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FARADAY RESONANCE IN THIN SEDIMENTARY LAYERS

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Abstract

It is well established that earthquake waves passing through soft alluvial valleys and sedimentary basins will generally amplify, so that signals are stronger, longer, and more complex on such sites than on adjacent crystalline rocks. One anomalous feature of the amplification there is the rather significant variability in the recorded strong ground motion, especially over relatively short lengthscales. This paper explains such short-scale variability in the amplification factor by proposing a novel seismic resonance mechanism in thin flat layers. The variability is attributed to the subharmonic generation of standing elastic waves on the surface of the deposit. Nodal regions of little horizontal shaking, or little destruction, are separated by antinodal regions of strong horizontal shaking, i.e. significant damage. Vertical ground shaking follows a similar pattern. The subharmonic surface waves are forced by the vertical component of an input seismic shaking at the base of the deposit, through a linear instability mechanism in the layer. A perturbation theory is worked out to describe the conditions for resonance, and a small shaking tank experiment is conducted to confirm some of the theoretical findings.

Key words: Layered media, sedimentary basins, seismic resonance, strong motion, surface waves, secondary resonance, shaking-table experiment, vertical shaking, stratification.

1 INTRODUCTION

Seismologists have long realized that damage from seismic shaking is appreciably affected by local ground properties. Amplitudes of earthquake waves recorded at sites on water-saturated soft ground may be more than ten times those recorded on nearby crystalline rock (Gutenberg 1957). It is also well known that ground effects may produce appreciable differences in duration and amount of shaking at localities only a fraction of a mile apart. Data collected from a number of recent, well-monitored earthquakes have only served to confirm and reinforce these ideas, sometimes in a very dramatic way. One celebrated example is the 1985 Mexico City earthquake. The source was near the coast some 400 km west of Mexico City. The quake caused little damage near the coast. As the seismic waves propagaged inland, they started to gradually attenuate. Nevertheless, on reaching the city the waves suddenly amplified, inducing severe shaking at some areas which continued to shake for several minutes after the seismic waves had passed. Another well-documented effect is the rather selective destruction associated with the Loma Prieta earthquake in the Oakland-San Francisco Bay Area of Northern California. Similar effects have been reported in the recent Northridge earthquake of Southern California, with a noted prominence of the vertical shaking component.

Early studies (e.g. Haskell 1960) have suggested a primary resonance mechanism to explain the observed amplification. Resonance would occur when the frequency of the incident earthquake wave matches one of the natural frequencies of a surface layer. In a simple 1-D analysis of a vertically propagating elastic shear wave, this resonance condition can be shown to imply that the thickness of the surface layer must be deeper than one-fourth the elastic wavelength. In spite of its simplicity, this classical 1-D flat-layer model has been used extensively by researchers, and in many cases it has yielded results in good agreement with observations.

After the unexpected results of the 1985 Mexico City earthquake, many seismologists realized that this classical model is seriously inadequate, and other more elaborate models began to gain prominence. The new models attempted to extend the 1-D treatment to two dimensions and three dimensions (e.g. Aki and Larner 1970, Bard and Bouchon 1980, Rial 1989, Whitman 1987). These and other studies examined the role of lateral heterogeneities of sedimentary basins in focusing seismic rays and in trapping of seismic energy at the boundary or edges of a basin. The analysis is necessarily numerical in order to deal with the complex multi-dimensional boundary conditions, and in many respects is rather basin-specific. However, a number of general conclusions may also be drawn. One important finding concerns the importance of local generation of surface waves as a significant part of the general basin response. These surface waves are assumed to be generated primarily by the basin's lateral boundaries.

In this paper, we propose a novel seismic resonance mechanism in a surface sediment layer, leading also to the local generation of surface waves, but with the important distinction that the generation is not caused by lateral boundaries. In fact, we are going to adopt a classical flat-layer geometry, with an infinite lateral extent, and perform a 2-D stability analysis of such a uniform layer. In dynamical terms, our proposed mechanism is referred to as a *secondary resonance* mechanism, as opposed to the much more well-known class of primary resonances, to which all of the above-mentioned studies belong. In a secondary resonance there is no direct forcing at the system's natural frequency. Rather, *new* harmonic oscillations in the sediment layer are generated via some inherent instabilities in the system. In particular, we will concentrate on the problem of generating *subharmonic* waves in a shallow sediment layer when it undergoes simple vertical shaking. One important property of this subharmonic resonance is that, unlike primary resonance which requires the forcing frequency to be close to certain discrete values (the natural frequencies), strong amplification will take place for *any* imposed frequency within a wide continuous range, as will be shown in this paper.

Subharmonic resonance is commonly referred to as Faraday resonance, acknowledging the famous scientist who was the first to discover it while performing experiments on water oscillation. In 1831, Faraday performed his famous experiment, where a shallow tank filled with water was made to oscillate uniformly in the vertical direction at a fixed frequency ω. At resonance the free water surface started to undulate violently in the form of stationary, or standing waves with clear nodal lines across the length and/or the width of the tank. The amplitude of the excited surface waves would be significantly larger, perhaps an order of magnitude larger than the amplitude of forced shaking at the base. It was also found that the frequency of the excited surface wave was $\omega/2$, or one-half of the shaking frequency. More than a century later a valid theory was developed to explain this subharmonic resonance phenomenon by Benjamin and Ursell (1954). The basic process is described by Mathieu's equation (Nayfeh and Mook 1979)

$$m \frac{d^2 x}{d t^2} + (\delta + \epsilon \cos 2 \omega t)\, x = 0 \qquad (1)$$

where x is the displacement of an equivalent one-dimensional spring-mass system, m is the mass, and $(\delta + \epsilon \cos 2\omega t)$ is the spring parameter which is modulating with time at a frequency of 2ω. It is well known that x becomes unstable if oscillating at the subharmonic frequency ω. In the case of vertical shaking, the spring parameter is proportional to the gravitational acceleration g. With respect to a coordinate system fixed on the shaking base, the effective gravity g_e will be periodically modulating at the imposed shaking frequency. Subharmonic surface-wave perturbation will then interact resonantly with this modulating gravity, giving rise to an exponentially-growing standing surface wave.

This Faraday resonance appears to offer a simple explanation of the amplification variability question. The variability is attributed to one of the basic properties of standing waves. Horizontal shaking has nodal regions of little oscillation, and hence little destruction, while antinodal regions one-quarter of a wavelength away are experiencing strong horizontal shaking, i.e. significant building damage. Vertical shaking has the same pattern, but is shifted in space by one-quarter wavelength. In this paper, a combined analytical-experimental approach is followed in order to examine Faraday resonance for earthquake waves. In the analytical part, a perturbation analysis is developed to describe the instability mechanism. In the experimental part, a small shaking-tank experiment is conducted in order to examine and verify the theoretical predictions.

2 MATHEMATICAL FORMULATION

We consider two-dimensional (plane-strain) dynamic soil response, and introduce the Cartesian coordinates (x,y) with the origin placed on the base of the soil layer, and y is positive upward as shown in Figure 1. The rigid base is assumed to oscillate vertically around a mean position, with a uniform amplitude W and a frequency 2ω. Therefore, the vertical displacement w_o of the base about its mean position is given by

$$w_o = W \sin 2\omega t \tag{2}$$

Following Benjamin and Ursell (1954), we fix the coordinate system on the oscillating base, so that the base is fixed at $y = 0$. This, however, will give rise to a modulating gravity, with an effective gravitational acceleration g_e in this oscillating coordinate system given by

$$g_e = g\,(1 - \epsilon \sin 2\,\omega t); \qquad \epsilon = 4W\omega^2/g \tag{3}$$

Typically, the peak of the forcing vertical acceleration during a seismic event is a small fraction of g; the fixed-frame gravitational acceleration, so that we will assume a small parameter epsilon, i.e.

$$\epsilon \ll 1 \tag{4}$$

As for the soil layer, we assume that the undisturbed soil layer has a constant thickness h which is much smaller than the P-wave length scale so that we may ignore compressibility effects. We further assume that the response of the layer to small disturbances will be viscoelastic, i.e. there will be both elastic and viscous components in the harmonic soil response to base shaking. We adopt the Voigt model for the viscoelastic behavior. This, in one-dimension, corresponds to a spring and dashpot connected in parallel. The spring is taken to obey Hook's law and the dashpot is drawn through a Newtonian viscous fluid. In a general viscoelastic model, both the elastic stiffness coefficient and the fluid viscosity coefficient are allowed to take on any value. Realistically, however, a soil layer is very likely to have a much weaker viscous component as compared to its elastic component. Therefore, we will restrict the analysis here to just the weak-viscosity range of the viscoelastic response. The equation of motion for a small disturbance in an incompressible viscoelastic layer is then given by (Kolsky 1963)

$$\frac{\partial^2 \boldsymbol{X}}{\partial t^2} = -\frac{1}{\rho}\nabla p + \frac{G}{\rho}\nabla^2 \boldsymbol{X} + \epsilon \nu \frac{\partial}{\partial t}\nabla^2 \boldsymbol{X} - g_e \boldsymbol{j} \tag{5}$$

where $\boldsymbol{X}$ are particle displacements, p is the pressure, ρ is the density, G the shear modulus of elasticity and $\epsilon\nu$ the kinematic viscosity. The small parameter ϵ in the definition of the kinematic viscosity parameter highlights the weak-viscosity assumption of the present study.

At the ground level, the free surface is allowed to undulate freely (Figure 1) with a vertical deflection $\eta(x, t)$ measured from its undisturbed horizontal position $y = h$. In particular, we assume that the free surface is slightly perturbed from its flat horizontal plane, and is undulating sinusoidally in the form of a small standing wave with a frequency ω (one-half that of the shaking base), i.e. we assume that

$$\eta = A \cos kx \sin \omega t; \qquad A \ll 1 \tag{6}$$

where k is the surface-wave wavenumber and A is the wave amplitude, assumed initially to be very small. The response of the soil layer to this perturbation is obtained through the solution of a non-homogeneous boundary-value-problem. The details of the solution is given in Foda and Chang (1995). The main result is an evolution equation for the amplitude A of the perturbation wave, to show if the wave will grow with time (unstable perturbation) or decay (stable perturbation):

$$\frac{\partial A}{\partial t} = (\lambda - \beta) A \tag{7}$$

Clearly, Equation 7 predicts that the small amplitude A will grow exponentially with time if the instability coefficient λ is larger than the damping coefficient β. Figure 2 shows a numerical example which resembles the situation for the 1985 Mexico City earthquake. Calculations for the growth rate of subharmonic waves are made for a surface layer of thickness h = 35m, density ρ = 1,700 kg/m^3, shear modulus G = 10^7 N/m^2 (shear wave velocity c_s = 77 m/s) and a viscosity coefficient ν = 0.01 m^2/s. These values are close to those reported as the average soil conditions at the heavily damaged sites in Mexico City during the 1985 quake (Seed et al. 1987). The Figure shows an active range of subharmonic wave generation in the wave period range from 0.8 to about 1.5 second. The corresponding range of wave period for the generating vertical shaking is obtained by dividing these values by half, i.e. from 0.4 to 0.75 second.

Figure 3 shows another numerical examle for the case of a very thin, soft layer of water-saturated sediment placed on bedrock. The shear modulus is 10^5 N/m^2, the density is 2,000 kg/m^3 (c_s = 7 m/s), the viscosity is 0.01 m^2/s, and the layer depth varies from 5m to 15m. Notice the extraordinary growth rates associated with these situations, several orders of magnitude higher than the values in Figure 2. This demonstrates the very high risk of destruction and ground failure associated with extremely soft, saturated soils. In real sedimentary basins, soil profiles are expected to show certain vertical stratification in the soil's properties. Such stratification is further expected to display a general softening trend as one approaches ground level, i.e. shear wave velocity would be lowest near the surface, and generally increases with depth. For such stratification profiles, and judging from the results in Figures 2 and 3, one may therefore expect that accounting for stratification in sedimentary basins, rather than using average soil properties, would result in an increase in predicted amplification of subharmonic surface waves. Confirmation of this expectation, however, is left to future studies.

3 EXPERIMENTAL STUDY

A small shaking-tank experiment was conducted at the Richmond Field Station of the University of California at Berkeley. The sediment bed is placed inside a 3(L) x 2(W) x 2(H) ft plexiglass tank mounted on top of a hydraulically-driven shaking table. The table is capable of oscillating in the vertical direction at a prescribed amplitude and a prescribed frequency. The prime mover is an electric motor (Electra Motors, model CA1542 LC, with 20 HP, 3 Phase, 60 rpm) which drives a positive displacement pump. A hydraulic cylinder is placed under the center of the shaking table, and its function is to initiate and maintain the table's vertical shaking.

In order to prevent twisting and lateral motion of the sediment tank, four bearing guides are placed at the four corners of the tank. The developed system is capable of providing a wide range of shaking frequencies and a maximum shaking amplitude of 1.5 inches.

Plastic beads of a uniform diameter of 3 mm and a density of 1,015 kg/m^3 are used to make a saturated sediment bed inside the plexiglass tank. Before placing the beads, an array of pore pressure transducers

(Data Instruments model AB/HP, and Kyowa model BP 500 GRS) are installed at designated locations inside the tank, fastened to thin steel wires that run from bottom to top of tank. Next, water is slowly poured into the tank, and finally the plastic beads are gently dropped at the water surface until reaching a desired depth. The excess water is then syphoned out, and the plastic bead surface is smoothed out. Further detailed discussion of the experimental set-up and procedure is given in Chang (1994).

Figure 4 shows a wave-gauge record of surface oscillation η during the generation of a subharmonic wave. Resonance occurred about thirteen seconds after the initiation of vertical shaking. Prior to resonance the bed surface was observed to oscillate at the same frequency as the imposed shaking, with a comparable oscillation amplitude of about 0.25 cm. As shown in the detailed enlargement in Figure 4 around that time, resonance has resulted in two prominent effects. One is the doubling of the period of oscillation as the subharmonic wave dominates the signal. The second is the significant amplification of surface oscillation (about a six-to-eightfold increase in the amplitude of oscillation in the inset of Figure 4). This record was taken at a location close to an antinode of the generated standing wave. Right at the antinode, however, the amplification would clearly be larger.

4 CONCLUSION

We have demonstrated in this paper, both theoretically and experimentally, that a surface sediment layer in vertical shaking is inherently unstable to subharmonic standing surface-wave perturbations. These subharmonic waves, when resonantly excited, can lead to very large amplification of ground shaking (both horizontal and vertical) relative to the amplitude of the forcing base shaking, regardless of how shallow the layer is. The amplification, however, is not uniform spatially, but follows a standing surface-wave pattern of intense amplifications at anti-nodal zones and intervening nodal zones of no or negligible amplifications.

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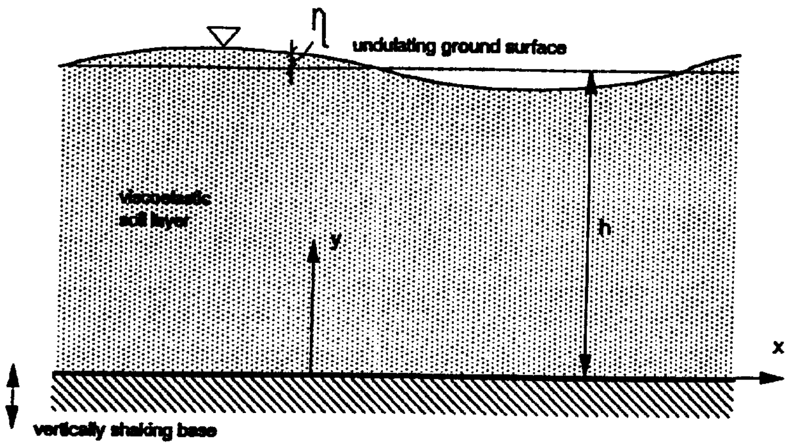

Figure 1: Definition sketch for our flat horizontal layer of infinite lateral extent in vertical shaking. The ground surface is allowed to undulate freely $\eta(x,t)$.

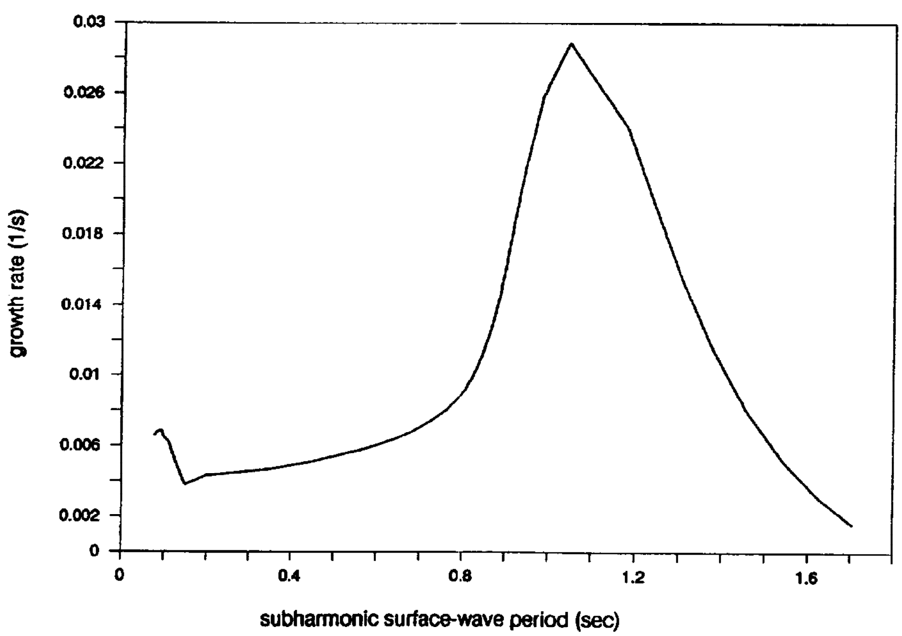

Figure 2: Growth rate of subharmonic waves in a layer of depth $h = 35$ m, shear-wave velocity $c_s = 77$ m/s, and viscosity $\nu = 0.01$ m^2/s.

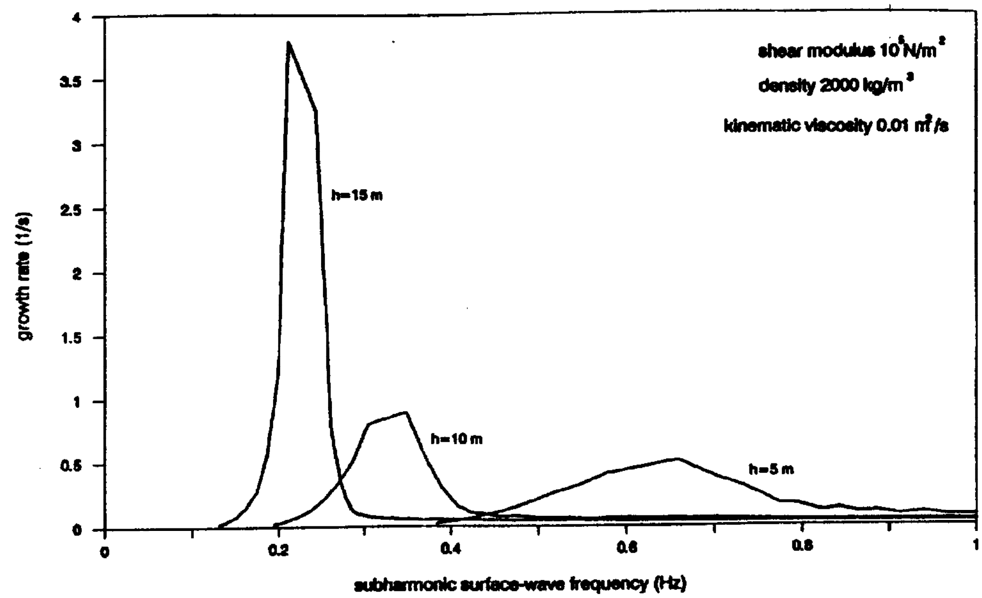

Figure 3: Growth rate of subharmonic perturbations in very soft and very shallow surface layers as a function of the perturbation frequency in Hz (1/2 the input shaking frequency).

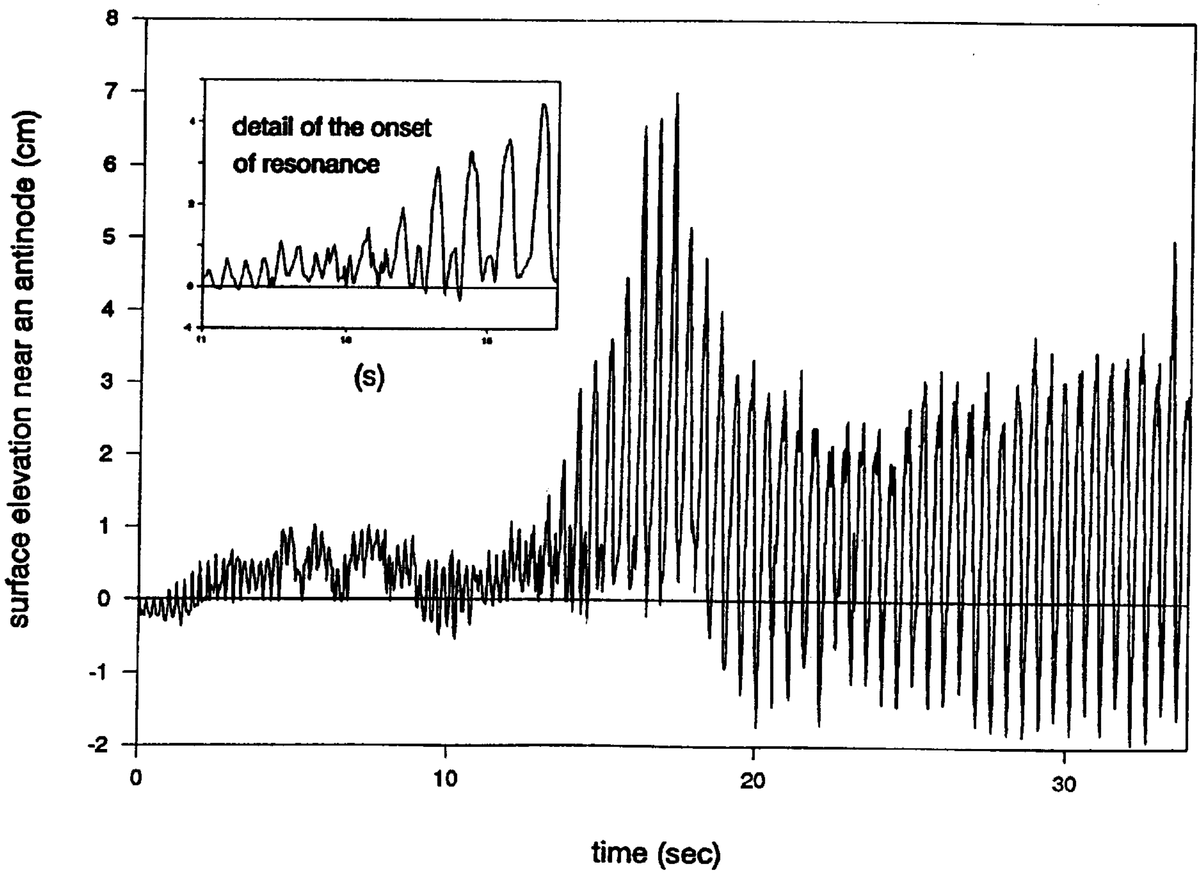

Figure 4: Sample record of vertical oscillation of bed surface during a subharmonic resonance run. Notice the resulting doubling of the period of oscillation.