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EFFECT OF THE POSITION AND NUMBER OF DAMPERS ON THE SEISMIC RESPONSE OF FRAME STRUCTURES

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SUMMARY

Little attention has been paid to evaluating the influence of the number and placement of dampers on the dynamic response, although many studies have been made of these systems. The objectives of this paper are: i) to asses how the variation of placement and number of dampers affect the seismic response of a frame structure, and ii) to evaluate a simplified method to analyze frame structures that have non-classical damping, in order to study how the error in the simplified method is influenced by placement of dampers. To fulfill these objectives, five-story moment resisting frames with two values of the fundamental period subjected to two earthquake ground motions were used. Several distributions of dampers varying number and location were considered, while maintaining the same amount of damping in each case. The results showed that the dampers placement influences significantly the structural response. A large number of dampers do not always leads to the best benefit in terms of drift reduction for all stories. Three dampers lead to the best overall benefit for all stories in this structure. If one damper is placed, this should be located at the first story in order to obtain the best overall drift reduction. The best damper placement is one damper per story; if the number of dampers is less than the number of stories, one damper per story beginning at the lowest story is the best choice. The simplified method is not recommended for a damper distribution concentrated in a few stories, because large errors in the structural response could be obtained. The analysis considering the simplified method may be used, without introducing significant errors, in the systems with a more uniform damping distribution, that is, one damper per story with the same damping constant.

INTRODUCTION

In recent years damping devices have been developed in order to reduce effectively the seismic response of structures subjected to earthquake excitation, Hanson, Soong, Constantinou [1, 2, 3]. The usefulness of the devices is a function of where they are located in the structure, Zhang [4]; when incorporated into a structure these devices can substantially increase the costs, then optimization of number and location of dampers is convenient. However, little attention has been paid to evaluating the influence of the number and placement of dampers on dynamic response, although a significant amount of experimental and analytical research has been done regarding the applicability of damping devices, Zhang, Aiken and Lai [5, 6, 7]. This problem was studied for idealized structure's configuration by Ashour and Hanson [8] by

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modeling a uniform shear beam; optimal damper location was shown to conform to the pattern of distribution that will result in maximizing the first mode damping ratio. Cheng and Pantelides [9] used the controllability index associated with each story of a multistory building, which provided the optimal location of the damper. Zhang and Soong [4] extended this index to address the damper location problem, leading to a sequential procedure for optimal placement of damper. Shukla and Datta [10] applied the controllability index based on the root-mean-square value of the interstory drift of multistory building frames to find optimal damper placement. More recently López-García [11] proposed a simplification of this methodology in order to make a procedure more practical and efficient for the design of optimal configurations of dampers.

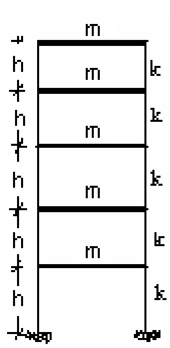
On the other hand, a structure with dampers is a non-classically damped system with complex eigenvectors, whose solution requires significantly more computational effort. For analytical convenience and to reduce computations, classical damping is usually assumed in the analysis, but this assumption may lead to unacceptable error in the response. Clough and Mojtahedi [12] described several analytical techniques in order to compare the responses of classically and non-classically damped systems. Hasselman [13], Warburton and Soni [14] have developed a criterion to estimate the errors induced in the case of closely spaced natural frequencies and indicated the influence of damping distribution. Veletsos and Ventura [15] presented a critical review of the mode superposition method with complex frequencies and modes of vibration for discrete systems; they concluded that depending on the excitation and the systems itself, the approximate solution involving the use of classical modes of vibration may be substantially in error. Prater and Singh [16] worked out some numerical indices to determine the extent of non-classical damping and the error induced when classical damping is assumed. More recently Grecco and Santini [17] compared the results of some techniques for the dynamics analysis of a non-classically damped linear system, and showed the effect of the amount of damping and of the extent of non-classical damping on the error. Goel [18] investigated the effect of neglecting the off-diagonal terms of the transformed damping matrix on the seismic response of asymmetric plan systems and identified the range of the system parameters for which this simplification can be used.

The objectives of this paper are: i) to asses how the variation of placement and number of dampers affect the seismic response of a frame structure, and ii)) to evaluate a simplified method to analyze frame structures that have non-classical damping, in order to study how the error in the simplified method is influenced by placement of dampers. Preliminary results concerning to the first objective are discussed in Tovar, [19].

SYSTEMS, ANALYSIS METHODS AND EARTHQUAKE GROUND MOTION

Systems

The system considered is the idealized linear five-story shear frame shown in Figure 1. Every story has the same value of mass and stiffness. The damping ratio was taken as 5% at each vibration mode. Two systems are defined: one with short fundamental period (T_1 = 0.20 sec) and other with long fundamental period (T_1 = 2.00 sec).



T_1(sec)	M(kgf sec^2/cm)	K(kgf /cm)	h(m)
0.20	16.00	195x10^3	3.00
2.00	1,600.00	195x10^3	3.00

Figure 1. The structural system and its properties

Analysis Methods

The equation of motion for a N degree of freedom system subjected to ground motion is:

$$\mathbf{M} \cdot \ddot{\mathbf{u}}(t) + \mathbf{C} \cdot \dot{\mathbf{u}}(t) + \mathbf{K} \cdot \mathbf{u}(t) = -\mathbf{M} \cdot \mathbf{r} \cdot \ddot{u}_g(t) \tag{1}$$

Where **M**, **C** and **K** are the mass matrix, damping matrix and stiffness matrix of the system, of size NxN respectively; **u** (t) is the column vector of the story displacements relative to the ground, **r** is a column vector of ones, and $\ddot{u}_g$ (t) is the ground acceleration.

The damping matrix for each system was obtained by assembling two matrices, Chopra [20]:

$$\mathbf{C} = \mathbf{C}_E + \mathbf{C}_A \qquad \mathbf{C}_E = \mathbf{M}\left[\sum_{n=1}^{N} \frac{2\xi_n \omega_n}{M_n} \phi_n \phi_n{}^t\right]\mathbf{M} \tag{2 a, b}$$

$\mathbf{C}_E$ is the damping matrix of the system without supplemental dampers defined by Equation (2b); ξ_n, ω_n and ϕ_n are the damping ratio, the natural frequency and the eigenvector of the nth vibration mode. $\mathbf{C}_A$ is the damping matrix due to supplemental damping. The stiffness matrix was calculated neglecting the stiffness of the dampers.

For system with classical damping, modes shapes and frequencies are obtained by solving the following eigenvalue-eigenvector problem:

$$(\mathbf{K} - \boldsymbol{\omega}^2\mathbf{M})\boldsymbol{\Phi} = \mathbf{0} \tag{3}$$

Where $\boldsymbol{\Phi}$ is the modal matrix and $\boldsymbol{\omega}$ is a diagonal matrix of the natural frequencies.

Equation (1) can be transformed into a set of N uncoupled differentials equations (second degree) only if the mode shapes of the system diagonalize the damping matrix **C**, that is, the result of the product $\boldsymbol{\Phi}^t\mathbf{C}\boldsymbol{\Phi}$ is a diagonal matrix. When this condition is not satisfied the system is non-classically damped. For system with classical damping modal damping ratios (ξ_n) and structural response (**u** (t)) are calculated by the following expressions:

$$\xi_n = \frac{\phi_n^t\ \mathbf{C}\phi_n}{2\omega_n\ \phi_n^t\mathbf{M}\phi_n} \qquad \mathbf{u}(t) = \sum_{n=1}^{N} \phi_n q_n(t) \tag{4 a, b}$$

An approximate solution of the response of systems with non-classical damping can be obtained neglecting the off-diagonal terms of the product $\boldsymbol{\Phi}^t\mathbf{C}\boldsymbol{\Phi}$ and using Eq. (1) to (4) as for the case of classical damping; this method will be called in this paper the simplified method.

For systems with non-classical damping the product $\boldsymbol{\Phi}^t\mathbf{C}\boldsymbol{\Phi}$ is not a diagonal matrix and the above method is not strictly valid. In this case, the modal damping ratios and the modal natural frequencies are calculated by solving the following characteristic value problem, Veletsos [15]:

$$(\mathbf{B} + \lambda\,\mathbf{A})\mathbf{Z} = \mathbf{0} \tag{5}$$

Z is a vector of 2N elements that contains the eigenvectors, **0** is the null vector of size 2N and

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix} \tag{6 a, b}$$

A and **B** are matrices of size 2Nx2N.
The solution of the Equation (5) leads to 2N complex-valued eigenvalues λ_n^* and eigenvectors ϕ_n^*. The complex eigenvalues appear in complex conjugate pairs in the form of

$$\lambda_n^* = -\xi_n^*\omega_n^* - i\omega_n^*\sqrt{1-\left(\xi_n^*\right)^2} \quad \text{and} \quad \tilde{\lambda}_n^* = -\xi_n^*\omega_n^* + i\omega_n^*\sqrt{1-\left(\xi_n^*\right)^2} \qquad \text{(7 a, b)}$$

In which ξ_n^* and ω_n^* are the apparent modal damping ratio and the apparent natural vibration frequency, respectively, associated with the nth modal pair, which can be obtained as:

$$\omega_n^* = \sqrt{\mathrm{Re}^2(\lambda^*{}_n) + \mathrm{Im}^2(\lambda^*{}_n)} \qquad \xi^*{}_n = \frac{-\mathrm{Re}(\lambda^*{}_n)}{\omega^*{}_n} \qquad \text{(8 a, b)}$$

Where Re and Im are the real and imaginary parts of the value within the parenthesis.
The response of the system considering non-classical damping is computed solving the coupled equations of motion (Equation (1)) by numerical integration.

Earthquake Ground Motions
Two ground motions were considered: the North-South (360º) component recorded during the 1940 El Centro earthquake, on soil, with a peak ground acceleration of 0.30g, and the North- East (21º) component recorded during the 1952 Kern County earthquake, Taft, on rock, with a peak ground acceleration of 0.16g.

EFFECT OF THE NUMBER AND PLACEMENT OF DAMPERS

Cases of study
Several distributions of one, three and five dampers in the structure were analyzed. To asses the effect of one damper, five cases were defined as shown in Figure 2. Each case is defined by the location of one damper at a particular story. Case 0 is defined as the system without dampers. The damper inclination was taken as 45º. The damper size C is the same for all cases and was calculated by an iterative procedure using Equations (5) to (8b), until a maximum value of damping ratio in the fundamental mode for Case a_1 was obtained. For the system defined in this paper, the maximum damping ratio in the fundamental mode was 13%. The resulting values of dampers size are shown in Table 1.

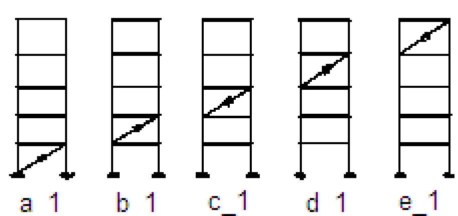


Figure 2. Cases of study to asses the effect of one damper's location

The cases defined to asses the effect of the three and five dampers are shown in Figures 3 and 4, respectively. The values of the damper constant C are indicated in Table 1. All dampers at each case have the same value of C. The sum of the damper's constants is the same for each building with the same period.

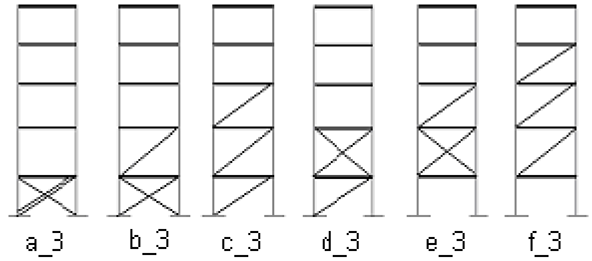


Figure 3. Cases of study to asses the effect of three damper's location

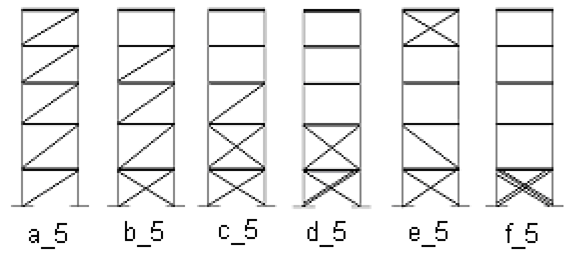


Figure 4. Cases of study to asses the effect of five damper's location

Table 1. Damper size C of each device

Nº of dampers	T = 0.20 sec	T = 2.00 sec
1	24,838.50 kgf sec/cm	248,385 kgf sec/cm
3	8,279.50 kgf sec/cm	82,795 kgf sec/cm
5	4,967.60 kgf sec/cm	49,676 kgf sec/cm

Drift reduction

The response is calculated in terms of maximum interstory drift, which is obtained during the earthquake ground motion. A parameter β_i^k is defined in order to evaluate the reduction of interstory drift due to the dampers:

$$\beta_i^k = \frac{\Delta_i^k}{\Delta_i^0} \qquad \beta_m^k = \frac{1}{5}\left(\sum_{i=1}^{5}\beta_i^k\right) \tag{9 a, b}$$

Where Δ_i^k is the maximum drift on story i in Case k, Δ_i^0 is the maximum drift at story i in Case 0. It can be noted that β_i^k represents the benefit in drift reduction at story i in Case k, and β_m^k is the mean value of β_i^k for all stories; β_m^k measures the overall benefit introduced by the dampers on the building at Case k. A lower value of β_m^k mean a higher drift reduction.

The β_m^k values were computed for each system and for each ground motion. Results are also calculated for the average for the two ground motion. The results for the structure with only one damper are shown in Figure 5.These results show that when the damper is localized at the first story, the greatest response reduction is obtained. This is especially true for the 0.20 sec period system (Figure 5a) where the average drift reduction for both seismic motions is about 40%. If the damper is located at the top story the average drift reduction is only 10%, that is, the damper has little effect on the response. For the 2.00 sec period system (Figure 5b) a 22% average drift reduction is achieved when the damper is at the first story (Case a_1), even though Case c_1 also shows an important drift reduction. The drift reduction pattern is similar for both seismic motions.

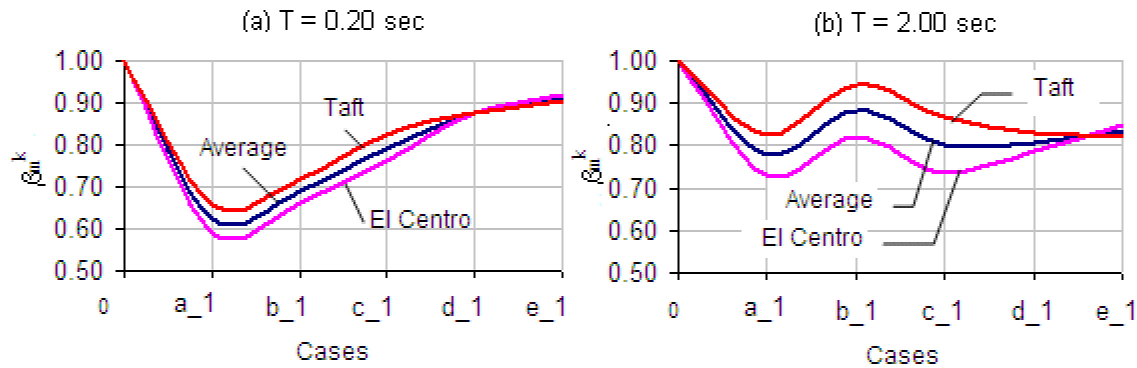


Figure 5. Effect of one damper's placement

The result for three dampers is shown in Figure 6. For the 0.20 sec system a 50% average drift reduction is observed at Case c_3 (Figure 6a). This is the greatest response reduction obtained compared with other damper placement. For the 2.00 sec system the drift is reduced at least 45% in Case c_3. The Case a_3 is the worst damper placement because the response is reduced only 20%. Thus, it can be concluded that a more uniform damper distribution, beginning at the first story, is the best dampers placement.

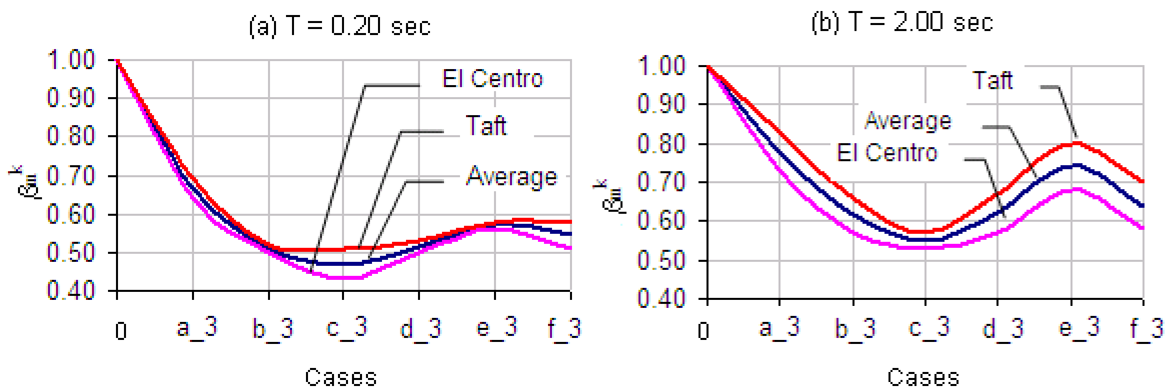


Figure 6. Effect of three damper's distribution

The results for five dampers placement is shown in Figure 7. For the 0.20 sec period system the greatest benefit in drift reduction is obtained in the Case c_5, being about 53% reduction, although cases a_5, b_5 and d_5 also present a significant drift reduction of about 50%. For the 2.00 sec system the best distribution of dampers is Case a_5 that shows an average drift reduction about 53%. For the five damper systems the best distribution is either the uniform one or a damper placement concentrated at the lower stories.

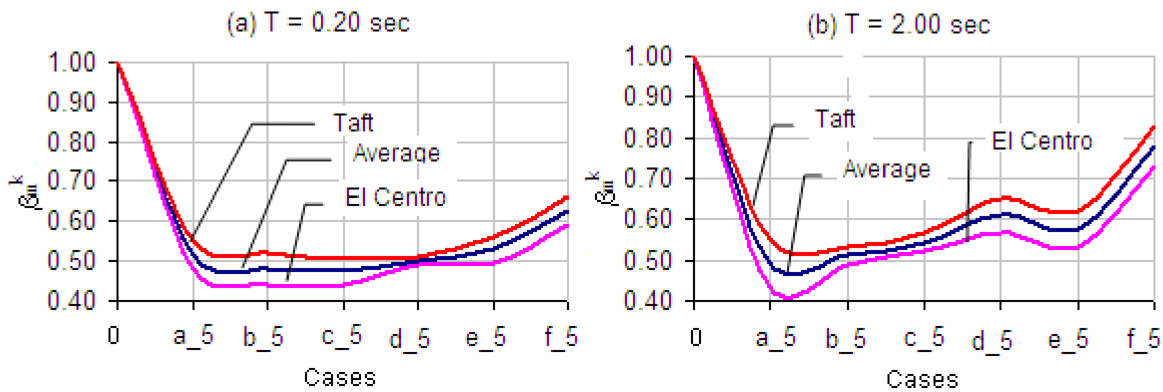


Figure 7. Effect of five damper's distribution

Figure 8 shows the values of β_i^k at each story for the best location of dampers, as was obtained in the previous discussion, that is, the placement that brings more drift reduction. If one would like to obtain a large drift reduction in the first story, only one damper at first story is enough. This is true for both systems (see Cases a_1 in Figures 8a and 8b). For the 0.20 sec period system a large drift reduction can be achieved in all stories with the configurations c_3 and c_5, three and five dampers, respectively. For the 2.00 sec period system the largest drift reduction in the upper stories requires to place one damper at each story (Case a_5).However, in order to achieve a uniform benefit for all stories, it is convenient to place dampers as in Case c_3, because the curve of β_i^k is almost a straight line. The best placement is that with a uniform distribution of dampers, beginning at the first story.

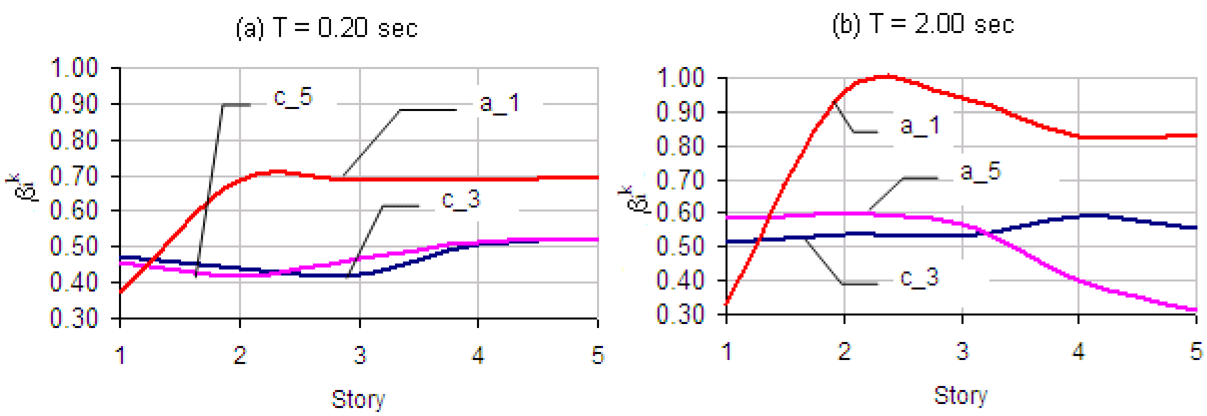


Figure 8. Values of β_i^k at each story for the best damper placement for 1, 3 and 5 dampers

EVALUATION OF THE SIMPLIFIED METHOD

The maximum interstory drift for the systems in the simplified method of analysis is calculated by solving the Equations (1) to (4) neglecting off-diagonal terms of the transformed damping matrix $\mathbf{\Phi^t C \Phi}$. The approximate response so calculated is denoted "R_{approx}" and is compared with the response "R_{exact}" for the system with non-classical damping that is computed by numerical integration of Eq. (1).

The ratio R_{approx}/R_{exact} measures the accuracy of the simplified method to estimate the "exact" response at each story and in each system. The average error of each system is calculated taking average for all stories of the absolute error in each interstory drift:

$$e = \frac{1}{5}\sum_{i=1}^{5}\left|1 - \frac{R_{approx}}{R_{exact}}\right| \tag{10}$$

Figure 9 shows the values of ratio R_{approx}/ R_{exact} at each story for the cases with only one damper defined in Figure 2, for El Centro ground motion and for systems with T = 0.20 sec and T = 2.00 sec. The simplified method can significantly underestimate or overestimate the "exact" response, depending on the story where the damper is located and the story where the drift is calculated. For example, when the damper is located at the top story (Cases e_1), the approximate response is about 1.80 the exact response in the top story, but only 0.80 in the lower stories. The values of ratio R_{approx}/ R_{exact} show larger overestimations for the 2.00 sec period system than for the 0.20 sec period system.

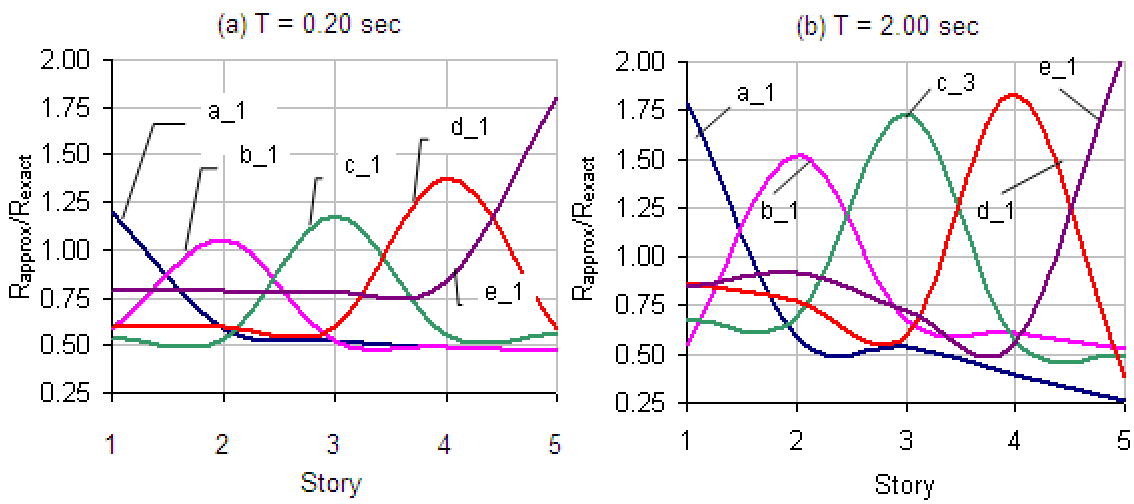


Figure 9. Values of ratio R_{approx}/R_{exact} for the Case with one damper defined in Figure 2. El Centro ground motion

Figure 10 shows the values of ratio R_{approx}/ R_{exact} at each story for the cases with three dampers defined in Figure 3. For the 0.20 sec period system the average error in Cases c_3 and f_3 is about 10% and 8%, respectively, which is significantly less than the other Cases. For the 2.00 sec period system these Cases, c_3 and f_3, have also less average error than the other cases, as for the 0.20 sec system, because they have a more uniform damping distribution, which is more similar to the behavior of classically damped systems. The Cases a_3 present the greatest average error of interstory drift, being about 43% for the 0.20 sec period system and 60% for the 2.00 sec period system. These cases have concentrated all the dampers at the first story, which leads to larger error because its extent of non-classical damping is higher.
In general the results for these Cases indicate underestimation in the response; this is particularly true in the uppers stories. The values of ratio R_{approx}/ R_{exact} show larger overestimations and underestimation for the 2.00 sec period system than for the 0.20 sec period system.

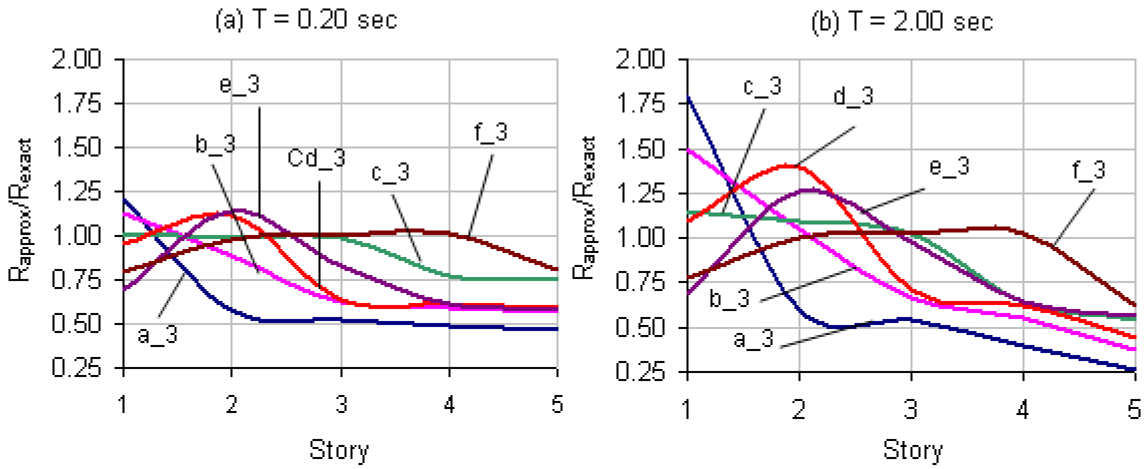


Figure 10. Values of ratio R_{approx}/R_{exact} for the case with three dampers defined in Figure 3. El Centro ground motion

Figure 11 shows the values of ratio R_{approx}/ R_{exact} at each story for the cases of five dampers defined in Figure 4. For both systems, the average error is about 3% when one damper is located at each story (Cases e_5), which leads to a uniform damping distribution where the damping matrix is very similar to the classical damping matrix. As expected, cases f_5 present the largest error, being about 43% at the top story, because all dampers are concentrated at the first story. In the upper stories the errors are in general less than one, indicating underestimation of the response. On the other hand, in the first story the errors are always greater than one, indicating overestimation of the response.

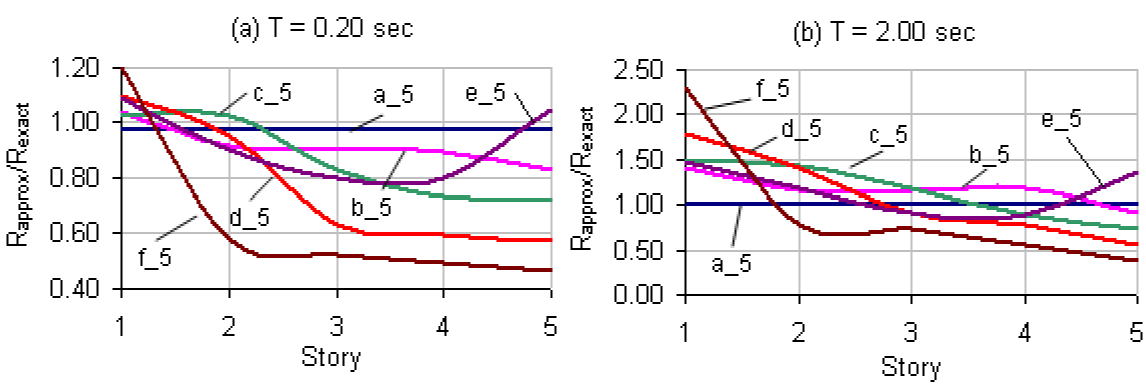


Figure 11. Values of ratio R_{approx}/R_{exact} for the case with five dampers defined in Figure 4. El Centro ground motion

Figure 12 shows the values of ratio R_{approx}/ R_{exact} at each story for the best location of dampers, as was defined in the previous discussion and shown in Figure 8; that is, the placement that brings the largest drift reduction. For the 0.20 sec period system the best placement damper is Case c_3 presenting an average error about 10%. For the 2.00 sec period system a_5 is the best case showing an average error of 0.20%, whereas Case c_3 presents an average error of about 21%. Finally, the worst case is a_1 where the average error is greater than 50% in both systems. It is concluded that in order to achieve the lowest average error for interstory drift, it is convenient to place more than one damper. The best placement is that with a uniform distribution of dampers, beginning at the first story.

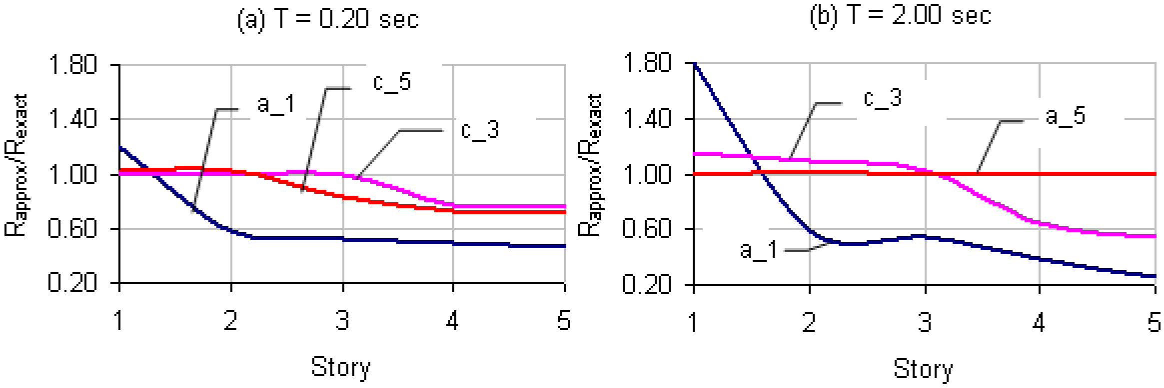


Figure 12. Values of ratio R_{approx}/R_{exact} for the best damper placement for 1, 3 and 5 dampers. El Centro ground motion.

CONCLUSIONS

This study investigated the effects of the number and placement of dampers in a five-story linear elastic frame structure subjected to two earthquake ground motion, and compared the response of the systems considering classical and non-classical damping for several damper distributions. This investigation has led to the following conclusions:

1. The dampers placement influences significantly the structural response. A large number of dampers do not always leads to the best benefit in terms of drift reduction for all stories. Three dampers lead to the best overall benefit for all stories in this structure.
2. When only one damper is placed this should be located at the first story in order to obtain the best overall drift reduction. The best damper placement is one damper per story; if the number of dampers is less than the number of stories, one damper per story beginning at the lowest story is the best choice.
3. The simplified method is not recommended for a damper distribution concentrated in a few stories, because large errors in the structural response could be obtained.
4. The analysis considering the simplified method may be used without introducing significant errors in the systems with a more uniform damping distribution, that is, one damper per story with the same damping constant.

ACKNOWLEDGEMENTS

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REFERENCES

1. Hanson R.D. and Soong T. S. Seismic Design with Supplemental Energy Dissipation Devices. Monograph n° 8. Earthquake Engineering Research Institute, Oakland, 2001.
2. Soong T.T and Dargush G. F. Passive Energy Dissipation System in Structural Engineering. John Wiley and Sons. Ltd., London (UK) and New York (USA), 1997.
3. Constantinou M., Soong T. T and Dargush G. Passive Energy Dissipation System for Structural Design and Retrofit. Monografh n° 1. Multidisciplinary Center for Earthquake Engineering Research, Buffalo N.Y, 1998.

4. Zhang R. and Soong T. Seismic Design of viscoelastics dampers for structural applications. Journal of Structural Engineering 1992. 118(5):1375-1392.
5. Zhang R., Soong T. and Mahmoodi P. Seismic Response of Steel Frame Structures with Added Viscoelastic Dampers. Eathquake Engrg. Struct. Dyn. 1989, 18(3), 389-396.
6. Aiken I., Kelly J. and Mahmoodi P. The Application of Viscoelastic Dampers to Seismically Resistant Structures. Proc. 4th U. S. Nat. Conf. on Earthquake Engrg. 1990., Earthquake Engineering Research Institute, Oakland, Calif., 3, 459-468.
7. Lai M., Chang K., Soong T.T, Hao D. and Yeh Y. Full Scale viscoelastically damped steel frame. J. Struct. Engrg 1995. ASCE, 121(10), 1443-1447.
8. Ashour S. and Hanson R. Elastic Response of buildings with supplemental damping. Report UMCE 87-1 1987, Dept. of Civ. Engrg. University of Michigan, Ann. Arbor, Mich.
9. Cheng F. and Pantelides C. Optimal placement of actuators for structural control. Tech. Rep. NCCER-88-0037 1988. Nat. Ctr. For Earthquake Engrg. Res. State University of New York, Buffalo. N.Y.
10. Shukla A. and Datta T. Optimal Use of Viscoelastic Dampers in Building Frames for Seismic Force. J. of Struct. Engrg. 1999. 125(4):401-409.
11. López D. A Simple Method for the Design of Optimal Damper Configurations in MDOF Structures. Earthquake Spectra 2001, 17(3):387-398.
12. Clough R. y Mojtahedi S. Earthquake response analysis considering non-proportional damping. Earthquake Engineering and Structural Dynamics 1976, vol. 4, 489-496.
13. Hasselman T. Modal coupling in lightly damped structures. AIAAJ 1976, 14, 1627-1628.
14. Warburton G. y Soni S. Errors in response calculations for non-classically damped structures. Earthquake Engineering and Structural Dynamics 1977, vol. 5, 365-376.
15. Veletsos A. y Ventura C. Modal analysis of non-classically damped linear systems. Earthquake Engineering and Structural Dynamics 1986, vol. 14, 217-243.
16. Prater G. y Singh R. Quantification of the extent of non-classical viscous damping in discrete vibratory systems. J. Sound Vib. 1986, 104, 109-125.
17. Greco A. y Santini R. Comparative study on dynamic analyses of non-classically damped linear system. Structural Engineering and Mechanics 2002, vol. 14(6),679-698
18. Goel R. Simplified analysis of asymmetric structures with supplemental damping. Earthquake Engineering and Structural Dynamics 2001, vol. 30, 1399-1416.
19. Tovar C., López O. A. y Tovar A. G. Efecto de la posición y número de amortiguadores en la repuesta sísmica de estructuras aporticadas. Memorias del VII Congreso Venezolano de Sismología e Ingeniería Sísmica 2003. Barquisimeto-Venezuela.
20. Chopra A. Dynamics of Structures. Theory and Applications to Earthquake Engineering. Prentice Hall. E.E.U.U. 2001, 729 pags.