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RELIABILITY OF HYSTERETIC STRUCTURE UNDER EARTHQUAKE EXCITATION

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ABSTRACT

The reliability of a hysteretic structure under earthquake excitations is investigated theoretically. The ground motion is modeled as a random pulse train with a broad-band evolutionary spectrum. Failure is considered to have occurred, once the structural response exceeds a prescribed critical state. The total energy in the structural system is approximated as a Markov process, and its governing equation is derived using a modified version of quasi-conservative averaging procedure. The reliability of the system as a function of time is obtained by using the numerical method of path-integration. Examples are given for illustration.

KEYWORDS

Reliability; hysteretic structure; earthquake excitation; Markov process; path-integration.

INTRODUCTION

Under strong earthquake excitations, a structure is likely to become nonlinear and inelastic. The term hysteresis is used to describe a type of inelastic behavior in which the restoring force depends not only on the instantaneous deformation, but also the past history of the deformation. Consider an engineering structure idealized as a single-degree-of-freedom system governed by

$$\ddot{X} + 2\zeta\dot{X} + (1-\alpha)X + \alpha Z = \xi(t) \tag{1}$$

where Z is a hysteretic force, α is a constant between 0 and 1, representing the level of hysteresis, and $\xi(t)$ is the horizontal ground acceleration. Equation (1) is cast in a non-dimensional form. The effect of vertical ground acceleration is neglected in the investigation, which is justified for most practical cases.

One widely used theoretical model for hysteretic forces has the form of

$$\dot{Z} = -\gamma\,|\dot{X}|Z|Z|^{n-1} - \beta\dot{X}|Z|^{n} + A\dot{X} \tag{2}$$

where A, n, γ and β are parameters. This type of model was proposed initially by Bouc (1967) and extended by Wen (1976, 1980), and it has the following desirable properties: (i) the deformation-force relationship is smooth, thus more amenable to analytical treatments, and (ii) the parameters in the model can be adjusted to match a real hysteresis behavior.

It is reasonable to model the earthquake ground acceleration $\xi(t)$ as a non-stationary stochastic process. A versatile model in which the nature of local geological features and seismic wave propagation can be incorporated is that of a random pulse train, given by

$$\xi(t) = \sum_{j=1}^{N(t)} Y_j h(t-\tau_j) \tag{3}$$

where τ_j is the random time at which the jth pulse arrives at a given site, Y_j is the random magnitude of the jth pulse, $N(t)$ is a Poisson process, $h(t-\tau_j)$ is a deterministic pulse shape function which may be determined from the knowledge of the physical features of the ground. For different j, the magnitudes Y_j are assumed to be independent, but have the same probability distribution as a random variable Y. It is known that such a $\xi(t)$ process possesses an evolutionary spectral density (Lin, 1986)

$$\hat{\Phi}(t,\omega) = \frac{1}{2\pi} \mathrm{E}[Y^2] \, |a(t,\omega)|^2 \tag{4}$$

where E[ ] denotes a statistical average,

$$a(t,\omega) = \int_{-\infty}^{\infty} h(u) \sqrt{\nu(t-u)} \, e^{-i\omega u} du \tag{5}$$

and $\nu(t)$ is the average arrival rate of the random pulses per unit time. We assume that $a(t, \omega)$ is slowly varying with time, so that the correlation function of $\xi(t)$ may be obtained as

$$R(t,\tau) = \mathrm{E}[\xi(t)\,\xi(t+\tau)] = \int_{-\infty}^{\infty} \hat{\Phi}(t,\omega) \, e^{i\omega\tau} d\omega \tag{6}$$

We will be concerned with the total energy (also called the energy envelope) of the system, and the system is considered to have failed if the energy envelope exceeds a prescribed safety level. The energy envelope is expected to vary slowly with time; namely, its relaxation time is long. If this relaxation time is much longer than the correlation time of the earthquake acceleration process, then the energy envelope may be approximated as a Markov stochastic process. In the present paper, a modified version of quasi-conservative averaging procedure is applied to derive a governing stochastic differential equation for such a Markov Process. The reliability function of the system (one minus the probability of failure) as a function of time is obtained by using the numerical procedure of path-integration. Numerical examples are given for illustration.

MODIFIED QUASI-CONSERVATIVE AVERAGING

Traditionally, hysteresis behavior of a dynamical system is interpreted in terms of a cyclic motion. In this case, the dissipated energy is represented by the well-known hysteresis loop. Fig. 1 shows several hysteresis loops corresponding to cyclic motions of different amplitudes. It is, therefore, reasonable to separate the hysteretic force Z in equation (1) into two parts as follows:

$$Z = h(X,\dot{X}) + u(X) \tag{7}$$

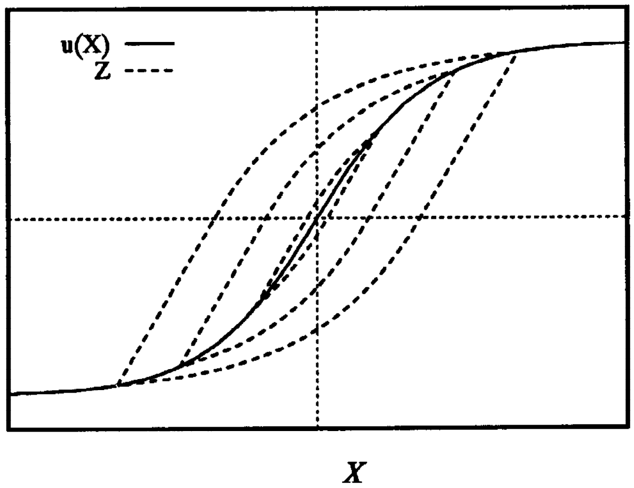

Fig. 1. The Bouc-Wen hysteresis model.

where $h(X,\dot{X})$ is an equivalent damping force, and $u(X)$ is an equivalent spring force. One reasonable choice for the equivalent spring force $u(X)$ is the so-called "backbone", which passes the extremities of all the hysteresis loops, as illustrated in Fig. 1. For a given set of parameters A, n, γ and β, this equivalent spring force can be obtained analytically or numerically from equation (2). The equivalent potential energy and the energy envelope of the system are then

$$U(X) = \frac{1}{2}(1-\alpha)X^2 + \alpha\int_0^X u(y)dy, \qquad \Lambda = \frac{1}{2}\dot{X}^2 + U(X) \tag{8}$$

We assume that (7) and (8) remain valid for non-cyclic motions.

Now letting

$$\begin{aligned} \mathrm{sgn}X\sqrt{U(X)} &= \sqrt{\Lambda}\cos\phi, \qquad 0 \le \phi < 2\pi \\ \dot{X} &= -\sqrt{2\Lambda}\sin\phi \end{aligned} \tag{9}$$

where sgn$X$ is the sign of X, equation (1) may be replaced by

$$\dot{\Lambda} = -4\zeta\Lambda\sin^2\phi + \alpha\sqrt{2\Lambda}\sin\phi\; h(\Lambda,\phi) - \sqrt{2\Lambda}\sin\phi\;\xi(t) \tag{10}$$

$$\dot{\phi} = -2\zeta\sin\phi\cos\phi + \frac{\alpha}{\sqrt{2\Lambda}}\left[h(\Lambda,\phi)\cos\phi + \frac{u(\Lambda,\phi)}{\cos\phi}\right] - \frac{\cos\phi}{\sqrt{2\Lambda}}\xi(t) \tag{11}$$

where $h(\Lambda,\phi)$ and $u(\Lambda,\phi)$ are obtained from $h(X,\dot{X})$ and $u(X)$ by replacing X and $\dot{X}$ by Λ and ϕ, respectively, according to (8). We note in passing that the energy envelope would remain constant if the excitation were periodic, and the system were performing a steady-state periodic motion.

In practice, the evolutionary spectral density of an earthquake excitation is expected to be of a similar order of magnitude as the total system damping. Then the energy envelope $\Lambda(t)$ is slowly varying with time. In this case, the procedure of modified quasi-conservative averaging (Roberts, 1982; Cai, 1994) is applicable, and the averaged $\Lambda(t)$ is approximately a Markov process governed by an Itô stochastic differential equation

$$d\Lambda = m(\Lambda,t)\,dt + \sigma(\Lambda,t)\,dB(t) \tag{12}$$

where $m(\Lambda, t)$ and $\sigma(\Lambda, t)$ are known as the drift and diffusion coefficients, respectively, and they are obtained as follows

$$m(\Lambda,t) = \left\langle -4\zeta\Lambda\sin^2\phi + \alpha\sqrt{2\Lambda}\sin\phi\, h(\Lambda,\phi)\right\rangle_t$$

$$+ \int_{-\infty}^{0} \left\langle \sin\phi(t+\tau)\sin\phi(t) + \cos\phi(t+\tau)\cos\phi(t)\right\rangle_t R(t,\tau)\, d\tau \tag{13}$$

$$\sigma^2(\Lambda,t) = 2\Lambda \int_{-\infty}^{\infty} \left\langle \sin\phi(t+\tau)\sin\phi(t)\right\rangle_t R(t,\tau)\, d\tau \tag{14}$$

In (13) and (14) $\langle\ \rangle_t$ denotes a time averaging procedure, defined as

$$\langle[\cdot]\rangle_t = \frac{1}{T}\int_0^T [\cdot]\, dt \tag{15}$$

in which T is the quasi-period corresponding to a total energy level Λ, obtained on the basis of a hypothetical undamped free motion

$$\ddot{X} + (1-\alpha)X + \alpha u(X) = 0 \tag{16}$$

At a given energy level Λ,

$$T = 4T_{1/4} = 4\int_0^a \frac{1}{\sqrt{2\Lambda - 2U(X)}}\, dX \tag{17}$$

where a is the amplitude corresponding to Λ, namely, a satisfies

$$U(a) = \Lambda \tag{18}$$

Since $\sin\phi$ and $\cos\phi$ are functions of X and $\dot{X}$ according to transformation (9), they are periodic functions with period T for a given Λ. As such, they can be expanded into Fourier series

$$\sin\phi(t) = \sum_{n=1}^{\infty} a_n \sin[(2n-1)\omega_T t] \tag{19}$$

$$\cos\phi(t) = \sum_{n=1}^{\infty} b_n \cos[(2n-1)\omega_T t] \tag{20}$$

where $\omega_T = 2\pi/T$, and coefficients a_n and b_n are calculated from

$$a_n = \frac{2}{T}\int_0^T \sin\phi(t)\sin[(2n-1)\omega_T t]\, dt = -\frac{2}{\sqrt{2\Lambda}\, T_{1/4}} \int_0^{T_{1/4}} \dot{X}\sin[(2n-1)\omega_T t]\, dt \tag{21}$$

$$b_n = \frac{2}{T}\int_0^T \cos\phi(t)\cos[(2n-1)\omega_T t]\,dt = \frac{2}{\sqrt{\Lambda}\,T_{1/4}}\int_0^{T_{1/4}} \sqrt{U(X)}\cos[(2n-1)\omega_T t]\,dt \tag{22}$$

Substitution of (19) and (20) into (13) and (14) leads to

$$m(\Lambda,t) = -\frac{2\zeta}{T_{1/4}}\int_0^{T_{1/4}} \dot{X}^2\,dt - \alpha\frac{A_r(\Lambda)}{T} + \frac{1}{2}\pi\sum_{n=1}^{\infty}(a_n^2+b_n^2)\,\hat{\Phi}[t,(2n-1)\omega_T] \tag{23}$$

$$\sigma^2(\Lambda,t) = 2\pi\Lambda\sum_{n=1}^{\infty} a_n^2\,\hat{\Phi}[t,(2n-1)\omega_T] \tag{24}$$

where $A_r(\Lambda)$ is the area of the hysteresis loop corresponding to the energy level Λ. When carrying out the integrations in (21), (22) and (23), X and $\dot{X}$ are obtained from equation (16) for the free motion. The calculation may be performed numerically over one quarter of a period.

RELIABILITY ANALYSIS

The probabilistic evolution of the energy process $\Lambda(t)$ is described by its probability density at time t on the condition that its value at an earlier time t' is known. This conditional probability density is called the transition probability density, and is governed by the Fokker-Planck equation (e.g., Lin, 1967)

$$\frac{\partial}{\partial t}q = -\frac{\partial}{\partial\lambda}[m(\lambda,t)\,q] + \frac{1}{2}\frac{\partial^2}{\partial^2\lambda}[\sigma^2(\lambda,t)q] \tag{25}$$

in which the unknown q is an abbreviation for $q(\lambda,t\,|\,\lambda',t')$, where λ is a possible value of $\Lambda(t)$ and the symbols λ' and t' behind the vertical bar indicate the condition $\Lambda(t') = \lambda'$. Since the evolutionary spectral density of the random excitation $\xi(t)$ is assumed to be slowly varying with time, the drift and diffusion coefficients will not change appreciably in a short time interval. For a short time step $\Delta t = t - t'$ and under the condition

$$[q(\lambda,t\,|\,\lambda',t')]_{t=t'} = \delta(\lambda-\lambda') \tag{26}$$

the solution for (25) is approximately Gaussian. Specifically,

$$q(\lambda,t\,|\,\lambda',t') = \frac{1}{\sqrt{2\pi\,\sigma^2(\lambda',t')\Delta t}}\exp\left\{-\frac{[\lambda-\lambda'-m(\lambda',t')\Delta t]^2}{2\sigma^2(\lambda',t')\Delta t}\right\} \tag{27}$$

If the initial probability density $p(\lambda,t_0)$ is known, then the probability density $p(\lambda,t)$ can be calculated successively as follows, using the short time solution (27),

$$p(\lambda,t) = \int_0^{\lambda_c} q(\lambda,t\,|\,\lambda',t')\,p(\lambda',t')\,d\lambda' \tag{28}$$

In the step-by-step calculation, the probability density $p(\lambda,t)$ is obtained at discrete points, and its values between these discrete points can be obtained by a suitable interpolation procedure.

We assume that the structure fails if its response displacement $X(t)$ exceeds an allowable level a_c, or equivalently, the total energy $\Lambda(t)$ exceeds the corresponding critical level λ_c, where the two are related as $\lambda_c = U(a_c)$. The critical level λ_c is an absorbing boundary for the energy process in the sense that a sample function is removed from the total population once it reaches λ_c. The above procedure is known as the path integration (e.g., Wehner and Wolfer, 1983; Naess and Johnsen, 1993). The reliability of the system at time t is the probability that the process $\Lambda(t)$ remains below the critical level λ_c. This is obtained as

$$R(t) = \int_0^{\lambda_c} p(\lambda, t)\, d\lambda \tag{29}$$

NUMERICAL EXAMPLES

The application of the above analytical/numerical procedure will now be illustrated in some numerical examples. For the earthquake excitation model (3), we select an average arrival rate for the random pulse train

$$\gamma(t) = t\left(1 - \cos\frac{\pi t}{30}\right), \quad 0 \le t \le 60 \tag{30}$$

and a pulse shape function

$$h(t-\tau) = \omega_g \exp[-\zeta_g \omega_g (t-\tau)]\left\{\frac{1-2\zeta_g^2}{(1-\zeta_g^2)^{1/2}} \sin[\omega_{gd}(t-\tau)] + 2\zeta_g \cos[\omega_{gd}(t-\tau)]\right\}, \quad t > \tau \tag{31}$$

where $\omega_{gd} = \omega_g \sqrt{1-\zeta_g^2}$, $\omega_g = 1$ rad/s, and $\zeta_g = 0.3$. It has been shown that the sample functions generated from this earthquake model have the general appearance of the 1985 Mexico City earthquake records (Lin and Yong, 1987). Fig. 2 depicts the evolutionary spectral density $\hat{\Phi}(t,\omega)$ of the ground acceleration at several different times. It is seen that the spectral density reaches its maximum at about t = 35s.

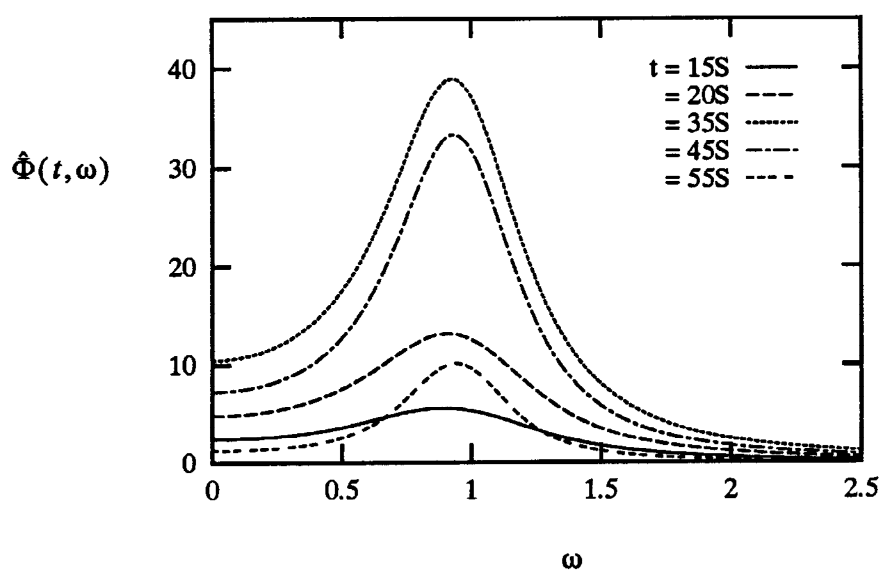

Fig. 2. Spectral densities of an evolutionary process at different times for $E[Y^2] = 1.0$.

Numerical results have been obtained for a structure described by equation (1) with damping ratio $\zeta = 0.1$. The parameters in equation (2) are taken as $n = 1$, $A = 1$ and $\beta = \gamma = 0.5$. The critical amplitude is assumed to be $a_c = 3$, and the reliability is calculated from (29), assuming that the structure is at rest initially, namely, its initial energy level is zero. Fig. 3 depicts the computed reliability functions vs time for a structure of strong hysteresis $\alpha = 0.9$. Several different values are assumed for the mean square magnitude of the random pulse: $E[Y^2] = 0.003, 0.005, 0.01, 0.02$ and 0.05. As expected, the structural reliability decreases with an increase in the excitation level. Except for small excitation intensities ($E[Y^2] = 0.003$ and 0.005 in Fig. 3), the survival probabilities are practically zero toward the end of the earthquake, as shown in Fig. 3. Fig. 4 depicts the calculated reliability functions for structures with different hysteresis levels: $\alpha = 0.1$, 0.3, 0.6 and 0.9. The small differences in the results indicate that the hysteresis level is not an important factor in these examples. A higher level hysteresis corresponds to a softer structure, as well as greater energy dissipation. These two effects are opposite, and they nearly cancel each other in this case.

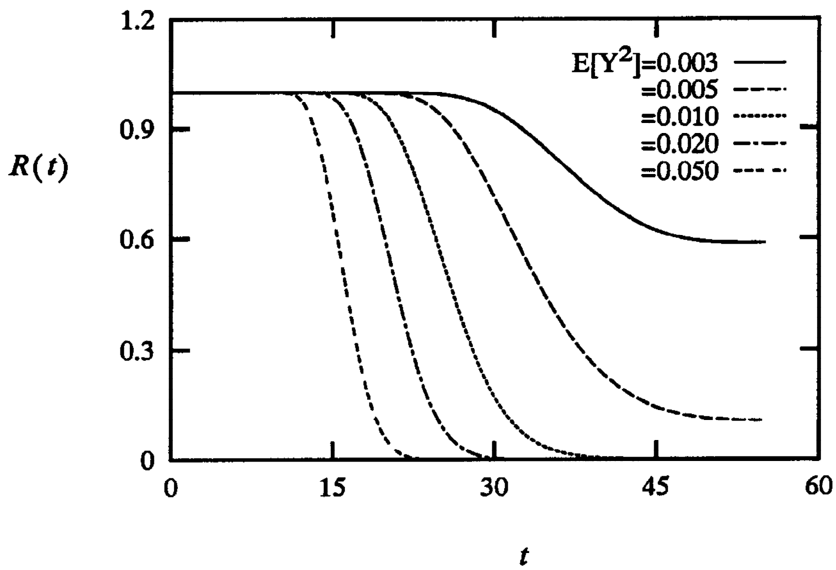

Fig. 3. Reliability for different values of $E[Y^2]$ at $\alpha = 0.9$.

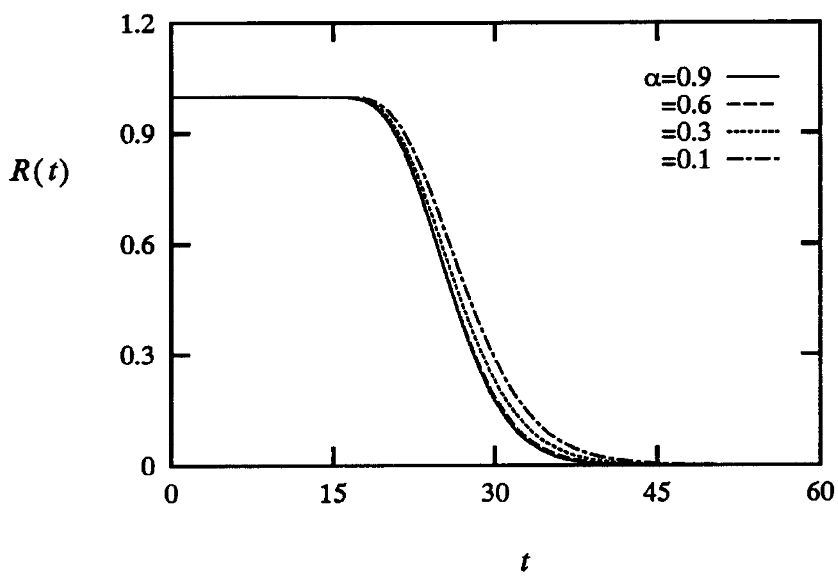

Fig. 4. Reliability for different values of α at $E[Y^2] = 0.01$.

CONCLUSION

Under the assumptions that the ground acceleration is a broad-band evolutionary process and that energy dissipation in a SDOF hysteretic structure is low, the response energy level in the structure may be approximated as a Markov process. The reliability of the structure can then be investigated in terms of the first-passage problem in the theory of stochastic process. By using a modified version of quasi-conservative averaging, in conjunction with the numerical path-integration method, the reliability function can be obtained numerically. The level of hysteresis is shown to be unimportant for the specific examples investigated in the present paper.

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