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SEISMIC DAMAGE DETECTION OF MULTI-STORY BUILDINGS USING VIBRATION MONITORING

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ABSTRACT

A three stage scheme is presented to detect the column damage of multi-story buildings by using ambient vibration measurement. First of all, the change in natural frequencies, damping ratios and mode shapes is identified from ambient vibration time series by a multivariate ARMA model. Then, damaged stories containing damaged columns are searched in terms of the change in curvature of mode shapes. Lastly, the location of damaged columns is pinpointed and the extent of damage is evaluated in terms of the change in stiffness and damping of each column by an inverse modal perturbation, taking account of the variability of identified modal properties. To verify the applicability of the scheme, a series of shaking table tests of a multi-story building experimental model have been performed. Based on the results identified by using random response data, the effectiveness and limitation of the damage detection scheme are discussed.

KEYWORDS

Damage detection, system identification, vibration monitoring, multi-story building, shaking table test, multivariate ARMA model, inverse modal perturbation.

INTRODUCTION

Most building structures continuously accumulate damage during their service life. The damage of building structures may be due to natural hazards such as earthquakes and wind storms and due to long duration aging. For the purpose of assuring seismic safety, it is necessary to monitor the damage as to its occurrence, its location and as to the extent of damage. Undetected damage may potentially cause more seismic damage and eventually catastrophic structural failure. In addition, the performance requirements of recent building structures become more complex and critical. Hence, a rapid structural damage detection is more essential. Information on the damage may be utilized to make a decision on whether repairs, reinforcement, partial replacement or demolition should be done after severe natural hazards or long duration usage.

Periodic monitoring of existing building structures using vibration measurement is one of the effective nondestructive tests to identify their damage states. Especially, vibration measurement under natural ambient excitations, such as micro-tremors and gentle winds, seems to be preferable. The reason is mainly due to the daily acquisition of measurement data, the easy installation of measurement instruments and the cost of artificial excitations such as vibration generator and explosion tests. For the purpose of damage detection, a variety of system identification techniques have been proposed and applied to existing structures (Natke *et al.*, 1988,1993; Hamamoto and Kondo, 1992,1994). The system identification may be carried out in the frequency domain or in the time domain. The time domain approach is superior to the frequency domain approach because of a good discrimination of modal properties between different modes of vibration. An autoregressive moving average (ARMA) model belongs to the time domain approach and has been successfully used to identify modal properties of structures (Gersch *et al.*, 1976). On the other hand, an inverse modal perturbation has been often used to detect the location and extent of damage by making use of the change in modal properties between damaged and undamaged states in the fields of offshore and space technology (Sandstorm and Anderson, 1982 ; Chen, 1988).

In this study, a three stage damage detection scheme is presented and applied to multi-story buildings. The first stage is to identify the change in modal properties such as natural frequencies, damping ratios and mode shapes by using a multivariate ARMA model at the system level. The second stage is to search for damaged stories by making use of the change in modal properties. The third stage is to pinpoint the location and extent of column damage by using an inverse modal perturbation at the element level. Based on the results identified by using random response data of shaking table tests, the effectiveness and limitation of the damage detection scheme are discussed.

DAMAGE DETECTION SCHEME

The schematic representation of a three stage damage detection scheme is shown in Fig.1. The determination of a mathematical model being in undamaged state is required to update the model in accordance with the parameter estimation using an equivalent linear model. If the damage affects a structural system to such an extent that the equivalent linear model is no longer applicable for the prediction of damage, it may still be used for the detection of damage. Once the initial mathematical model is determined, the subsequent procedure may consist of periodic and nonperiodic monitoring.

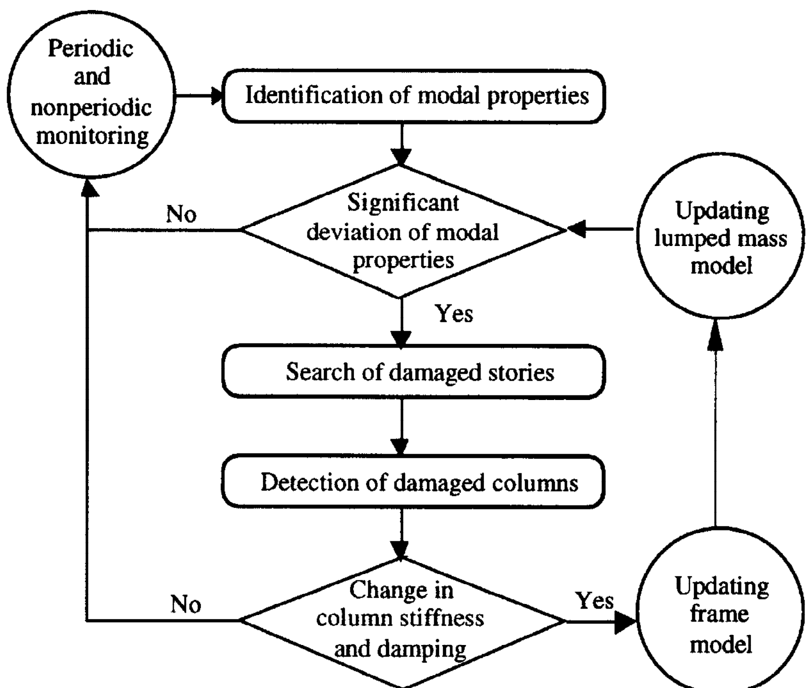

Fig.1 Three stage damage detection scheme

A significant deviation in natural frequencies or damping ratios from the undamaged state indicates the possible occurrence of damage. When the significant deviation is not observed, vibration monitoring is repeatedly continued. If the significant deviation is observed, the change in mode shapes is used to search for damaged stories. Then, the change in all modal properties is used to locate damaged columns in the damaged stories and to evaluate the extent of damage quantitatively. Based on the results, the initial mathematical model is replaced by the updated one. First the frame model is updated and then translated into the lumped mass model. The above procedure is iterated until a decision making is made on whether repair, reinforcement, partial replacement or demolition should be done.

CHANGE IN MODAL PROPERTIES

A multivariate autoregressive moving average (ARMA) model is used to identify the change in modal properties due to damage.

The multivariate autoregressive, AR(p), model is fitted to the time series of structural response under ambient excitations. The multivariate AR(p) model is described by

$$\{Y_t\} - \sum_{k=1}^{p} [\Psi_k]\{Y_{t-k}\} = \{X_t\}, \tag{1}$$

in which $\{X_t\}$ is the discrete time series vector of a white noise random excitation, $\{Y_t\}$ is the discrete time series vector of a stationary random response, and $[\Psi_k]$ is the autoregressive coefficient matrix. Writing down eq.(1) from $p+1$ to M with respect to t and using a linear least squares method, matrix $[\Psi_k]$ can be determined.

The multivariate AR(p) model is converted to a multivariate ARMA(n,m) model by applying an inverse function method (Pandit and Wu, 1983). The multivariate ARMA(n,m) model is described by

$$\{Y_t\} - \sum_{k=1}^{n} [\Phi_k]\{Y_{t-k}\} = \{X_t\} - \sum_{k=1}^{m} [\Theta_k]\{X_{t-k}\}, \tag{2}$$

in which $[\Phi_k]$ is the autoregressive coefficient matrix and $[\Theta_k]$ is the moving average coefficient matrix.

The transfer function of the multivariate ARMA (n,m) model is put into a parallel form realization by using a partial fraction expansion. Using Z-transform operator, the transfer function of the multivariate ARMA(n,m) model is obtained as

$$[H(Z)] = \left[[I] - \sum_{k=1}^{m} [\Theta_k] Z^{-k}\right]\left[[I] - \sum_{k=1}^{n} [\Phi_k] Z^{-k}\right]^{-1}, \tag{3}$$

in which $[\ I\ ]$ is the identity matrix. We first determine the poles, *i.e.*, the roots of denominator polynomial in eq.(3). For a stable system, the poles are all in complex conjugate pairs and have modulus less than one, *i.e.*, the poles are all located inside the unit circle in the complex plane. By combining the pair of terms corresponding to the pair of complex conjugate poles, the transfer function of the multivariate ARMA(n,m) model can be rewritten as

$$[H(Z)] = \sum_{i=1}^{N} \left(\frac{[R_i]Z}{Z-\alpha_i} + \frac{[R_i^*]Z}{Z-\alpha_i^*} \right), \tag{4}$$

in which N is the number of degrees-of-freedom, α_i and α_i^* are the *i-th* pair of complex conjugate poles, and $[R_i]$ and $[R_i^*]$ are the *i-th* pair of complex conjugate residues given by

$$[R_i] = \frac{H(Z)(Z-\alpha_i)}{Z}\Big|_{Z=\alpha_i}, \quad [R_i^*] = \frac{H(Z)(Z-\alpha_i^*)}{Z}\Big|_{Z=\alpha_i^*}. \tag{5}$$

The form of eq.(4) is called a parallel form realization of the transfer function and may be described by the corresponding mechanical model. The highest power of eq.(3) is equal to twice the number of modes of vibration. Equation (4) shows that the transfer function, $[H(Z)]$, is equal to the sum of multiple second order filters. Each second order filter represents a single-degree-of-freedom damped oscillator and corresponds to a mode of vibration. The relationships between the *i-th* damped natural frequency, ω_i, and damping ratio, h_i, and the *i-th* pair of complex conjugate poles, α_i and α_i^*, are given by

$$\omega_i = \sqrt{\log\alpha_i \log\alpha_i^*} / \Delta t, \quad h_i = -\left(\log\alpha_i + \log\alpha_i^*\right)\cdot\left(2\omega_i\Delta t\right), \tag{6}$$

in which Δt is the time interval of discrete time series. The pairs of complex conjugate residues correspond to mode shape coefficients. The absolute value of complex residues is the amplitude of mode shape coefficient. The argument of complex residues is used to judge on whether the mode shape coefficient is positive or negative.

The change in modal properties between undamaged and damaged states is determined. The change in the *i-th* natural frequency, damping ratio and mode shape between damaged and undamaged states may be given by

$$\Delta\omega_i = \omega_{0i} - \omega_{ti}, \quad \Delta h_i = h_{ti} - h_{0i}, \quad \{\Delta\varphi_i\} = \{\varphi_{ti}\} - \{\varphi_{0i}\}, \tag{7}$$

in which the subscripts 0 and t denote undamaged and damaged states, respectively, and Δ represents the change in modal properties between undamaged and damaged states. The change in mode shapes may be simply expressed in terms of undamaged mode shapes as follows:

$$\{\Delta\varphi_i\} = \sum_{j=1, j\neq i}^{N} \beta_{ij}\{\varphi_{0j}\}, \tag{8}$$

in which β_{ij} denotes the participation of the *j-th* mode to the change in the *i-th* mode.

SEARCH OF DAMAGED STORIES

Damaged stories are searched by comparing damaged mode shapes with undamaged ones if the change in magnitude of damage is reasonably large. However, when the damage is small, it is not an easy task to distinguish the change in mode shapes due to damage. In this study, the curvature of mode shapes is introduced as a damage indicator for locating damaged stories. The curvature of mode shapes is much more sensitive to damage than mode shapes themselves. On the other hand, the variability of the curvature of mode shapes becomes considerably large and the useful information on damage tends to be disturbed. To overcome this problem, a long recorded time series are broken into a number of segments and each segment is used to calculate the sample mean and variance of the curvature of mode shapes. The mean offsets the randomness of each sample, while the variance represents the variability of all samples.

A building structure is idealized as the lumped mass model. The change in curvature of mode shapes in undamaged and damage states is given as

$$CURV_{ij} = \left|C_{ij,t} - C_{ij,0}\right|, \tag{9}$$

in which C_{ij} is the curvature of mode shapes in the *i-th* mode at *j-th* lumped mass, subscripts 0 and t represent damaged and undamaged states, respectively. C_{ij} may be expressed by the central difference expression as

$$C_{ij} = \left(\phi_{i,j-1} - 2\phi_{i,j} + \phi_{i,j+1}\right)/h^2, \tag{10}$$

in which ϕ_{ij} is the mode shape amplitude in the *i-th* mode at *j-th* lumped mass, and h is the distance between the j-1 and j+1 th lumped masses. By calculating the change in curvature of mode shapes at each lumped mass, we can find the damaged column which is connected to the lumped mass with a large magnitude of C_{ij}.

DETECTION OF DAMAGED COLUMNS

An inverse modal perturbation is applied to detect the location and extent of damaged columns. The modal properties identified by a multivariate ARMA model are used to obtain the change in stiffness and damping of each column.

To determine the change in dynamic characteristics in such a way that a small change in modal properties is satisfied, a linearized model of structural system is constructed. A structural system in an undamaged state is described by the system of equation

$$[M_0]\{\ddot{X}\} + [C_0]\{\dot{X}\} + [K_0]\{X\} = 0, \tag{11}$$

in which $[M_0]$, $[C_0]$ and $[K_0]$ represent the mass, damping and stiffness matrices, respectively, and $\{X\}$ is a vector of structural displacement. At some later time, the structure is damaged in one or more locations and the resulting equation of motion becomes

$$[M_t]\{\ddot{X}\}+[C_t]\{\dot{X}\}+[K_t]\{X\}=0, \tag{12}$$

During the intervening period, the stiffness and damping matrices of structural system are changed as follows:

$$[K_t]=[K_0]-[\Delta K], \quad [C_t]=[C_0]+[\Delta C], \tag{13}$$

The mass matrix is assumed to be unchanged throughout the service life, *i.e.*, $[M_0]=[M_t]$.

The relationship between the change in modal properties and the change in stiffness and damping of structural system is derived. Substituting eqs.(7) and (13) into eq.(12), making use of usual orthogonal restrictions, and disregarding more than second order terms, the change in modal properties may be given as

$$\Delta\lambda_i=-\frac{1}{M_{0i}}\left[\{\phi_{0i}\}^T[\Delta K]\{\phi_{0i}\}+\lambda_{0i}\{\phi_{0i}\}^T[\Delta C]\{\phi_{0i}\}\right], \tag{14}$$

$$\{\Delta\phi_i\}=-\sum_{j=1,j\neq i}^{2N}\frac{\{\phi_{0j}\}}{M_{0j}(\lambda_{0i}-\lambda_{0j})}\left[\{\phi_{0j}\}^T[\Delta K]\{\phi_{0i}\}+\lambda_{0i}\{\phi_{0i}\}^T[\Delta C]\{\phi_{0i}\}\right], \tag{15}$$

in which M_{0i} is the *i-th* generalized mass in the undamaged state.

The relationship between the change in modal properties and the change in stiffness and damping of each column is derived. Decomposing the change in stiffness and damping of structural system, $[\Delta K]$ and $[\Delta C]$, into the change in stiffness and damping of each column, $[\Delta K_e]$ and $[\Delta C_e]$, *i.e.*,

$$[\Delta K]=\sum_{e=1}^{M}[\Delta K_e]=\sum_{e=1}^{M}[K_{e0}]\ \alpha_{ek}, \quad [\Delta C]=\sum_{e=1}^{M}[\Delta C_e]=\sum_{e=1}^{M}[C_{e0}]\ \alpha_{ec}, \tag{16}$$

in which M is the number of columns in the damaged stories, $[K_{e0}]$ and $[C_{e0}]$ are stiffness and damping matrices of undamaged columns, respectively, and α_{ek} and α_{ec} represent the stiffness and damping reduction ratios. Substituting eq.(16) into eqs.(14) and (15), we can write as

$$\Delta\lambda_i=-\frac{1}{M_{0i}}\left[\sum_{e=1}^{M}\{\phi_{0i}\}^T[K_{e0}]\{\phi_{0i}\}\ \alpha_{ek}+\lambda_{0i}\sum_{e=1}^{M}\{\phi_{0i}\}^T[C_{e0}]\{\phi_{0i}\}\ \alpha_{ec}\right], \tag{17}$$

$$\{\Delta\phi_i\}=-\sum_{e=1}^{M}\left(\sum_{j=1,j\neq i}^{2N}\frac{\{\phi_{0j}\}}{M_{0j}(\lambda_{0i}-\lambda_{oj})}\{\phi_{0j}\}^T[K_{e0}]\{\phi_{0i}\}\right)\alpha_{ek}$$
$$-\sum_{e=1}^{M}\left(\sum_{j=1,j\neq i}^{2N}\frac{\lambda_{oi}\{\phi_{0j}\}}{M_{0j}(\lambda_{0i}-\lambda_{oj})}\{\phi_{0j}\}^T[C_{e0}]\{\phi_{0i}\}\right)\alpha_{ec}. \tag{18}$$

Thus, the change in modal properties is expressed by the change in stiffness and damping of each column in the damaged stories.

Equations (17) and (18) may be arranged as the following matrix form:

$$\{\Delta R\}=[S]\{\alpha\}, \tag{19}$$

in which $\{\Delta R\}$ is the vector consisting of $\Delta\lambda_i$ and $\{\Delta\phi_i\}$, $[S]$ is the sensitivity matrix and $\{\alpha\}$ is the vector of α_{ek} and α_{ec} . The change in stiffness and damping of each column is determined, taking account of the variability of identified modal properties. The objective function weighted by the variability of modal properties is given by

$$J = (\{\Delta R\} - [S]\{\alpha\})^T [W](\{\Delta R\} - [S]\{\alpha\}), \tag{20}$$

in which [W] is the weighting matrix given by

$$[W] = \begin{bmatrix} 1/\sigma_1^2 & & & 0 \\ & 1/\sigma_2^2 & & \\ & & \ddots & \\ 0 & & & 1/\sigma_r^2 \end{bmatrix}, \tag{21}$$

where σ_i^2 is the *i-th* sample variance of $\{\Delta R\}$ and r is the number of modal properties to be taken. The optimal value $\{\hat{\alpha}\}$ may be obtained by minimizing eq. (20) as

$$\{\hat{\alpha}\} = [S_v]\{\Delta R\}, \tag{22}$$

in which

$$[S_v] = ([S]^T [W][S])^{-1} [S]^T. \tag{23}$$

EXPERIMENTAL VERIFICATION

A series of shaking table tests have been performed on a multi-story building experimental model as shown in Fig.2. The damage is represented by replacing columns from the member with ϕ 6 to ϕ 3 as shown in Fig.3. An undamaged state(Case0) and two damaged states(Cases1 and 2) are considered; one and two column members are replaced in the 1st story, respectively. Nearly white noise time series are generated in two orthogonal horizontal directions and used as the excitations to the experimental models at the base. Structural responses are measured at diagonal corners of the 1st and 5th floors by acceleration sensors. Sampling time interval is selected as 0.01 sec.

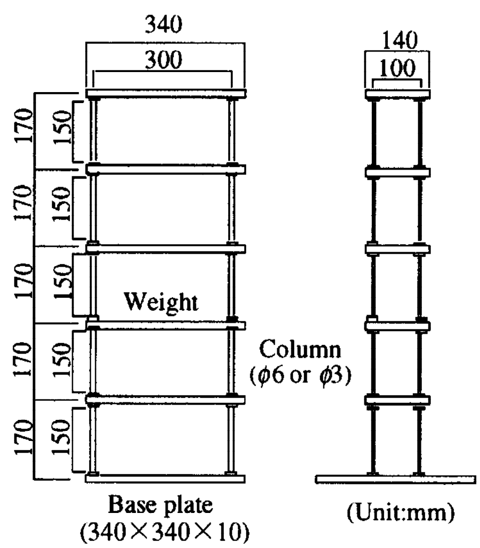

Fig.2 Experimental model

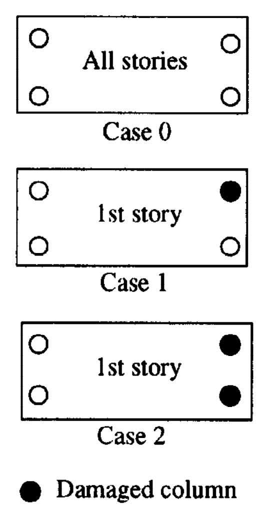

Fig.3 Undamaged and damaged states

The natural frequencies, damping ratios and mode shapes of the experimental models are identified using two orthogonal horizontal components of acceleration response. AR(19) and ARMA(10,9) models are used. Table 1 shows the sample means and coefficients of variation of identified natural frequencies and damping ratios. It is observed that the identification of natural frequencies is much more robust (small variability) than that of damping ratios. In what follows, damping ratios are not used in the detection of column damage because of their large variability.

Table1 Identification of natural frequencies and damping ratios

Mode	Frequency(Hz)			Damping ratio		
No.	Case0	Case1	Case2	Case0	Case1	Case2
1st	8.11	7.66	7.22	0.0078	0.0084	0.0110
	(0.008)	(0.007)	(0.011)	(0.714)	(0.730)	(1.006)
2nd	24.08	23.20	22.24	0.0038	0.0047	0.0037
	(0.003)	(0.007)	(0.003)	(0.746)	(0.943)	(0.911)
3rd	38.41	37.51	36.48	0.0041	0.0056	0.0025
	(0.003)	(0.003)	(0.004)	(0.613)	(0.430)	(0.792)

( ) : Coefficient of variation

Figure 4 shows the identified mode shapes in the first three modes for each case. It seems to be difficult to pinpoint the damaged stories because the change in mode shapes is small between undamaged and damaged states. Figure 5 shows the sample mean and variability of the change in curvature of mode shapes at each lumped mass. It is observed that the change in curvature of mode shapes becomes large at the damaged 1st story except for the 3rd mode in Case2.

Figure 6 shows the stiffness reduction ratio of each column in the 1st story for different combinations of the 1st and 2nd translational/torsional modes. In Case1, the location of a damaged column is not clearly pinpointed for

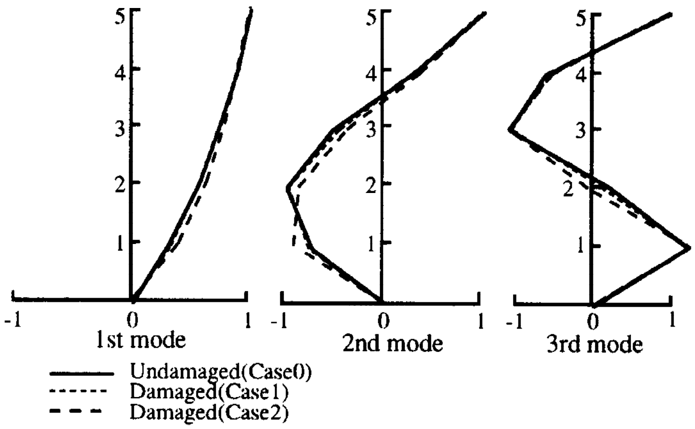

Fig.4 Identified mode shapes

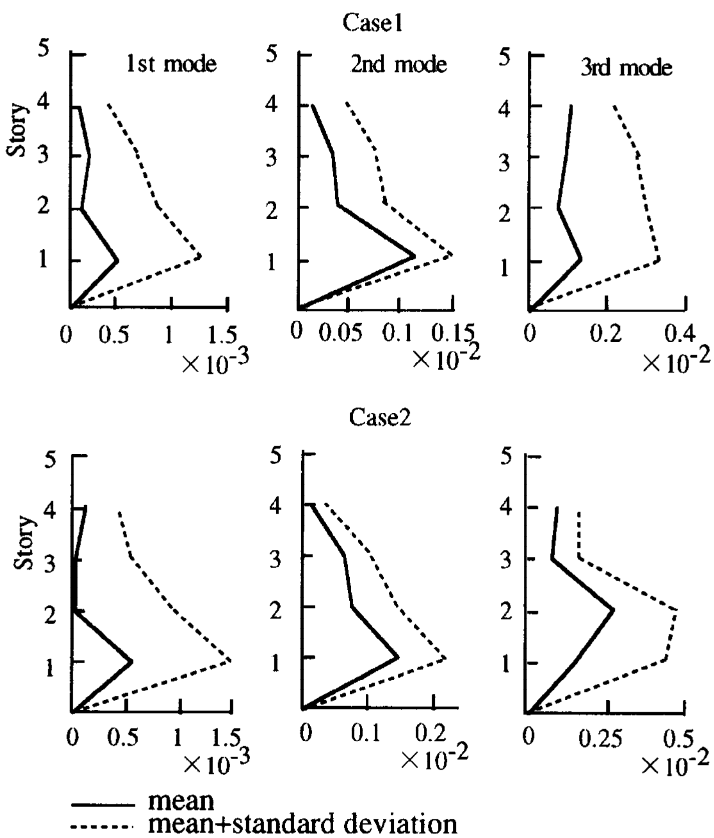

Fig.5 Vertical distribution of change in curvature of mode shapes due to damage

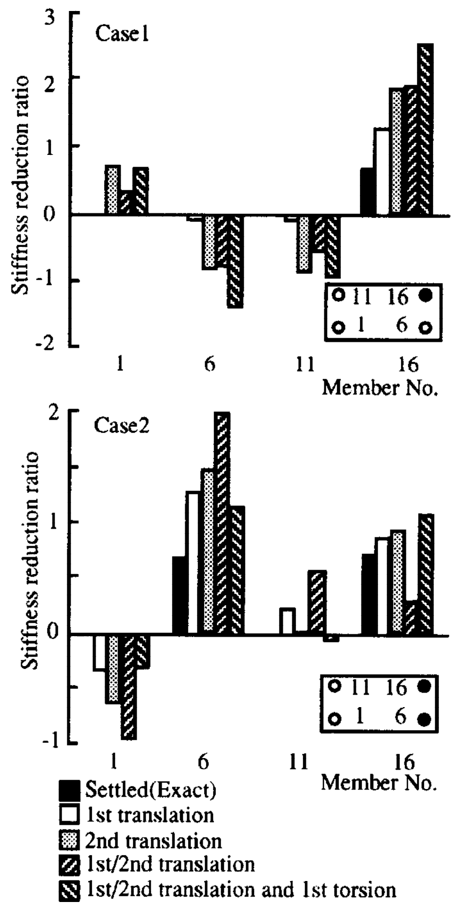

Fig.6 Comparison of stiffness reduction ratios for different combinations of mode

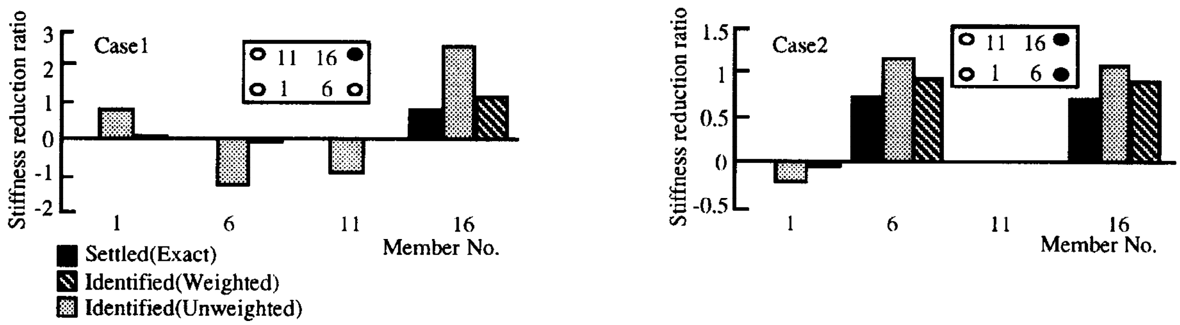

Fig.7 comparison of stiffness reduction-ratios by unweighted and weighted least squares method any combination of modes. In Case2, the location of two damaged columns is roughly pinpointed as members 6 and 16 by using all modes and, however, the extent of damage is not evaluated accurately for both members. Figure 7 shows the results identified with and without the weighting matrix to take account of the variability of modal properties by using all modes. It is clearly observed that the weighting matrix considerably improves the accuracy of identification. In Case1, member 16 can be clearly located as the damaged member and the extent of damage is reasonably evaluated. In Case2, members 6 and 16 can be clearly located as the damaged members and the extent of damage is reasonably evaluated for both members.

CONCLUSIONS

A damage detection scheme using vibration monitoring has been developed for multi-story building structures. The scheme is comprised of three stages: identification of modal properties, search of damaged stories and detection of damaged columns. Firstly, the change in modal properties of structural system is identified by using a multivariate ARMA model. Secondly, damaged stories are searched in terms of the change in curvature of mode shapes. Thirdly, the location and extent of column damage is pinpointed in terms of the stiffness reduction of each member by using an inverse modal perturbation. Based on the experimental verification, the following conclusions are obtained.

1. The proposed three stage damage detection scheme is a promising method to detect the column damage of multi-story buildings systematically.

2. The damping ratio is amplitude dependent and has a large variability. The mode shape is insensitive to column damage when the magnitude of damage is small. Consequently, among modal properties, natural frequencies are preferable as a damage indicator to detect the possible occurrence of damage because of their sensitivity and robustness.

3. The change in curvature of mode shapes is a better indicator to search for damaged stories because of their sensitivity in comparison with mode shapes themselves .

4. The column damage detection is heavily dependent on the accuracy and robustness of identified modal properties. The inverse modal perturbation with variance weighted least squares is effective to pinpoint the location and extent of column damage.

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