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FAST SESMIC STRENGTH ANALYSIS OF MASONRY BUILDINGS WITH UNKNOWN DATA FOR VULNERABILITY PURPOSES

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ABSTRACT

In the paper the authors purpose a procedure to investigate the seismic capacity of the masonry buildings based on the limit state analysis theory with a minimal need of data from field surveys. The method yields to evaluate a conventional maximum lateral strength the building are capable to resist. The effort is lumped on simplifying the set of data that are necessary to get the result, keeping arbitrary assumptions into very narrow limits. In this sense the "vulnerability index", that is identified in the limit horizontal load of the floor supported by the walls, is *conventional*, in that it is not calculated on the basis of real data, but from a set of data that is produced by a codified procedure based on an application of Jaynes' principle.

KEYWORDS

Maximum Entropy Limit Analysis Capacity Index Lateral Strenght Vulnerability

INTRODUCTION

Much of the seismic damage to existing, possibly old, masonry buildings depends on poor connections between structural elements, excessive thrust from arches and vaults, badly scarfed floors and so on. Nevertheless most codes of practice, and Italian standards in particular, require that a standard level of load bearing capacity against horizontal forces is provided to any building to be repaired, unless the works can be looked at as a very partial measure, that is looked at as a simple "improvement" of the aseismic capacity of the structure. The paper aims at developping a procedure able to yield a conventional lateral strength for buildings, with a minimal need of data from field surveys. Such lateral strength is viewed at as a "quality index" (the inverse of the vulnerability index) of this requirement. A certain conclusion is that if this lateral strength is poor, the building is not safe. The opposite is not guaranteed, but it is highly probable that in this case light works can produce significant improvement and ensure survival after moderately intense quakes. The evaluation of lateral strength is based on very simple structural analysis, essentially referred to limit analysis. The effort is lumped on simplifying the set of data that are necessary to get the result, keeping arbitrary assumptions into very narrow limits. In this sense the "vulnerability index", that is identified in the limit horizontal load of the floor supported by the walls, is *conventional*, in that it is not calculated on the basis of real data, but from a set of data that is produced by a codified procedure based on an application of Jaynes' principle.

THE ALGORITHM

Let consider a floor supported by a set of n masonry walls. If, as already announced in the Introduction, one is interested in the maximum horizontal load-carrying capacity, one should consider an active horizontal force **F** on the floor, assumed known both in direction and location, and express the equilibrium equation of the floor

$$\begin{cases} \sum_{i=1}^{n} T_{ix} = F_x = F\alpha_x \\ \sum_{i=1}^{n} T_{iy} = F_y = F\alpha_y \\ \sum_{i=1}^{n} (T_{ix} y_i - T_{ix} y_i) = F_x y_C - F_y x_C = F(\alpha_x y_C - \alpha_y x_C) \end{cases} \quad (1)$$

where F_x , F_y denote the Cartesian components of the force vector **F**; x_C , y_C are the co-ordinates of the centre of the application of **F**; x_i , y_i are the co-ordinates of the centroid of the i-th wall; T_{ix} and T_{iy} are the components of the shear force T_i in the i-th wall; α_x and α_y are the directors of **F**, and F is the intensity of the action (Fig. 1).

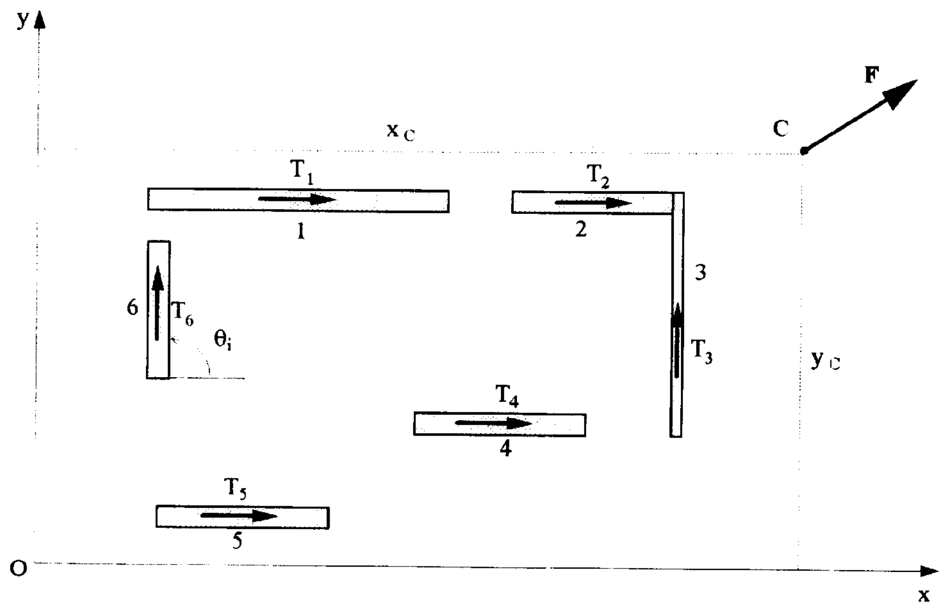

Fig. 1 - A generic set of masonry walls supporting the floor

Based on the principles of Limit Analysis, one should search for the maximum value of F respective of eqs (1) and of the admissibility conditions

$$\begin{cases} T_i \le T_{0i} \\ -T_i \le T_{0i} \end{cases} \quad \forall i = 1, \ldots\ldots, n \quad (2)$$

T_{oi} denoting the limit shear force in the i-th wall.

In vulnerability analyses, data are often not so detailed as to be able to perform the above calculations. It is conceivable however, that some data can be reliably assumed. This is the case for the location, angle θ_i on the axis X and length ℓ_i of walls, that can be inferred even from building plants at a very small scale, while the thickness b_i is usually very difficult to be detected.

Therefore, it must be assumed that the above limit analysis shall be done with very poor information about these data.
As a preliminary statement, one can consider that the expected wall thickness at the j-th floor in a m-storeys building is strongly recommended by existing *rules of the art* (Alberti 1450, Benvenuto 1981, Vituvio 1st cent. B.C.). It is possible, therefore, to assume a *standard reference wall* with any length "ℓ" and the thickness "b" conforming to the rule. If T_0 is the limit shear force in the standard wall and τ_0 is the limit shear stress in the masonry, one can write

$$\frac{T_{0i}}{T_0} = \frac{\tau_0 \ell_i b_i}{\tau_0 \ell b_h} = \frac{\ell_i b_i}{\ell b_h} = \lambda_i \hat{x}_i \Rightarrow T_{0i} = \lambda_i T_0 \hat{x}_i \quad \text{with} \quad \lambda_i = \frac{\ell_i}{\ell} \quad \text{and} \quad \hat{x}_i = \frac{b_i}{b} \tag{3}$$

where it is expected that $\hat{x}_i$ is the realization of a random variable $\tilde{x}$ with expected value $E[\tilde{x}] =$.
After position (3), the equilibrium eqs. (1) can be written

$$\begin{cases} \sum_{i=1}^{n} T_{0i} \frac{T_i}{T_{0i}} \cos(\theta_i) = T_0 \sum_{i=1}^{n} \lambda_i \cos(\theta_i) \hat{x}_i \frac{T_i}{T_{0i}} = F\alpha_x \\ \sum_{i=1}^{n} T_{0i} \frac{T_i}{T_{0i}} \sin(\theta_i) = T_0 \sum_{i=1}^{n} \lambda_i \sin(\theta_i) \hat{x}_i \frac{T_i}{T_{0i}} = F\alpha_y \\ \sum_{i=1}^{n} T_{0i} \frac{T_i}{T_{0i}} [\cos(\theta_i) y_i - \sin(\theta_i) x_i] = T_0 \sum_{i=1}^{n} \lambda_i [\cos(\theta_i) y_i - \sin(\theta_i) x_i] \hat{x}_i \frac{T_i}{T_{0i}} = F(\alpha_x y_C - \alpha_y x_C) \end{cases} \tag{4}$$

Introducing the new random variables

$$\tilde{p}_i = \begin{cases} \frac{T_i}{T_{0i}} \tilde{x}_i & \text{if} \quad T_i \geq 0 \\ 0 & \text{if} \quad T_i < 0 \end{cases} \quad ; \quad \tilde{p}_{i+n} = \begin{cases} 0 & \text{if} \quad T_i \geq 0 \\ -\frac{T_i}{T_{0i}} \tilde{x}_i & \text{if} \quad T_i < 0 \end{cases} \tag{5}$$

eqs.(4) become

$$\begin{cases} \sum_{i=1}^{n} \lambda_i \cos(\theta_i)\, \tilde{p}_i - \sum_{i=1}^{n} \lambda_i \cos(\theta_i)\, \tilde{p}_{i+n} = \tilde{s}\alpha_x \\ \sum_{i=1}^{n} \lambda_i \sin(\theta_i)\, \tilde{p}_i - \sum_{i=1}^{n} \lambda_i \sin(\theta_i)\, \tilde{p}_{i+n} = \tilde{s}\alpha_y \\ \sum_{i=1}^{n} \lambda_i [\cos(\theta_i) y_i - \sin(\theta_i) x_i]\, \tilde{p}_i - \sum_{i=1}^{n} \lambda_i [\sin(\theta_i) x_i - \cos(\theta_i) y_i] \tilde{p}_{i+n} = \tilde{s}(\alpha_x y_C - \alpha_y x_C) \end{cases} \tag{6}$$

with $\tilde{s} = \tilde{F}/T_0$ that is the random value of the limit multiplier $s = F/T_0$, the randomness depending on the uncertainty in the thicknesses. Taking expectations on both members, one gets the following equations for the expected $\tilde{p}_k$'s and limit force multiplier

$$\begin{cases} \sum_{i=1}^{n} \lambda_i \cos(\theta_i)\, p_i - \sum_{i=1}^{n} \lambda_i \cos(\theta_i)\, p_{i+n} = s\alpha_x = sq_1 \\ \sum_{i=1}^{n} \lambda_i \sin(\theta_i)\, p_i - \sum_{i=1}^{n} \lambda_i \sin(\theta_i)\, p_{i+n} = s\alpha_y = sq_2 \\ \sum_{i=1}^{n} \lambda_i [\cos(\theta_i) y_i - \sin(\theta_i) x_i]\, p_i - \sum_{i=1}^{n} \lambda_i [\sin(\theta_i) x_i - \cos(\theta_i) y_i] p_{i+n} = s(\alpha_x y_C - \alpha_y x_C) = sq_3 \end{cases} \tag{7}$$

where $p_k = E[\tilde{p}_k]$ and $s = E[\tilde{s}]$.

Keeping in mind that $E[\tilde{x}] =$ and the definitions (5) for $\tilde{p}_k$ (k = 1,...,2n), the conditions for admissibility are

$$\begin{aligned} &0 \le p_k \le 1 \qquad \forall k = 1,\dots\dots\dots,2n \\ &p_i \; p_{i+n} = 0 \qquad \forall i = 1,\dots\dots\dots\dots,n \end{aligned} \tag{8}$$

Eq. (7) can be expressed in the compact form

$$\mathbf{C}\,\mathbf{p} = \mathbf{q}\,s \tag{9}$$

Given the matrix **C**, depending only on the *known* geometry of the structure -taking advantage from the first (8)- one can solve the problem to find the limit strength in the floor with unknown wall thickness, by applying the Principle of the Maximum Entropy, in order to cover the uncertainty in the distribution of the wall thicknesses, that inficiates the expectation that s is the maximum complying with eqs. (7) in the respect of the constraints (8), as would be suggested by classical L.A.

To this aim, let transform eqs. (9), by the position, $v_{ij} = \dfrac{c_{ij}}{q_i}$,to the following form

$$\begin{cases} \sum_{j=1}^{2n} v_{1j}\; p_j = s \\ \sum_{j=1}^{2n} v_{2j}\; p_j = s \\ \sum_{j=1}^{2n} v_{3j}\; p_j = s \end{cases} \tag{10}$$

By subtracting the 2nd and the 3rd of eqs. (10) from the 1st one gets

$$\begin{cases} \sum_{j=1}^{2n} \left(v_{1j} - v_{2j}\right)\, p_j = 0 \\ \sum_{j=1}^{2n} \left(v_{1j} - v_{3j}\right)\, p_j = 0 \end{cases} \tag{11}$$

that, denoting by $g_{ij} = v_{ij} - v_{i+1,j}$ (i =1,2), can be expressed in the form

$$\left\{ \sum_{j=1}^{2n} g_{ij}\; p_j = 0 \qquad (i = 1,2) \right. \tag{12}$$

while the expected failure multiplier (the *quality index* for vulnerability purposes) remains given by the first (10). Note that $g_{ij} = -\, g_{i,j+n}$.

Thus, the quality index can be found after solving the following problem

$$\begin{aligned} &\textit{find the maximum of} \qquad S = -\sum_{k=1}^{2n} p_k \ln(p_k) \\ &\text{under the conditions} \qquad \mathbf{G}\,\mathbf{p} = \mathbf{q}\,s \\ &\qquad\qquad\qquad\qquad\quad\; p_i\; p_{i+n} = 0 \qquad \forall i = 1,\dots\dots\dots n \end{aligned} \tag{13}$$

The condition $0 \leq p_k \leq 1 \quad \forall k = 1,.........,2n$ is implicitly satisfied by solving the above problem.
The problem (13) can be solved by investigating the Lagrangian functional, that by introducing the coefficients β_r $(r = 1,2)$ and μ_i $(r=1,.....,n)$ is as follows

$$\mathscr{L}_S = -k\sum_{k=1}^{2n} p_k \ln(p_k) - \sum_{r=1}^{2}\beta_r\left(\sum_{k=1}^{2n} g_{rk} \ln(p_k)\right) + \sum_{i=1}^{n}\mu_i (p_i \; p_{i+n}) \tag{14}$$

whose the stationarity conditions are

$$\begin{aligned} \frac{\partial S}{\partial p_m} &= -[\ln(p_m)+1] - \sum_{r=1}^{3}\beta_r g_{rm} \ln(p_k) + \mu_m p_{m+n} = 0 \\ & \qquad\qquad\qquad\qquad\qquad\qquad m \leq n \\ \frac{\partial S}{\partial p_{m+n}} &= -[\ln(p_{m+n})+1] - \sum_{r=1}^{3}\beta_r g_{r,m+n} + \mu_m p_m = 0 \end{aligned} \tag{15}$$

Combining eqs. (15) with the condition $p_m \; p_{m+n} = 0$ one derives, respectively

$$p_m = \begin{cases} 0 & \text{se } p_{m+n} \neq 0 \\ \overline{p}_m & \text{se } p_{m+n} = 0 \end{cases} \quad ; \quad p_{m+n} = \begin{cases} \overline{p}_{m+n} & \text{se } p_m = 0 \\ 0 & \text{se } p_m \neq 0 \end{cases} \tag{16}$$

where

$$\begin{cases} \overline{p}_m = \exp\left[-\left(1+\sum_{r=1}^{3}\beta_r g_{rm}\right)\right] \\ \overline{p}_{m+n} = \exp\left[-\left(1+\sum_{r=1}^{3}\beta_r g_{r,m+n}\right)\right] \end{cases} \tag{17}$$

So the problem turns on finding the β_r values, which can be done by transforming the problem (13) in its dual.
Moreover in order to satisfy the condition $p_m \; p_{m+n} = 0$, one introduces the Heavyside function H and with the position

$$\begin{aligned} S_m &= -p_m \ln(p_m) \\ S_{m+n} &= -p_{m+n} \ln(p_{m+n}) \end{aligned} \tag{18}$$

after observing that the choice in eqs.(16) is determined by which one between p_m and p_{m+n} yields the largest contribution to the Entropy, one can write

$$\begin{aligned} p_m &= H(S_m - S_{m+n})\overline{p}_m \\ p_{m+n} &= H(S_{m+n} - S_m)\overline{p}_{m+n} \end{aligned} \tag{19}$$

Therefore the dual of problem (13) can be written as *find min* mum of $$\mathscr{D}(\beta_1,\beta_2)=\sum_{i=1}^{n}H(S_i-S_{i+n})\overline{p}_i+\sum_{i=1}^{n}H(S_{i+n}-S_i)\overline{p}_{i+n} \tag{20}$$

The condition of stationarity is

$$\frac{\partial\mathscr{D}}{\partial\beta_k}=-\sum_{i=1}^{n}H(S_i-S_{i+n})\frac{\partial\overline{p}_i}{\partial\beta_k}-\sum_{i=1}^{n}\frac{\partial H(S_i-S_{i+n})}{\partial\beta_k}\overline{p}_i-\sum_{i=1}^{n}H(S_{i+n}-S_i)\frac{\partial\overline{p}_{i+n}}{\partial\beta_k}-\sum_{i=1}^{n}\frac{\partial H(S_{i+n}-S_i)}{\partial\beta_k}\overline{p}_{i+n} \tag{21}$$

where

$$\frac{\partial\overline{p}_i}{\partial\beta_k}=-\overline{p}_i\,g_{ki}\ ;\quad \frac{\partial\overline{p}_{i+n}}{\partial\beta_k}=-\overline{p}_{i+n}\,g_{k,i+n}\ ;\quad \frac{\partial H(S_{i+n}-S_i)}{\partial\beta_k}=\frac{\partial H(S_i-S_{i+n})}{\partial\beta_k}=0\ ^{1} \tag{22}$$

whence, substituting the eqs. (22) in the eq:(21) and remembering the eq. (17), the eq. (21) become

$$\frac{\partial\mathscr{D}}{\partial\beta_k}=-\sum_{i=1}^{n}g_{ki}\,p_i-\sum_{i=1}^{n}g_{k,i+n}\,p_{i+n}=-\sum_{i=1}^{2n}g_{ki}\,p_i \tag{23}$$

Once the values of the β_r are found, the eqs (17) and. (10) provide the value of s that maximizes the Entropy.

CONCLUSIONS

The proposed procedure should allow examination of large urban areas with relatively small expense, yielding results having clear meaning in relation to the probability of survival. The quality index is, in fact, expressed as the lateral collapse load, making very easy the comparison with the action.

The method shows the possibility to be implemented in seismic risk analysis procedures. The basic idea to balance some ignorance of the data with compatible "minimum disorder" hypothetical data sets is very attractive, and is not confined to the application presented herein, but may be that more effective procedures for seismic vulnerability can be formulated on this basis.

Moreover the method proposed present the following advantages:

- does not strictly depend on the survey data collected soon after an earthquake
- produces a *conventional* parameter of vulnerability suitable for comparison with the main standard vulnerability methods
- can be easily combined with computerized procedure based on automatic reading of the main geometry of the buildings on plants of large urban settlements
- could be used to produce, in the next step of the research, a sort of continuos typology scale not confined in the classes proposed by the macro- seismic scales and closer to the real variety of the structure building types.

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[1] In fact $\frac{\partial H(S_i-S_{i+n})}{\partial\beta_k}=\delta(S_i-S_{i+n})\cdot\frac{\partial(S_i-S_{i+n})}{\partial\beta_k}=0$. The coefficient of the δ - impulse is 0 for $S_i=S_{i+n}$. In fact

$$S_i=-p_i\ln(p_i)=-\frac{e^{-\alpha}}{e}(-1-\alpha)=\frac{1+\alpha}{e}e^{-\alpha}$$

$$S_{i+n}=-p_{i+n}\ln(p_{i+n})=-\frac{e^{\alpha}}{e}(-1+\alpha)=\frac{1-\alpha}{e}e^{\alpha}$$

with $\alpha=\sum_{r=1}^{2}\beta_r g_n$

and $\delta\neq 0$ for $S_i=S_{i+n}\Rightarrow(1+\alpha)e^{-\alpha}=(1-\alpha)e^{\alpha}\Rightarrow\alpha=\frac{e^{\alpha}-e^{-\alpha}}{e^{\alpha}+e^{-\alpha}}=\mathrm{tgh}(\alpha)\Rightarrow\alpha=0$

whence one derives $\frac{\partial(S_i-S_{i+n})}{\partial\beta_k}=0$ for $S_i=S_{i+n}$.

REFERENCES

Alberti L.B., De Re Aedificatoria

Benvenuto E. ,(1981), La Scienza delle Costruzioni ed il suo Sviluppo Storico. Sansone, Bologna, Italy

Breyman, G.,A., (1926), Trattato Generale di Costruzioni Civili. **Volume I** , Costruzioni in Pietra , Vallardi, Milan

Franciosi, V., (1980) Calcolo a Rottura , Liguori, Naples

Massonet, C., Save, M., (1967) Calcolo Plastico a Rottura delle Costruzioni . Centre d'Information de L'ancier, Buxelles

Templeman A. B. and Li X. S. , (1985) Entropy Duals, in Engineering Optimization **9**, 2, 107-119

Templeman A. B., (1992) Entropy and Civil Engineering Optimization, in Optimization and Artificial Intelligence in Civil and Structural Engineering, **Volume I**, 87-105. Kluwel Academic Publishers, Printed in Netherland,

Vituvio, De Architettura