RESEARCH ON STRUCTURE VULNERABILITY EXPRESSION WAY BASED ON SEISMIC MOTION PARAMETER

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ABSTRACT :

The probability of each damage grade of structure in given intensity is fitted continuously by the use of beta distribution function. According to the corresponding relation between intensity and seismic motion parameter, the beta probability density distribution of structure damage ratio and the probability of every damage grade in each acceleration peak value are calculated based on seismic peak as seismic motion parameter in logarithm interpolation way. Thus the tradition step distribution way of structure vulnerability which is expressed by intensity versus earthquake damage grade is converted to continuous distribution way expressed by seismic motion parameter compared to destruction ratio. The practical data showed that this transformed approach is feasible and provides the reference for earthquake damage prediction and earthquake economic loss evaluation.

KEYWORDS: Beta distribution function, intensity, vulnerability matrix, earthquake damage grade, expectation, variance

1. INTRODUCTION

For a long time, the building vulnerability assessment is always based on seismic intensity in China. That is, every damage level probability of different types of buildings in all the intensity is obtained on the basis of statistical analysis of earthquake damage data and expressed by building vulnerability matrix [1-4]. According to the earthquake-site investigations and the earthquake damage prediction research, a great deal of basic information is accumulated for earthquake damage prediction work in future. A number of the region vulnerability matrixes of typical buildings are established in many cities.

However, the intensity itself means a rough grade concept evaluating faintly seismic ground motion intensity with structure earthquake damage and it is not a physical quantity. There is interdependence relation between the seismic intensity and the earthquake damage macroscopic phenomena. With the development of anti-seismic design level and the change of structure types in china, the scale of intensity itself has been seen tremendous changes and is a contradiction recursive definition logically. There is great uncertainty with the change of the ages [5]. Therefore, associated with the development of the earthquake risk research, the intensity has elimination tendency. Such as in 2001 the seismic zoning map whose parameters are the peak acceleration and characteristic cycle is promulgated and implemented in China. In addition, with the development of disaster risk assessment technology and the need of social development for nearly 10 years, the earthquake insurance industry is imperative in China. Since the beginning of the 1990s the approach of earthquake insurance in China used the traditional structure earthquake damage assessment method which was based on the intensity appeared too rough, it should be converted into earthquake damage assessment based on seismic parameters.

On the other hand, due to long-term research on earthquake damage prediction and the abundance data accumulated from actual earthquakes in China, the building vulnerability is mostly expressed as the matrix of

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Foundation item: Key research project of China Earthquake Administration (0303008).
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intensity-probability of damage levels. It is the first choice for all to use these valuable data fully. However, because the intensity is step, it is not gained from a simple method of interpolation that the probability distribution of structural damage level under any seismic intensity (that is, any continuous seismic parameters value) obtains from intensity.
An approach of calculation vulnerability curve of buildings which takes brick building for example according to the relative value between the earthquake resistance force of building and seismic intensity is put forward by Yin Zhiqian etc [8]. The various parameters of this approach require a lot of shear strength experiments. It provides a way for the seismic vulnerability curve analysis of building types which are not experienced earthquake. By means of the building vulnerability assessment approach in earthquake damage assessment software which is used widely in United States and based on vulnerability matrix, the parameters of resistance curve and vulnerability curve is inversed using hybrid algorithm with the combination of genetic algorithm and simplex algorithm,. Then the approach of earthquake damage prediction of stock buildings is put forward based on seismic parameters by Tao Zhenru etc.

In this paper, the probability distribution of building damage level in given intensity is continuous fitted with Beta probability density distribution function. The peak acceleration as ground motion parameters, the vulnerability distribution of building is calculated in log- interpolation way according to the relation of intensity and peak acceleration. Thus the tradition step distribution way of building vulnerability which is expressed by intensity versus earthquake damage grade is converted to continuous distribution way expressed by peak seismic acceleration compared to destruction ratio. This conversion is not only simple and rapid, but also damage state expressed by continuous damage ratio in theory. At the same time, the probability density of building damage ratio in given seismic parameter is converted into that of damage grade, forming building vulnerability expressed by seismic parameter versus damage grade. And this paper provides the reference for earthquake damage prediction and the fine calculation of earthquake economic loss evaluation in insurance industry.

2. THE EXPRESSION WAY OF BUILDING VULNERABILITY BASED ON SEISMIC PARAMETERS

2.1. The Characteristic of Beta Distribution Function

Here firstly introduce the math formula and its nature of beta distribution density function [10].

If the distribution density of random variables x is（$a>0, b>0$）

$$BE(x;a,b) = \begin{cases} x^{a-1}(1-x)^{b-1}/B(a,b) & 0\le x\le 1 \\ 0 & x<0 \text{ 或 } x>1 \end{cases} \tag{2.1}$$

Then X obeys the beta distribution with the parameters b and a. Here,

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt \tag{2.2}$$

The expectation of Beta density distribution is:

$$E(x) = \frac{a}{a+b} \tag{2.3}$$

The variance is:

$$Var(x) = \frac{ab}{(a+b)^2(a+b+1)} \tag{2.4}$$

Obviously, the Beta density distribution function accords with total probability distribution in the interval of $x \in [0, 1]$. That is

$$\int_0^1 BE(x,a,b)\,dx = 1 \tag{2.5}$$

It is the same nature that the sum of the probability of various damage levels is 1 for certain intensity. But in the interval of $x \in [0, 1]$., when its parameters b and a are not less than 1 at the same time, Beta density distribution functions have Single-peak nature. Therefore, if the structural damage ratio Dr replaced by than x, the building vulnerability is fitted by the Beta density distribution function. Here, damage ratio Dr is the continuous variable.

2.2.The Method of The Building Vulnerability Based On The Intensity Versus Damage Level Converting Into That Based On Seismic Parameter Versus Damage Ratio

The method of the building vulnerability based on the intensity converting into that based on seismic parameters is divided into three steps: (1) The probability distribution of earthquake damage level in each intensity in the building vulnerability matrix is fitted to Beta density distribution function expressed with the continuous variable of damage ratio Dr in each intensity; (2) According to the relationship of intensity and the ground motion parameters, the building loss expectations and variance corresponding to the goal seismic parameters is calculated through certain interpolation way. Then the parameters of Beta distribution function are gained, so the Beta density distribution of building damage ratio in the goal seismic parameters is obtained. (3) The continuous Beta Density distribution of building damage ratio in the goal seismic parameters reduced to the probability distribution of various earthquake damage levels in the target ground motion parameters.

2.2.1 The calculation of the Beta density distribution function of structural damage ratio D_r in i intensity.

When the damage level is expressed by continuous damage ratio Dr in the vulnerability matrix, the probability density distribution of each damage level is:

$$f_{i,j} = \frac{P_{i,j}}{\Delta \mathrm{Dr}_{i,j}} \tag{2.6}$$

Where, $f_{i,j}$ means the probability density of j level damage region for intensity i; $P_{i,j}$ means the probability of j level damage for intensity i; $\Delta \mathrm{Dr}_{i,j}$ means the D_r interval of j level damage for intensity i; j means damage levels.

Then, the expectations E_i of damage ratio for i intensity is:

$$E_i = \sum_{j=1}^{5} \int f_{i,j} D_r dD_r \tag{2.7}$$

The variance σ_i^2 is:

$$\sigma_i^2 = \sum_{j=1}^{5} \int f_{i,j}(D_r - E_i)^2 dD_r \tag{2.8}$$

By simultaneous equations (2.3) and (2.4), the parameters a_i and b_i values of the Beta distribution function for intensity i by fitting curve are:

$$a_i = \frac{E_i^2 - E_i^3}{\sigma_i^2} - E_i \tag{2.9}$$

$$b_i = (1 - E_i)(\frac{E_i - E_i^2}{\sigma_i^2} - 1) \tag{2.10}$$

So the continuous Beta probability density distribution of building damage ratio Dr for intensity i is gained. And with the method, the probability density distribution of building damage in the intensity of 6, 7, 8, 9, and 10 can be obtained.

2.2.2 The calculation of the Beta density distribution of building damage ratio in the goal seismic parameters.

Seismic parameters can be the peak acceleration of ground motion or other parameters. Supposing the ground motion parameters, the expectations and the standards of building damage corresponding to the intensity i and the intensity i+1 are A_i, E_i, σ_i and A_{i+1}, E_{i+1}, σ_{i+1}. For the arbitrary seismic parameters value A, when $A_i \le A \le A_{i+1}$, the appropriate mathematical interpolation approach can be found to achieve the expectations E_A and the standards σ_A of building damage ratio for the seismic parameter A in accordance with the relationship of the intensity and the ground motion parameters. It is assumed that the seismic parameter is the peak acceleration. As the relationship of the intensity and the peak acceleration is basically accordance with log-linear relation. The expectations E_A and the standards σ_A of building damage ratio for the seismic peak acceleration A can be obtained by log-interpolation way when $A_i \le A \le A_{i+1}$.

$$E_A = \frac{(E_{i+1} - E_i)(\lg A - \lg A_i)}{(\lg A_{i+1} - \lg A_i)} + E_i \tag{2.11}$$

$$\sigma_A = \frac{(\sigma_{i+1} - \sigma_i)(\lg A - \lg A_i)}{(\lg A_{i+1} - \lg A_i)} + \sigma_i \tag{2.12}$$

The value a_A and b_A is given from the value E_A and σ_A, then the Beta density distribution function $BE(D_r; a_A, b_A)$ of building damage ratio is obtained for the peak acceleration value A referring to the formula (2.9) and (2.10).

2.2.3 The calculation of each damage level probability of building for the arbitrary seismic parameters value A.

The continuous Beta density distribution function of building damage ratio is obtained for the arbitrary seismic parameters value A according to step (2). Then the probability of building j level damage is achieved for the given seismic parameter A according to the following formula.

$$p_{A,j} = \int_{D_{rJ1}}^{D_{rJ2}} BE(D_r; a_A, b_A) dD_r \qquad (2.13)$$

Where, $p_{A,j}$ means the probability of j damage for seismic parameters A; $D_{r,j1}$ means the lower limit of building damage ratio belonging to j damage level; $D_{r,j2}$ means the upper limit of building damage ratio belonging to j damage level; j means five damage levels including well, slightly damage, middle damage, serious damage and destruction.

Above three steps, the building vulnerability matrix based on intensity is converted into that based on the ground motion parameters. It should be noted that the building vulnerability data from earthquake in China is often in the range of intensity 6 to intensity 10. However it is no statistical data for below intensity 5, and for below intensity 5 the building is intact level which did not contribute to economic losses in the earthquake damage prediction. Therefore, the probability of intact is 100% and that of other levels are just zero for the smaller peak acceleration corresponding to intensity 5. It is no influence on the economic loss prediction by interpolation way. However, there is great effect on the economic loss prediction by extrapolation way for the above intensity 10. So it should do further careful study on the extrapolation way.

3. ANALYSIS OF EXAMPLES

3.1. The Fitting Beta Distribution of Building Vulnerability for The Different Intensities.

The vulnerability matrix of the multi-storey masonry structure in certain regional is shown in table 3.1. Then the probability density of damage ratio for 6-10 intensity in all grades interval is shown in table 3.2 according to formula (2.6).

Table 3.1 The vulnerability matrix of multi-storey masonry structure (%)

Intensity /I	Intact D_r=0.0～0.1	Slight damage D_r=0.1～0.3	Medium damage D_r=0.3～0.5	Serious damage D_r=0.5～0.7	Collapse D_r=0.7～1.0
6	88	12	0	0	0
7	28	61	11	0	0
8	6	27	58	9	0
9	1	5	22	59	13
10	0	0	3	30	67

Table 3.2 The probability density distribution of damage ratio

Intensity /I	Intact D_r=0.0～0.1	Slight damage D_r=0.1～0.3	Medium damage D_r=0.3～0.5	Serious damage D_r=0.5～0.7	Collapse D_r=0.7～1.0
6	8.80	0.60	0.00	0.00	0.00
7	2.80	3.05	0.55	0.00	0.00
8	0.60	1.35	2.90	0.45	0.00
9	0.10	0.25	1.10	2.95	0.43
10	0.00	0.00	0.15	1.50	2.23

From formula (2.7) to formula (2.10), the values including the expectations, the standards of building damage ratio for each intensity and the fitting Beta distribution density function with the parameters of a, b values are calculated in table 3.3. The peak acceleration corresponding to the intensity in the seismic zoning map which is promulgated and implemented in China is also listed in Table 3.3.

Table 3.3 the peak acceleration corresponding to intensity, the expectation and standard deviation of damage ratio, the a, b values of beta distribution density function

Intensity /I	The peak acceleration /Gal	the expectation of damage E	The standard deviation of damage σ	The *a* value	The *b* value
6	50	0.073	0.059	1.338	16.991
7	100	0.185	0.114	1.973	8.692
8	200	0.348	0.147	3.295	6.173
9	400	0.568	0.174	4.013	3.052
10	700	0.767	0.152	5.170	1.575

The probability distribution and the density of Beta fitting distribution of building damage ration for five intensities are shown in figure 1

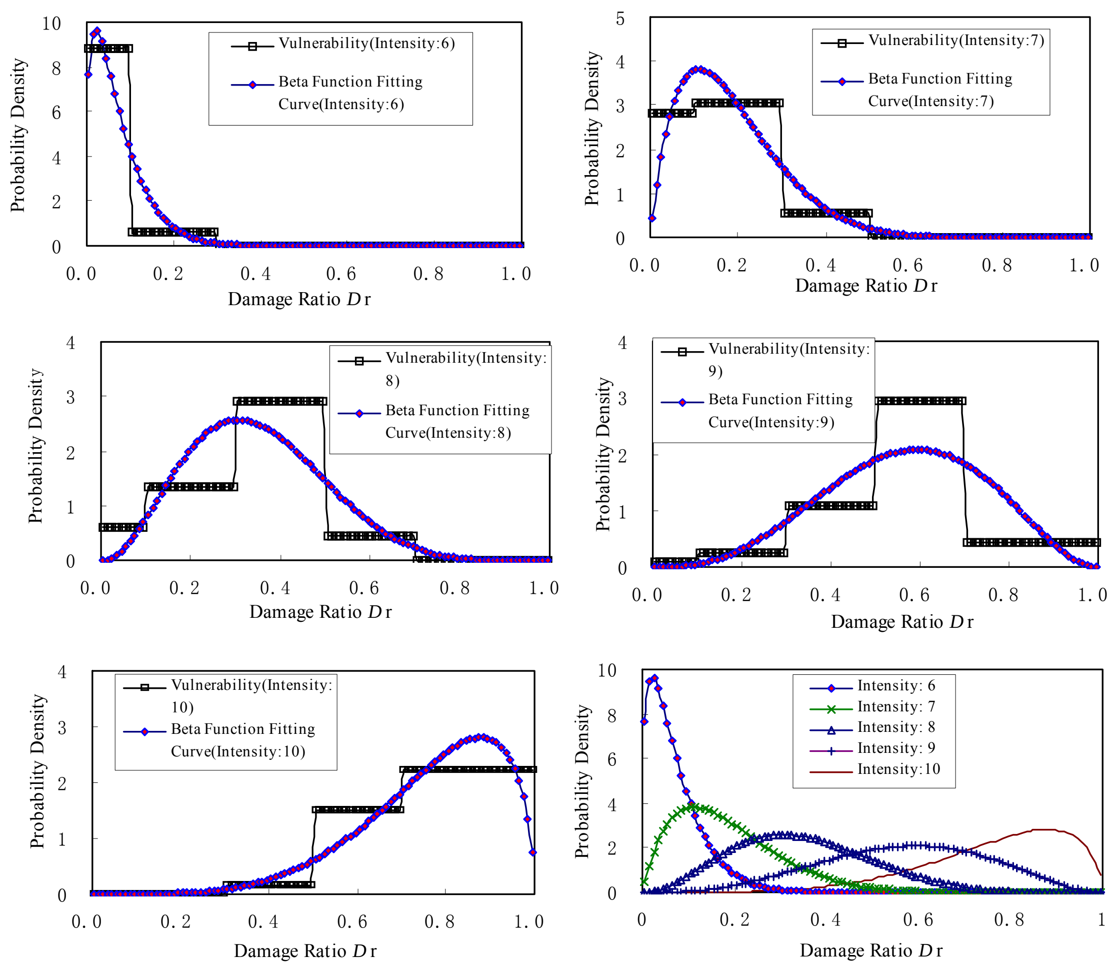

Figure 1 the probability density distribution of building damage with different intensities.

It can be seen from figure 1 that the density distribution of building damage ratio in vulnerability matrix fitted with Beta density distribution function is basically able to manifest the distribution nature of building damage ratio for different intensity. And it is obviously error in some individual results. Such as in the serious damage area of intensity 9 degree the probability density fitted by Beta distribution function is less than the actual probability density.

3.2. The Calculations of The Beta Probability Density Distribution of Building Damage Ratio and The Probability Distribution of Various Damage Levels for The Arbitrary Seismic Peak Acceleration.

Assuming that the probability of all damage levels for the peak acceleration of 600 gal are calculated. Reference to Table 3.3, the peak acceleration is between intensity 9 and intensity 10, according to formula (2.11) and (2.12), the expectation and standard of building damage ratio for the peak acceleration of 600 gal by log- interpolation way are separately:

$$E = \frac{(0.767 - 0.568)(\lg 600 - \lg 400)}{(\lg 700 - \lg 400)} + 0.568 = 0.712$$

$$\sigma = \frac{(0.152 - 0.174)(\lg 600 - \lg 400)}{(\lg 700 - \lg 400)} + 0.174 = 0.158$$

By formula (2.9) and (2.10), the parameters b and a of Beta distribution function are:

$$a = \frac{0.712^2 - 0.712^3}{0.158^2} - 0.712 = 5.13 \; ; \; b = (1 - 0.712)(\frac{0.712 - 0.712^2}{0.158^2} - 1) = 2.07 .$$

Figure 2 shows the probability density curve of damage ratio for the seismic peak acceleration of 600 gal. And as compared the probability density curves of building damage ratio for intensity 9 and intensity 10 are also given in figure 2.

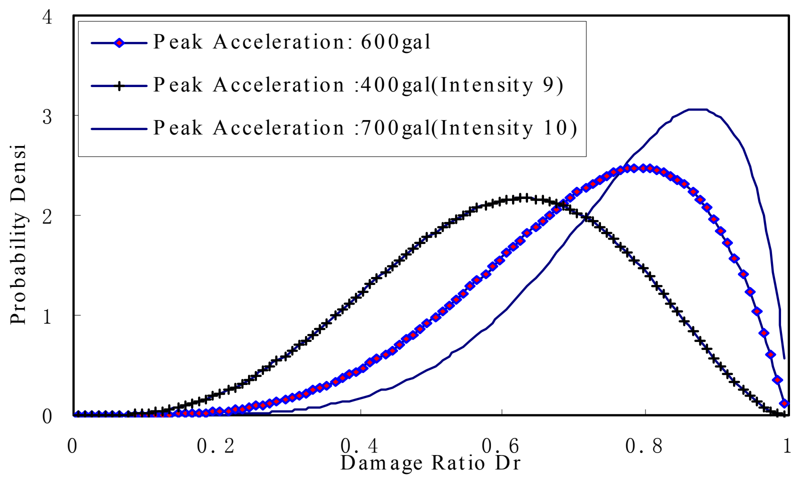

Figure 2 The probability density of building damage at the peak acceleration of 600gal

According the calculation of the formula (2.13), the probability of five damage levels at the peak acceleration of 600 gal is shown in table 3.4.

From figure 3.2 and table 3.4, the probability density curve of damage ratio and the probability of damage levels are accordance with our expectation by use of the interpolation.

Table 3.4 the probability of five damage grades at the peak acceleration of 600gal (%)

The peak acceleration /Gal	Intact D_r =0.0～0.1	Slight damage D_r =0.1～0.3	Medium damage D_r =0.3～0.5	Serious damage D_r =0.5～0.7	Collapse D_r =0.7～1.0
600	0	0	10	32	58
400 (intensity 9)	1	5	22	59	13
700 (intensity 10)	0	0	3	30	67

4. CONCLUSION

In order to make earthquake damage assessment on buildings based on the intensity convert into that based on the ground motion parameter, the probability distribution of building damage is fitted by beta function in this paper. The result shows: the probability density distribution of building vulnerability for different intensity which is fitted by Beta function accords with the probability distribution characteristics of building vulnerability as a whole. According to the relationship of the intensity and the ground motion parameters, the building vulnerability distribution for arbitrary seismic parameter is obtained in interpolation way. It is simple and feasible that the building vulnerability matrix expressed by intensity versus the probability of building damage grade is converted into that expressed by seismic versus the probability of building damage grade through the use of Beta distribution function. It should be noted that the extrapolation way in which the building vulnerability for above intensity 10 is converted into the building vulnerability which is expressed by seismic parameter should do further careful study.

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