13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 3149

DEMAND CAPACITY FOR UNSEATING PREVENTION CABLES OF BRIDGES

Kazuyuki IZUNO[1], Shinobu TAKENO[2], Hisashi NAKAO[3] and Hiroshi KOBAYASHI[4]

SUMMARY

The design of connecting devices for preventing bridge girders from becoming unseated during strong seismic motion is extended to include consideration of the velocity response of the bridge. The demand strength and cross-sectional area of the connecting cable are derived based on conservation of energy considerations. The demand capacity of the connecting cable is also defined for the worst case that the girder falls from the pier. The installation of shock absorber with optimum stiffness based on its deformation limit and the cable stiffness is found to reduce both the stress on the cable and the required cross-sectional area.

INTRODUCTION

All road bridges in Japan have been fitted with a seismic unseating prevention system to prevent bridge girders from falling during an earthquake. In the current design of Japanese highway bridges [1], the necessary strength of the system is prescribed to be 1.5 times the reaction force for the dead load of the bridge girder. In other words, the capacity is stipulated in terms of the dead load alone, with no consideration of the dynamic response of the bridge system.

With the emergence of the seismic unseating prevention system as an important issue in Japan, much research has been conducted in recent years [2]. However, most studies have focused on specific factors in bridge collapse such as cable capacity, while research on the design procedure for the unseating prevention system itself remains limited. The bridge unseating prevention system has been designed based on theoretical considerations under many assumptions because it is difficult to evaluate the operation of the system during an actual earthquake. It is necessary to ensure that the system functions adequately during earthquakes.

This research deals with the connecting-cable-type seismic unseating prevention system for highway bridges as shown in Fig. 1, presenting a rational design method based on the velocity conditions the

[1] Professor, Ritsumeikan University, Shiga, Japan. Email: izuno@se.ritsumei.ac.jp
[2] Engineer, CTI Engineering CO.,Ltd., Tokyo, Japan. Email: takeno@ctie.co.jp
[3] Graduate Student, Ritsumeikan University, Shiga, Japan. Email: rv005997@se.ritsumei.ac.jp
[4] Professor, Ritsumeikan University, Shiga, Japan. Email: kobayash@se.ritsumei.ac.jp

connecting device is expected to operate under. The proposed design method involves calculating the demand sectional area of the connecting cable from the absorbed strain energy for the expected girder velocities. The required stiffness of the shock absorber is then derived in consideration of the abrupt increase in cable force as the shock absorber reaches its deformation limit. In this study, the cable is treated as elastic assuming that the final device should not yield under easily supposed conditions.

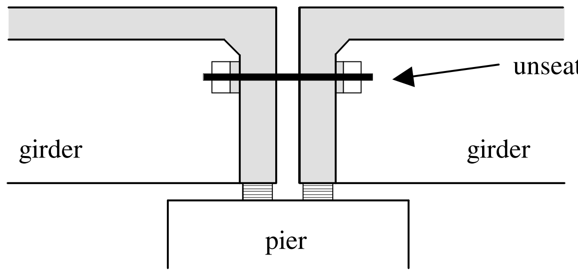

Fig. 1 Connecting-cable-type seismic unseating prevention system.

DEMAND CAPACITY OF CONNECTING DEVICE

Demand capacity of cable based on conservation of energy considerations

The behavior of the connecting cables during an earthquake is treated here as a simple vibrating problem, and the demand capacity of the cables is derived based on the law of the conservation of energy.

In order to discuss the demand capacity of the connecting device, it is first necessary to determine the expected movement of girders at which the device will operate. However, it is difficult to predict the response of the device during earthquakes with any degree of reliability.

In essence, the connecting device will be expected to operate under conditions in which adjacent girders move further than the allowed marginal displacement of the connecting device during a major earthquake. To simplify the problem, it is supposed that no earthquake load acts following the operation of the seismic unseating prevention system.

From the definition of impulse in physics, the load acting on the cable will be proportional to the relative velocity of the adjacent girders. Therefore, it is useful to treat the velocity response of the girders explicitly in this design approach.

Figure 2 shows a general schematic of the model used for analysis. Girders with mass m_1 and m_2 are supposed to be moving with speed v_1 and v_2 before the operation of the connecting device. A cable with spring constant k is used as a connecting device that connects both girders until the girders reach the same velocity v_0, as shown in Fig. 3. There is no guarantee that the girder stops at that time, however, the connecting cable supports the maximum force when the relative velocity becomes zero.

The following equation can be constructed from the law of energy conservation, designating cable extension as δ.

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}k\delta^2 + \frac{1}{2}\left(m_1 + m_2\right)v_0^2 \tag{1}$$

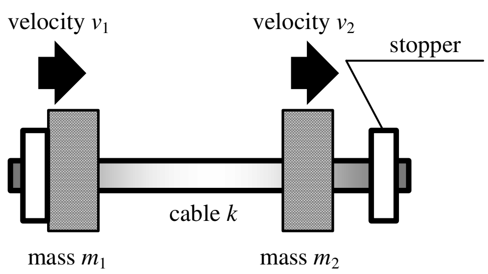

Fig. 2 Schematic model of connecting device before operation.

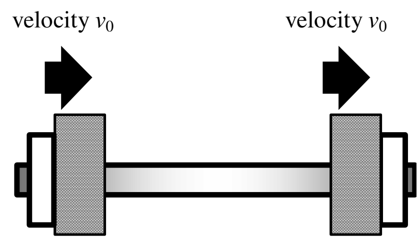

Fig. 3 Schematic model of connecting device in operation.

From the momentum conservation law, the velocity v_0 just after the connecting cable operates can be derived as follows.

$$v_0 = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \tag{2}$$

When v_0 is eliminated from Eqs. (1) and (2), the maximum cable deformation can be derived as follows, designating the relative velocity of the adjacent girders as $V_c = v_1 - v_2$.

$$\delta = \sqrt{\frac{M}{k}} V_c \tag{3}$$

where M is a reduced mass given by

$$M = \frac{m_1 m_2}{m_1 + m_2} \tag{4}$$

The reduced mass allows the description of the motions of the two masses to be reduced to that of an equivalent single mass.

The spring constant of the connecting cable is defined by the following equation in terms of cable length L, cross-sectional area A, and Young's modulus E.

$$k = \frac{EA}{L} \tag{5}$$

The maximum deformation of the cable is then derived as follows by substituting Eq. (5) into Eq. (3).

$$\delta = \sqrt{\frac{LM}{EA}} V_c \tag{6}$$

Therefore, the maximum stress acting on the cable becomes

$$\sigma = E\frac{\delta}{L} = V_c \sqrt{\frac{EM}{AL}} \tag{7}$$

In this paper, this stress is noted as σ, indicating that it is calculated based on the law of energy conservation. To design σ so as not to exceed the yield stress, the cross-sectional area of the cable (A) needs to satisfy the following equation.

$$A \geq \frac{ME}{L}\left(\frac{V_c}{\sigma_y}\right)^2 \tag{8}$$

Demand capacity of cables for free fall of girder

The equation of motion for a falling girder (shown in Fig. 4) is defined by the following equation in terms of girder length ℓ, girder mass m, angle θ and gravity acceleration g.

$$I\ddot{\theta} = mg\ell/2 \tag{9}$$

where I is moment of inertia for a girder:

$$I = m\ell^2/3 \tag{10}$$

The falling velocity v and the falling distance x are expressed as follows with zero initial angle and zero initial angular velocity at time $t = 0$.

$$v = \ell\dot{\theta} = 3gt/2 \tag{11}$$

$$x = \ell\theta = 3gt^2/4 \tag{12}$$

The velocity after falling x is as follows.

$$v = \sqrt{3gx} \tag{13}$$

After falling $x = \delta_0$, the connecting cable is assumed to begin operating. The equation of motion for falling girder with the operation of the connecting cable is as follows.

$$\frac{m\ell^2}{3}\ddot{\theta} + k\ell\theta \times \ell = mg \times \frac{\ell}{2} \tag{14}$$

where k is the cable stiffness.

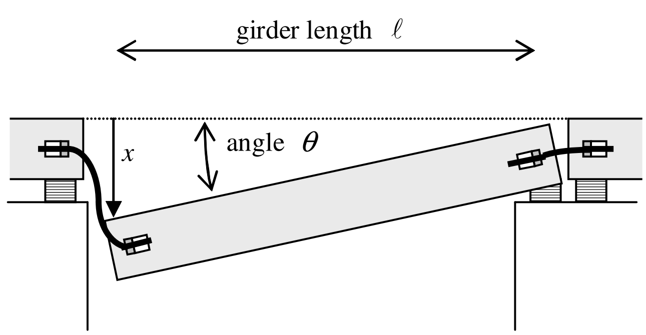

Fig. 4 Schematic diagram of falling girder.

The equation (14) is modified to the next equation (15).

$$\ddot{\theta}+\frac{3k}{m}\left(\theta-\frac{mg}{2k\ell}\right)=0 \tag{15}$$

Then, the falling angle θ is derived as follows with the initial condition of $\theta=0$ and $\dot{\theta}=\sqrt{3g\delta_0}$ at time $t=0$.

$$\theta=\frac{mg}{2k\ell}+\frac{1}{\ell}\sqrt{\frac{mg\delta_0}{k}}\sin\sqrt{\frac{3k}{m}}t-\frac{mg}{2k\ell}\cos\sqrt{\frac{3k}{m}}t \tag{16}$$

Therefore, the elongation of the unseating prevention cable *y* and the force applied to the cable *F* are as follows.

$$y=\frac{mg}{2k}+\sqrt{\frac{mg\delta_0}{k}}\sin\sqrt{\frac{3k}{m}}t-\frac{mg}{2k}\cos\sqrt{\frac{3k}{m}}t \tag{17}$$

$$F=ky=\frac{mg}{2}+\sqrt{mgk\delta_0}\sin\sqrt{\frac{3k}{m}}t-\frac{mg}{2}\cos\sqrt{\frac{3k}{m}}t \tag{18}$$

The cable force *F* of Eq. (18) can be described using the dead load of the girder R_d.

$$F=R_d\left\{1+\sqrt{1+2\beta}\times\cos\left(\sqrt{\frac{3k}{m}}t+\phi\right)\right\} \tag{19}$$

where $\phi=\tan^{-1}\sqrt{2\beta}$, and β is a coefficient expressed as a function of cable stiffness *k*, free-fall displacement of the girder δ_0 and the reaction force for the dead load of the girder R_d:

$$\beta=\frac{k\cdot\delta_0}{R_d} \tag{20}$$

The maximum cable force $F_{\max}$ becomes α times the reaction force R_d for the dead load of the girder, as follows.

$$F_{\max}=\alpha R_d \tag{21}$$

The coefficient α is given by

$$\alpha=1+\sqrt{1+2\beta} \tag{22}$$

From Eq. (22), α is 2 or more irrespective of the value of β, though the current design of Japanese highway bridges specifies the necessary strength of the system to be 1.5 times the reaction force for the dead load of the bridge girder.

DEMAND CAPACITY OF SHOCK ABSORBER IN CONNECTING DEVICE

Modeling of shock absorber

The authors [3] verified in the previous paper that the cross-sectional area of the connecting cable considering the demand capacity may be much greater than that prescribed in the current design manual. The insertion of a shock absorber is an effective means of reducing the sectional area of the connecting cable by reducing the demand capacity. A procedure for designing connecting cables with shock absorbers is proposed in this section.

A shock absorber is modeled as a spring element in a similar manner to the cable model in the previous section. The spring constant of the cable is designated k_1, and that of the shock absorber is designated k_2. The shock absorber is connected in series with the cable, providing the connecting device with a synthetic spring constant K given by

$$K = \frac{k_1 k_2}{k_1 + k_2} \tag{23}$$

Therefore, the deformation δ of the entire connecting device is expressed as

$$\delta = V\sqrt{\frac{M}{K}} \tag{24}$$

The load P that acts on the connecting device is then as follows.

$$P = K\delta = V\sqrt{MK} \tag{25}$$

Using P given by Eqs. (25), the deformation of the cable Δ_1 and that of the shock absorber Δ_2 are expressed as

$$\Delta_1 = \frac{P}{k_1} = \frac{PL}{EA} \tag{26}$$

$$\Delta_2 = \frac{P}{k_2} \tag{27}$$

Influence of the stiffness ratio between cable and shock absorber

The ratio of cable stiffness to shock absorber stiffness is defined as $r_k = k_2 / k_1$. The stiffness of the shock absorber is then expressed in terms of r_k as

$$k_2 = r_k \frac{EA}{L} \tag{28}$$

The relative demand capacity of the cable with a shock absorber compared to the case without a shock absorber then becomes

$$\frac{P}{F_E} = \sqrt{\frac{r_k}{1 + r_k}} \tag{29}$$

where F_E is the demand capacity of the cable derived from σ given by Eqs. (7).

Figure 5 shows the relationship between r_k and the demand capacity of the cable expressed by Eqs. (29). The vertical axis represents the relative demand capacity of the cable with a shock absorber. A very soft buffer is the most effective, with the relative demand capacity increasing as the stiffness ratio increases (about 0.7 at $r_k = 1$). Therefore, insertion of the softest available shock absorber is effective for reducing the demand capacity of the cable, regardless of the required size of the shock absorber.

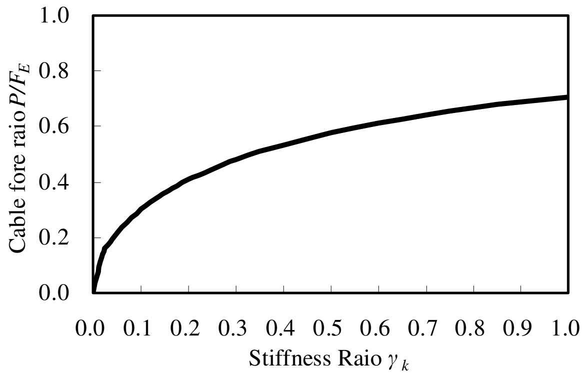

Fig. 5 Relationship between relative cable force P/F_E and stiffness ratio r_k.

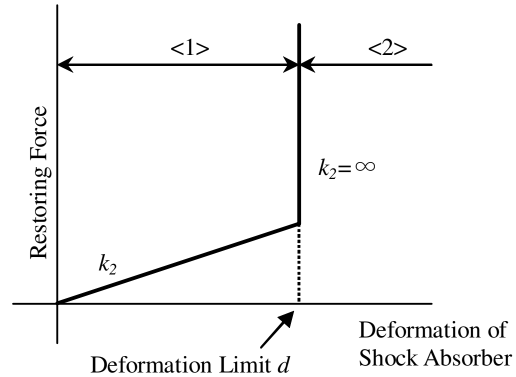

Fig. 6 Assumed force-displacement relationship of shock absorber.

Influence of shock absorber deformation limit

Equation (29) demonstrated the effectiveness of a soft shock absorber for reducing the demand capacity of the cable. However, it is expected from Eqs. (27) that a soft material will result in large deformation of the shock absorber itself. Therefore, the compression deformation limit of the material used for the shock absorber should be considered.

The rubber material commonly used for shock absorbers exhibits a compressional deformation limit due to a limited deformation capacity with nonlinear hardening behavior. Shock absorbers formed from metal springs also exhibit a deformation limit because of the limited compression between coil turns.

Figure 6 shows the force-displacement relationship for an arbitrary linear shock absorber with deformation limit d. In the deformation range from zero to d (the range <1> in Fig. 6), the stiffness of the shock absorber is modeled as k_2. When the deformation exceeds the deformation limit (the range <2> in Fig. 6), the stiffness is modeled as infinite.

For cable deformation Δ when the shock absorber reaches the deformation limit d, the following equation can be derived from the conservation of energy.

$$\frac{1}{2}k_1\Delta^2 + \frac{1}{2}k_2 d^2 = \frac{1}{2}MV^2 \tag{30}$$

The deformation of the cable Δ can then be expressed as

$$\Delta = \sqrt{\frac{1}{k_1}\left(MV^2 - k_2 d^2\right)} \tag{31}$$

The deformation δ of the entire connecting device then becomes the sum of the deformation of the shock absorber d and the cable deformation Δ:

$$\delta = d + \sqrt{\frac{L}{EA}\left(MV^2 - k_2 d^2\right)} \tag{32}$$

The maximum load P_d that acts on the connecting device can then be derived as follows from the cable stiffness and the final deformation of the cable.

$$P_d = \frac{EA}{L}\Delta = \sqrt{k_1\left(MV^2 - k_1 r_k d^2\right)} \tag{33}$$

If the shock absorber does not reach the deformation limit d, the load P acting on the connecting device is that given by Eqs. (25). Equation (33) therefore applies when the calculated deformation is greater than the deformation limit of the shock absorber. Equation (25) indicates that the larger the stiffness ratio r_k, the larger the load P that acts on the connecting device, whereas Eqs. (33) shows that larger r_k results in smaller P.

Influence of deformation limit on cable response

This section discusses the relationship between the stiffness ratio and the deformation limit under conditions of fixed input energy into the connecting device.

First, the relationship between the load applied to the cable (cable force) and the deformation limit of the shock absorber is examined for a fixed value of r_k. As mentioned above, the variation in cable force differs depending on whether the shock absorber has reached its deformation limit. Figure 7 shows the relationship between cable force and deformation limit. Below the deformation limit of the shock absorber (the range <1> in Fig. 7), the cable force remains constant. However, when the deformation of the shock absorber reaches its deformation limit (the range <2> in Fig. 7), the cable force begins to increase. This behavior is exaggerated when the deformation limit is relatively small. Therefore, to ensure that the cable load does not increase undesirably, the shock absorber should operate under the expected conditions within its deformation limit.

Next, the case of fixed deformation limit of the shock absorber is examined. Figure 8 shows the relationship between cable force and stiffness ratio of cable to shock absorber. The variation in cable force is governed by an inequality relation between the response and the deformation limit of the shock absorber. That is, when the response of the shock absorber is within its deformation limit, the cable force increases with the stiffness ratio (the range <1> in Fig. 8). In contrast, when an external load greater than the design load causes the shock absorber to reached its deformation limit, the cable force increases abruptly with decreasing stiffness ratio (the range <2> in Fig. 8). The figure also demonstrates that the cable force changes instantaneously when the shock absorber reaches its deformation limit.

An optimum stiffness ratio exists at which the cable force becomes minimum for the given design velocity of the connecting device and the deformation limit of the shock absorber. The optimum stiffness ratio is derived from Eqs. (18) and (26) for condition $P = P_d$ as follows.

$$r_k = \frac{1}{2}\left(-1 + \sqrt{1 + \frac{4MV^2}{k_1 d^2}}\right) \tag{34}$$

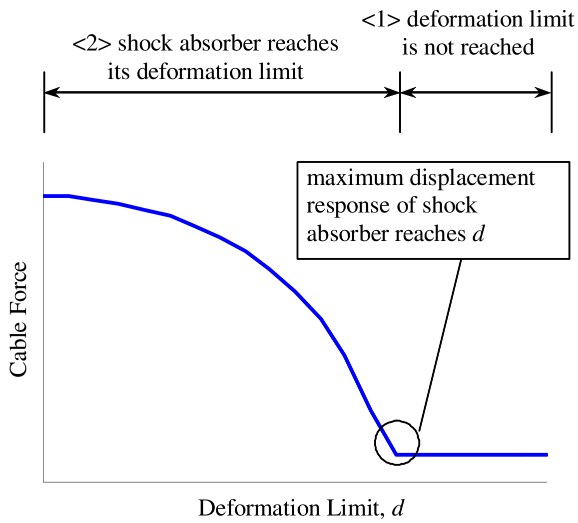

Fig. 7 Schematic diagram showing the effect of the deformation limit of a shock absorber.

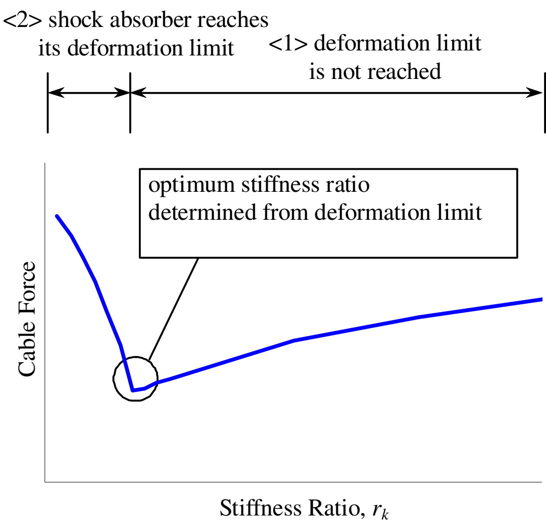

Fig. 8 Schematic diagram showing the effect of the stiffness ratio r_k.

According to Eqs. (27), the optimum stiffness ratio decreases as the deformation limit increases, and a higher design velocity for the connecting device requires a stiffer shock absorber.

This analysis reveals that it is necessary to consider the stiffness ratio and the deformation limit in the design of shock absorbers for the connecting device. Although softer shock absorbers are effective under a designated design load, it is important to note that the cable force may increase abruptly when a larger load is applied to the device causing the shock absorber to reach its deformation limit.

Furthermore, though the optimum stiffness of the shock absorber can be calculated from Eq. (34), the limitation of the stiffness exists to design a real shock absorber. It is difficult to realize the hard spring with long deformation limit d with rubber material. Even a helical spring cannot afford the ideal shock absorber.

For example, if the deformation limit d=500 mm is necessary, the hardest stiffness of the usual helical spring is about k_2=10 kN/m. Fig. 9 shows the schematic diagram of the helical spring. The stiffness of 10 kN/m and the deformation limit of 500 mm is available with 12 cycle windings of the steel helical spring with its free height H=750mm, the average coil diameter D=250mm, the wire diameter ϕ=20mm and the coil angle γ=10 degree. If the optimum stiffness from Eq. (34) is harder, a new type of shock absorber is needed.

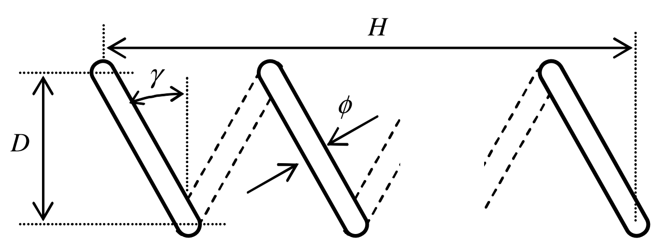

Fig. 9 Schematic diagram of helical spring.

CONCLUSIONS

This research extended the factors involved in calculating the demand capacity of a connecting cable for preventing bridge girders from becoming unseated to include the expected velocity response of the girder during a major earthquake.

The demand cross-sectional area of the cable was calculated based on conservation of energy considerations involving the cable stiffness, girder mass, and girder velocity at which the connecting device is designed to become operational (the design velocity).

The installation of a shock absorber was found to reduce both the required cable cross-sectional area and the stress acting on the cable. With a shock absorber installed, the cable force remained constant below the deformation limit of the shock absorber but increased abruptly when the deformation limit was reached. The optimum stiffness of the shock absorber was then proposed based on the cable stiffness and the deformation limit of the shock absorber.

The proposed calculation scheme is a more rational design method for the specification of connecting devices capable of preventing girder fall during huge earthquakes. As part of future work, it will be important to develop a method for setting an appropriate design velocity for the connecting device, define the reduced mass for continuous girder bridges, and clarify the energy balance at the time of a connecting cable operation. Data for setting the design velocity, for example, could be derived from the maximum recorded pounding velocities for bridges, or a pounding velocity spectrum [4] may be adopted.

ACKNOWLEDGEMENT

This research was supported in part by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture of Japan. The authors thank Mr. Kohei Kamata, Ms. Hiromi Ohno, Mr. Tatsuya Kenzaki and Mr. Hiroshi Kawarabayashi of Ritsumeikan University graduate students for their help in numerical simulations.

REFERENCES

1 Japan Road Association. "Design Specification for Highway Bridges, Part V: Seismic Design." 2002 (in Japanese).
2 Japan Society of Civil Engineers, Kansai Branch. "Report on unseating prevention system of bridges with shock absorber installed." 2001 (in Japanese).
3 Takeno S, Izuno K. "Demand strength for unseating prevention cables of bridges based on relative velocity response spectra." Proceeding of the JSCE Earthquake Engineering Symposium 2001: 26: 1165-1168 (in Japanese).
4 Takeno S, Izuno K. "Estimation of relative velocity response considering pounding between adjacent bridges during earthquake." Journal of Structural Mechanics and Earthquake Engineering, JSCE 2001: 668/I-54: 163-175 (in Japanese).