquad (18.a)$$

$$\lambda_{nk}^{tor} = M_0 \frac{k_1 k_2 k_\psi A g \beta_{nk} 2(X_{nk}^0 \Delta_{2x}^{hor} L_x^{-1} + Y_{nk}^0 \Delta_{2y}^{hor} L_y^{-1})}{H(\omega_{nk}^2)_b R_{nk}(r^2 + R_{nk}^2)} \qquad (18.b)$$

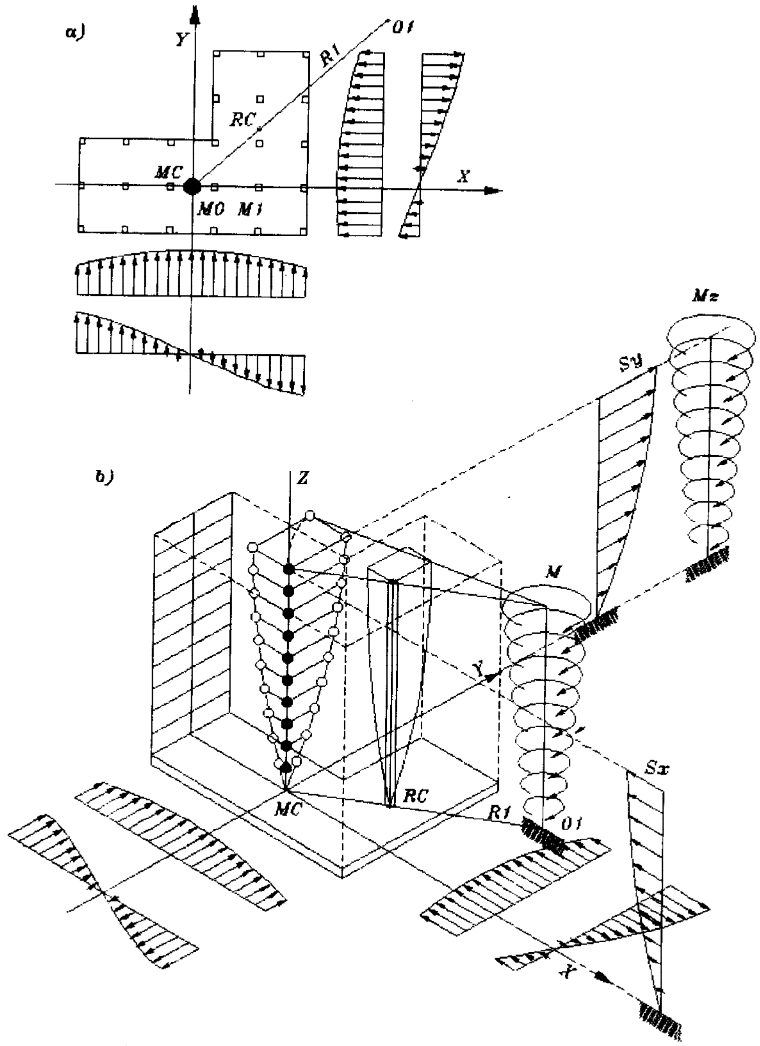

Fig.4 Seismic effects on asymmetric building

Seismic load applied on each frame will be distributed proportional of the natural oscillations of these frames and the expressions will have the form:

$$(S^x_{nk})_i = \frac{(S^x_{nk})_b}{(\omega_n^2)^x_b}(\omega_n^2)^x_i + \frac{(M^z_{nk})_b}{(\omega_n^2)^{tor}_b}(\omega_n^2)^x_i(e_y - b_i)$$

$$(S^y_{nk})_i = \frac{(S^y_{nk})_b}{(\omega_n^2)^y_b}(\omega_n^2)^y_i + \frac{(M^z_{nk})_b}{(\omega_n^2)^{tor}_b}(\omega_n^2)^y_i(e_x - b_i)$$

$$(M^z_{nk})_i = \frac{(M^x_{nk})_b}{(\omega_n^2)^{tor}_b}(\omega_n^2)^{tor}_i$$

Seismic loads due to torque are calculated analogically.

CONCLUSIONS AND RESULTS

The following conclusions are drawn from the research:

- The rotational components of earthquake ground motions are mostly revealed in strong soil and least - in sight one. Consequently, the rigid building works rather better when it is erected on soft soil and, reverse, the slender building behaves well on soft soil and badly - on strong soil.

- Circular ground components are depended on the soil characteristics as well as building parameters. They are also related to the magnitude of translational acceleration.

- When considering three degrees of freedom for each level of the structure, there are three centers of rotations and consequently, three centers of rotation. In asymmetric building the system of three differential equations are linked to each other and each oscillation mode has translational and torsional movement. In symmetric structure two centers are located in the infinity and on is coincided with the center of mass and the center of rigidity, thus the system of three differential equations are separated and the oscillation modes are clear two - translational and one - torsional.

- Proposed method can be easily adapted to each Design Code. In this paper as an example, the Russian Design Codes are presented.

The following results are made:

- The design models of soil was worked out, which enables the determination of the spatial earthquake effect using wave theory. Some of them are not considered in many Design Codes.

- The real values of normalized torsional accelerations for different seismic zones are determined.

- Using proposed method software programs were developed for the deign of the building on earthquake spatial effect.

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