RANDOM FUNCTION MODEL RESEARCH ON STRONG GROUND MOTION

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ABSTRACT:

Since a stochastic process may be represented perfectly by a family of functions with random parameters, an original random function model, in this context, for earthquake ground motion accelerograms is proposed. The new model contains two parts: one is constructed for describing the Fourier amplitude spectrum based on the physical theory; the other behaves the Fourier phase spectrum employing the empirical theorem. The Fourier amplitude spectrum model (FASM) is originated from the dynamic equation of a single-degree-of-freedom system, featuring the amplitude spectrum of the recorded accelerograms in principles. However, the Fourier phase spectrum model (FPSM) is built on the trend patterned accumulated phase (or unwrapped phase) spectrum which can be denoted by a series of linear polynomials or trigonometric functions. Finally, the model has been justified by the real accelerograms.

KEYWORDS: strong ground motion, Fourier amplitude spectrum, phase unwrapping, random function

1. INTRODUCTION

Due to the complex nature of the formation of seismic waves and their travel path before reaching the recording station, a stochastic approach may be most suitable for generating artificial accelerograms. In this regard, different stochastic models, both stationary and nonstationary have extensively been used in literatures to simulate earthquake ground motions. The stationary filtered white noise model of Kanai-Tajimi (Tajimi H., 1960), in this context, has been the favorite model for many researchers and engineers. Nevertheless the stochastic earthquake models expressed in the power spectrum density function (PSDF) are phenomenological description of the randomness in accelerograms, which can not establish the physical relationship between the sampling accelerogram and its influence factors. On the other hand, the stochastic seismic analysis based on the PSDF focuses on the statistical moments of responses, whereas much valuable probability information has been hidden in the classical random vibration theory. The probability density functions of responses at any instant time, however are in need, to access the dynamic reliability of the structures reliably. Recently, a family of probability density evolution method (PDEM) has been developed Jie Li and Jianbing Chen. (2003, 2004, 2006)., which is applicable to stochastic response analysis of MDOF systems and tends to assess the dynamic reliability of general systems.

It is necessary to apply the PDEM to the stochastic seismic analysis that the joint probability density function of the stochastic accelerogram process should be known in advance. As we know, the power density spectrum function is the second moment expression for a stationary random process, which is not enough for non-Gaussian processes to carry out the probability density function analysis by the PDEM. Theoretically, we can obtain any higher moments of the general random process through the statistics method, but in reality it is very difficult to get the required higher moments more than the second moments.

Revisiting the random function definition to the random process, an analytical random function has been put forward to express the randomness of the earthquake acceleration process. It is well known that the accelerograms is too complicated to express in a simple analytical function. On the other hand, the Fourier transform could map the time signals to the counterpart of the frequency domain, so a stochastic process for the acceleration probably can be described employed the analytical function model of the frequency domain. In the present random function model the random process exhibits the uniformed function style and the difference among the samples of the random process origins from the variability of the parameters in the random function. The outstanding charm of the random model is that the probability density function of the random process may be expressed as the multiplication of the probability density functions of the random parameters, on the premise of the independent random variables.

The random function model is divided into two parts, one is the random Fourier amplitude function and the other is the random Fourier phase function. There are 11 parameters in the random function model, three of them employed in the Fourier amplitude function and others utilized in the Fourier phase function. These parameters should be identified from the accelerograms, where the randomness of the stochastic process can be mirrored. The assumption of the parameters being independent underlies the complete probability space established by the parameter identification and statistics. Once the identified probability density functions of the random acceleration process obtained, the PDEM can be utilized to calculate the response probability density functions in next step.

2. BASIC PRINCIPLE OF STOCHASTIC PROCESS

A stochastic or random process $X(t)$ is defined as a parameterized family of random variables X with the parameter t. In mathematics there are two kind of definition to the stochastic process, one is from the joint probability density function and the other is from the random function representation.

Consider a random process $X(t)$, one could define the nth-dimensional joint distribution of $X(t)$ by

$$F_X(x_1,\cdots,x_n;t_1,\cdots,t_n) = P\{X(t_1)\le x_1,\cdots,X(t_n)\le x_n\}$$

The complete characterization of $X(t)$ requires knowledge of all the joint distributions as $n\to\infty$.

Unfortunately, it is difficult to get the higher-order moments more than the second-order moment, so we have to resort to the mean or the autocorrelation function of a random process. In practical, the power spectra are widely used to describe the stochastic process.

However, let ξ denote the random outcome of an experiment. A stochastic process $X(t)$ is a rule for assigning every ξ to a specified function $X(t,\xi)$. Thus, a stochastic process is a family of time-dependent functions hinging on the parameter ξ or, equivalently, a function of t and ξ. A stochastic process which can be completely specified by the random variables defines a predictable process.

In the following part, we attempt to transform a stochastic acceleration process into a predictable process which contains 11 random variables only.

3. RANDOM FUNCTION MODEL FOR EARTHQUAKE ACCELERATION

It is difficult to express the accelerogram by a simple analytical function in time domain due to the remarkable irregularity of the acceleration. Fortunately the Fourier transform can map the signal of the time domain to the counterpart of the frequency domain. Accordingly, an original random function model will be outlined in this paper by means of establishing the random Fourier amplitude function and the random Fourier phase function.

3.1. Random Fourier amplitude function model

It is well known that the relationship between the Fourier amplitude spectrum and the power spectral density can be expressed as

$$S(\omega)=\frac{1}{T}E\left[\left|F(\omega)\right|^2\right] \tag{3.1}$$

where $S(\omega)$ is the power spectral density; $F(\omega)$ is the Fourier amplitude spectrum of a sample; $|\ |$ means the magnitude of the complex value; E is the ensemble-expectation operator and T is the duration of the samples (Lin Y K and Cai G Q, 1995).

Therefore, the Fourier amplitude model of earthquake accelerations may have a form similar to the widely used power spectral density. Following this idea, three kinds of Fourier amplitude model has been deduced and the merits and disadvantages of each model has been compared in (Zihui An and Jie Li, 2007). In the latter part the Fourier amplitude model proposed by Matsuda and Asano (2006) using the power spectral density will be introduced in detail.

The power spectral density function for this model is given in the form

$$S_f(\omega)=\frac{\omega_g^2\omega^2}{(\omega_g^2-\omega^2)^2+4\xi_g^2\omega_g^2\omega^2}S_g \tag{3.2}$$

where ω_g is the natural circular frequency of the site; ξ_g is the damping factor of the site; S_g is spectral intensity of the white noise. The computation model is given in the figrue1.

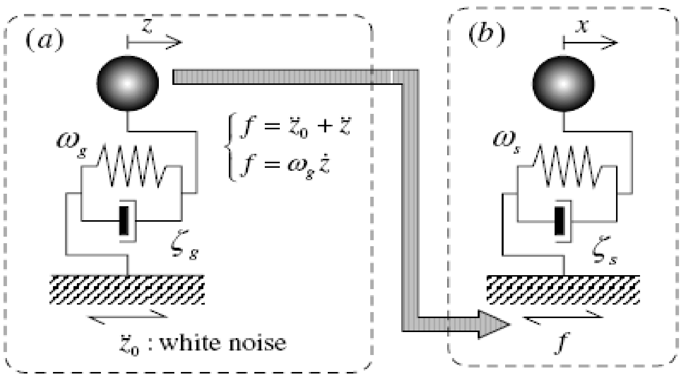

Figure 1 (a) SDOF system to define PSDF model and (b) SDOF structural model

This Fourier amplitude model can be deduced from figure1. It is well known that the pseudo-acceleration can be given by the relative velocity

$$\ddot{z}(t)+\ddot{z}_0(t)=\omega_g\dot{z}(t) \tag{3.3}$$

where $\ddot{z}(t)$ denotes the relative acceleration, $\ddot{z}_0(t)$ is the bedrock input acceleration, ω_g is the natural frequency of the SDOF system and $\dot{z}(t)$ is the relative velocity of the mass. Here the equation of motion of the SDOF system will be rewritten using the relative acceleration, velocity and displacement as

$$\ddot{z}+2\xi_g\omega_g\dot{z}+\omega_g^2 z=-\ddot{z}_0 \tag{3.4}$$

The Fourier transforms on the above equation yields

$$\frac{Z(i\omega)}{Z_0(i\omega)} = \frac{\omega^2}{\omega_g^2 - \omega^2 + 2\xi_g\omega_g(i\omega)} \tag{3.5}$$

Substituting Eqn. 3.3 in Eqn. 3.5 yields the transfer function

$$H(i\omega) = \frac{\omega_g(i\omega)\bullet Z(i\omega)}{-\omega^2 Z_0(i\omega)} = \frac{\omega_g(i\omega)\omega^2}{-\omega^2(\omega_g^2 - \omega^2 + 2\xi_g\omega_g(i\omega))} \tag{3.6}$$

then the Fourier amplitude model can be written as

$$F(\omega) = \frac{\omega_g\omega}{\sqrt{(\omega_g^2 - \omega^2)^2 + 4\xi_g^2\omega_g^2\omega^2}} \cdot S_g \tag{3.7}$$

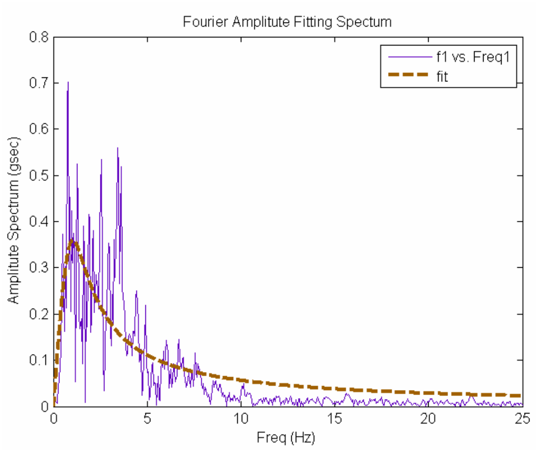

Figure 2a Fourier amplitude spectrum model for LIVERMORE record

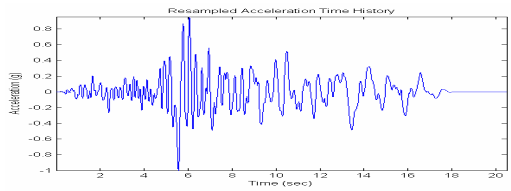

Figure 2b LIVERMORE record

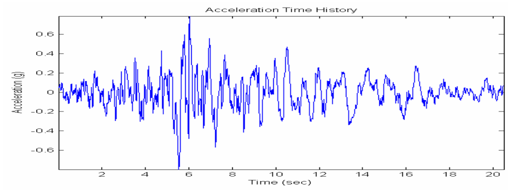

Figure 2c Simulated wave by modeled Fourier amplitude spectrum and phase spectrum of record

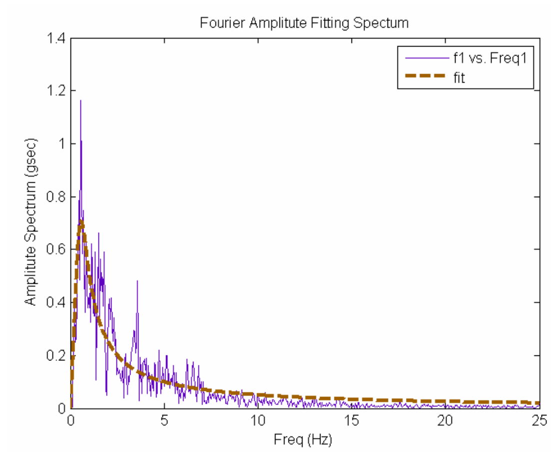

Figure 3a Fourier amplitude spectrum model for SUPERSTITION HILLS record

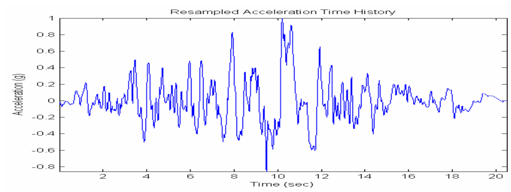

Figure 3b SUPERSTITION HILLS record

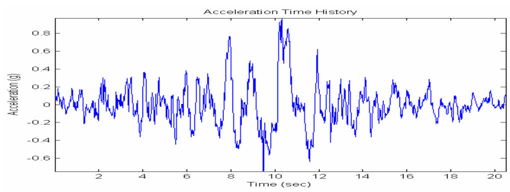

Figure 3c Simulated accelerogram by modeled Fourier amplitude spectrum and Fourier phase spectrum of record

The Fourier amplitude model has been applied to a great amount of earthquake records, and the comparison between the Fourier amplitude spectrum and the modeled Fourier amplitude spectrum shows good fitness. For illustration, Figure 2a illustrates the fitness between the Fourier amplitude spectrum of the accelerogram recorded at the LIVERMORE earthquake and the corresponding model curve. To clear the influence on the waveform, Figure 2b gives the recorded accelerogram and Figure2c presents the simulated record where the model substitutes the Fourier amplitude. Figure 3a, 3b and 3c picture another example of the accelerogram recorded at the SUPERSTITION HILLS earthquake. Good fitness and narrow confidence interval suggest that the Fourier amplitude model possesses most of outstanding properties contained in the Fourier amplitude spectra of the records. Table 1 labels the identified parameters and the 95% confidence bound on the two records.

Table 1 Fourier amplitude spectrum parameters' evaluation and confidence interval

Record name	ω_g (confidence interval)	ξ_g (confidence interval)	S_g (confidence interval)
LIVERMORE 01/24/80 CDMG STATION 57064	0.9935 (0.938, 1.049)	0.6622 (0.5948, 0.7296)	0.4623 (0.4269, 0.4977)
SUPERSTITION HILLS 11/24/87 USGS STATION 5051	0.5817 (0.5592, 0.6042)	0.5726 (0.5277, 0.6174)	0.7941 (0.7488, 0.8393)

3.2. Random Fourier phase function model

The dilemma in establishing the Fourier phase function model is the dramatic fluctuation in the Fourier phase spectrum of the accelerograms. For illustration, Figure 4 shows a phase spectrum of a typical record. In a former study, Shinozuka and Jan (1972) suggested that phase spectrum of earthquake record can be expressed as the random variables which are independently and uniformly distributed in the range $(-\pi,\pi)$. Ohsaki (1979) pointed out that although the phase in a recorded accelerogram appears to be uniformly distributed, they are far from independent. The distribution of the phase difference of the contiguous frequency components was founded to be similar in shape to the envelope of the time history. From then on, serial probability distributions have been put forward by different researchers including Normal distribution (Ohsaki, 1979, Kubo, 1990), Lognormal distribution (Zhu & Feng, 1992) and Beta distribution (Thrainsson & Kiremidjian, 2002). Though the nonlinearity of the accelerogram can be simulated in some degree by means of the phase difference distribution, the distribution model is still a statistical method to the phase spectrum. Since the distribution can be achieved by different phase assembles, the probability model cannot realize the mapping between the record and a couple of parameters.

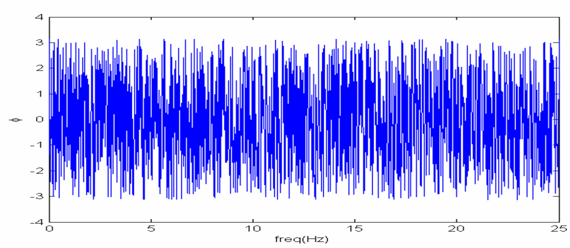

Figure4 FOUREIR phase spectrum of typical recorded strong ground

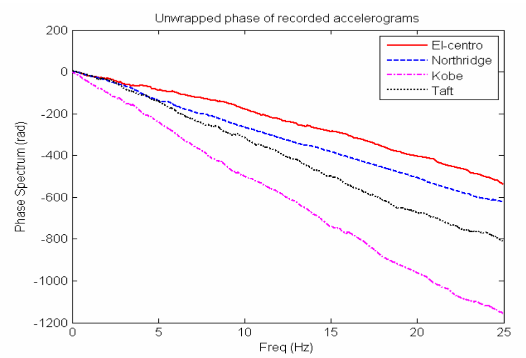

Figure5 Unwrapped Phase of four recorded Acceloragrams

In most studies the phase spectrum of the accelerogram is defined as the principle value of the inverse trigonometric function, which is in the interval of $(-\pi,\pi)$. However, the result of the trigonometric function remains constant by adding or subtracting the $2k\pi$ from the principle value of the phase. So can we add or subtract some $2k\pi$ from the normal phase to form a trended phase spectrum?

Alternately, the phase may be unwrapped by computing the relative phase between adjacent samples of the spectrum. If the phase is sampled at a rate sufficiently great to assure that it never changes by more than π between samples, a correction sequence $C(k)$ is added to the modulo 2π phase sequence *P(k)* where *C(k)* is

$$C(0)=0$$
$$C(k)=\begin{cases} C(k-1)-2\pi, & \text{if } P(k)-P(k-1)>\pi \\ C(k-1)+2\pi, & \text{if } P(k-1)-P(k)>\pi \\ C(k-1), & \text{otherwise} \end{cases} \tag{3.8}$$

then

$$\phi(k)=P(k)+C(k) \tag{3.9}$$

The phases may be added to achieve a cumulative (unwrapped) phase of each point. From the correction we can see that the cumulative phase is the counterpart of the principal phase.

For illustration, let us consider the phase curves of four recorded ground motions, i.e., (i) North-south component recorded at Imperial Valley Irrigation District substation in El Centro, California, during the Imperial Valley, California earthquake of May, 18, 1940, and (ii) Northridge, California earthquake of Jan 17, 1994 recorded at Sylmar County Hospital parking lot in Sylmar, California, (iii) Hyogo-ken Nanbu (Kobe) earthquake of Jan. 17, 1995 recorded at Kobe Japanese Meteorological Agency (JMA) station, and (iv) Kern County, California Earthquake of July 21, 1952 recorded at Taft Lincoln school tunnel, S69E Component. The phase curves of theses accelerograms , as computed in accordance with Eqns. 3.8 and 3.9, are shown in Fig. 5. It can be seen that the phase curves for all recorded time histories exhibit an almost monotonic downward trend. Similar trend has been observed in the case of many more recorded accelerograms. This suggests the possibility of modeling the phase curve by a generic analytical mathematical function. Shrikhande & Gupta (2001) considered the possibility of modeling unwrapped phase angels or the phase curve of an earthquake ground motion by a piecewise-linear curve and superimposing this with zero-mean Gaussian residual phases to introduce sample-to-sample variations within an ensemble. However the piecewise-linear curve need too much parameters to model a record and the Gaussian residual phase does not satisfy the mapping uniquely relationship between the record and the model. In order to overcome the shortcoming of the piecewise-linear curve model, we utilize the linear polynomial to model the monotonic trend and use the sum of sinusoidal functions to simulate the fluctuations around the linear line. The model we proposed can be expressed as

$$\begin{aligned}\Phi = p_1 f + p_2 + p_3 \sin(p_4 f) + p_5 \sin(2 p_4 f) + \\ p_6 \sin(3 p_4 f) + p_7 \sin(4 p_4 f) + p_8 \sin(5 p_4 f)\end{aligned} \tag{3.10}$$

where Φ is the cumulative phase, f is the frequency and $p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8$ are the undetermined coefficients, which should be identified from the earthquake record.

We have applied the preceding analytical function model proposed to two recorded accelerograms, the LIVERMORE earthquake occurred at January 24, 1980 recorded at CDMG STATION 57064, and the SUPERSTITION HILLS earthquake of November 24, 1987 at USGS STATION 5051. The identified coefficients from the records are labeled in Table 2. Figure 6a shows the fitting curve of the cumulative phase of the LIVERMORE record, and Figure 6b is the original record, Figure 6c is the synthetic acclelerogram by the modeled phase spectrum and the original amplitude spectrum. Figures 7a, 7b and 7c are the result of the

SUPERSTITION HILLS record. It is seen from the above two figures that the synthetic accelerogram is similar to the original record in the peak ground motion, the amount and the sequence of the spikes. We have to point out that the amplitude spectrum of the record applied to the synthetic wave is to reduce the impact on the difference from the application of the amplitude spectrum model.

Table2 Coefficients of Fourier Phase Spectrum

参数	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
LIVERMORE 01/24/80	-26.9491	14.1150	6.5057	1.0676	-2.3765	-0.4107	-0.2163	0.5187
SUPERSTITION HILLS 11/24/87	-7.7182	25.4433	6.7309	1.0143	-1.6725	-0.7536	0.3925	-3.7063

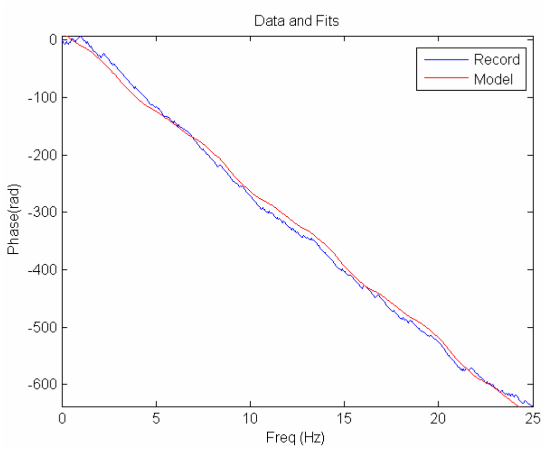

Figure 6a Fourier phase spectrum model for LIVERMORE record

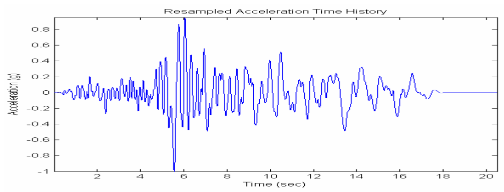

Figure 6b LIVERMORE record

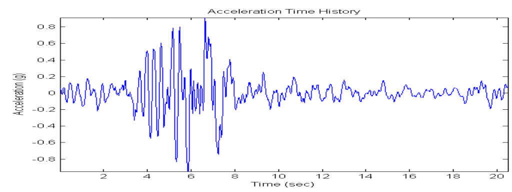

Figure 6c Simulated accelerogram by Fourier amplitude spectrum of record and modeled Fourier phase spectrum

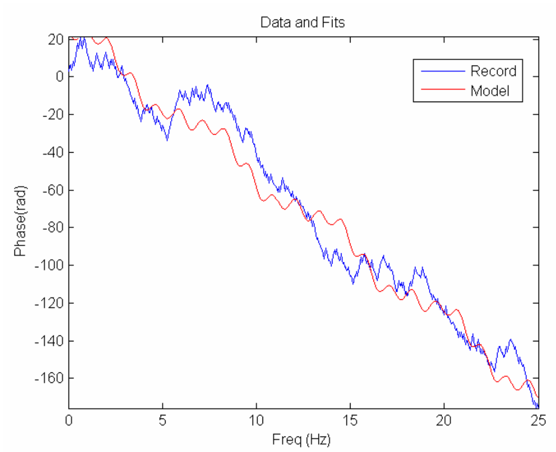

Figure 7a Fourier phase spectrum model for SUPERSTITION HILLS record

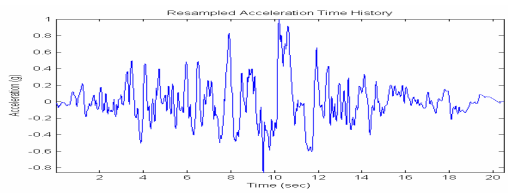

Figure 7b SUPERSTITION HILLS record

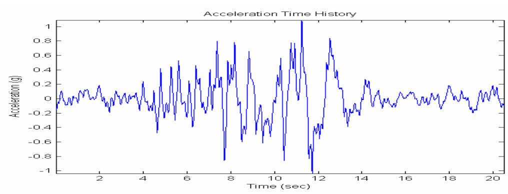

Figure 7c Simulated accelerogram by Fourier amplitude spectrum of record and modeled Fourier phase spectrum

In summary the Fourier model of the accelerograms can be model by Eqns. (3.7) and (3.10). In other words, any earthquake ground acceleration has been established a mapping relationship with the 11 model parameters (ω_g、ξ_g、S_0、p_1、p_2、p_3、p_4、p_5、p_6、p_7、p_8). Therefore the variation between the samples of an ensemble of accelerograms can be described by the variation of the model parameters. Furthermore the random function model can be incorporated with the number-theoretic technique to generate the ensemble of stochastic earthquake time histories. The statistics of the random parameters of the model and how to generate the stochastic earthquake accelerations will be discussed in another paper.

4. CONCLUDING REMARKS

A new stochastic model has been proposed on the basis of the random function definition of the random process. There involves two main parts contained in this paper:

(1) The basic theorem and method for establishing the stochastic process have been deeply explored, indicating that it is reasonable reflecting the randomness in stochastic process as the variation of the parameters in the random function model.

(2) The random function model includes two parts: one is the Fourier amplitude model according to the concept of pseudo-acceleration; and the other is the Fourier phase model based on the trended cumulative phase.

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