# SCATTERING OF SH-WAVE BY A SHALLOW-EMBEDDED CIRCULAR LINING STRUCTURE AND ITS SURROUNDING BEELINE CRACK AND GROUND MOTION

**Yang Zailin [1], Xu Meijuan [2] and Sun Baitao [3]**

[1]*Associate Professor, School of Civil Engineering, Harbin Engineering University, Harbin. China*
[2]*Postgraduate, School of Civil Engineering, Harbin Engineering University, Harbin. China*
[3]*Professor, Institute of Engineering Mechanics, China Earthquake Administration, Harbin, China*
*Email: yangzailin00@163.com, meijuan242@163.com , sunbt@iem.net.cn*

**ABSTRACT :**

The problems of SH-wave scattering, which are caused by a shallow-embedded circular lining structure and beeline crack of arbitrary length and arbitrary position, are studied in this paper beyond the field of linearly elastic dynamic mechanics. The methods of Green's function, complex variables and multi-polar coordinates are used here. Firstly a suitable Green's function is constructed, which is an essential solution to the displacement field for the elastic space possessing circular lining structure while bearing out-of-plane harmonic line source load at arbitrary point. Then using the Green's function and the method of crack-division, the crack is established: reverse stresses are inflicted along the crack, that is, out-of-plane harmonic line source loads, which are equal in the quantity but opposite in the direction to the stresses produced for the reason of SH-wave scattering by crack or circular lining structure, are loaded at the region where crack will appear, thus the crack can be made out. Then expressions of displacement and stress are established while crack and inclusion are both in existent. Finally, according to some numerical examples, the horizontal surface displacement is discussed to the case of different parameters just like the wave number of incident wave, the incident angle, the length of crack,and so on.

**KEYWORDS:** scattering of SH-wave, shallow-embedded circular lining structure, beeline crack, Green's Function, crack-division, ground motion

## 1. INTRODUCTION

Seismic wave scattering by inclusion and crack is a very important and challenging problem in the field of earthquake engineering and strong motion seismology, and an important issue in earthquake engineering and strong motion seismology is to describe and analyze the displacement amplitudes and the relative phases of motions of infrastructures on or nearby the ground surface. In this paper, the interaction of inclusion and crack impacted by SH-wave is investigated, and the horizontal surface displacement is given. The model of lining structure is common in many projects and has practical value and significance. The scattering of SH-wave, which is caused by half-space circular lining structure, had been studied(Qi Hui *et al* 2003). At present, most studies concerning scattering of SH-wave by the inclusion and crack focus on the radial crack, which originates from the boundary of inclusion and along the radius. In fact, when the forces are applied to the composite materials containing inclusions, the cracks are often found in the vicinity of the inclusions(Li Hongliang and Liu Diankui 2004; Liu Diankui and Lin Hong 2004). The purpose of this paper is to study scattering of SH wave by an interacting mode Ⅲ crack with any position and a circular lining structure in half space(Yang Zailin *et al* 2006), which can supply some beneficial references to the strength designing and non-destructive inspection of composite materials.

## 2. GREEN'S FUNCTION

This problem can be seen as the problem of the defending of earthquake. The model of a shallow-embedded circular lining structure and a crack is given according to Figure 1. $\rho_{\mathrm{I}}$ and $C_{\mathrm{I}}$, $\rho_{\mathrm{II}}$ and $C_{\mathrm{II}}$ are the mass density and the wave velocity of the medium and the inclusion, respectively. There are three coordinates: XOY , X'O'Y' and X"O"Y", there is a connection between them:

$$
\begin{aligned}
x' &= x\cos\beta_0 + y\sin\beta_0 \\
y' &= y\cos\beta_0 - x\sin\beta_0 \\
x'' &= x \qquad y'' = y - h_1 \\
h_3 &= \frac{(h_2 + b\sin\beta_0)}{\cos\beta_0}
\end{aligned}
\tag{2.1}
$$

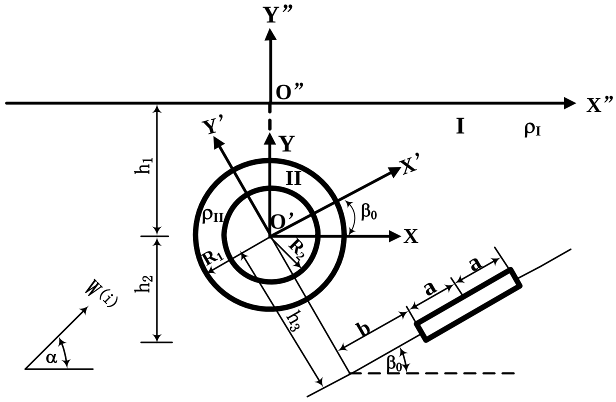


Figure 1 The infinite half space model with a circular lining structure and a crack impacted by SH wave

### *2.1. Governing Equation*

In the isotropic medium, the easiest scattering problem for the elastic wave is the scattering of SH-wave. The displacement $W(x,y,t)$ impacted by the incident wave is vertical to the *xy* plane, it is irrespective with the *z* axis. The displacement $W(x,y,t)$ should satisfy:

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} + k^2 W = 0 \tag{2.2}$$

in which: $k=\omega/c_s$ , $\omega$ is the circular frequency of the displacement function, $c_s = \sqrt{\mu/\rho}$ stands for the velocity of the shear wave. $\rho$ and $\mu$ are the mass density and shear modulus of the medium respectively.
The corresponding stress components can be expressed as:

$$\tau_{xz} = \mu\frac{\partial W}{\partial x}, \tau_{yz} = \mu\frac{\partial W}{\partial y} \tag{2.3}$$

Introducing a couple of new complex variables $z = x+iy, \overline{z} = x-iy$ , in the complex plane $(z,\overline{z})$ :

$$\frac{\partial^2 W}{\partial z\partial\overline{z}} + \frac{1}{4}k^2 W = 0 \tag{2.4}$$

The corresponding stress can be expressed as:

$$\tau_{xz} = \mu\left(\frac{\partial W}{\partial z} + \frac{\partial W}{\partial \overline{z}}\right), \tau_{yz} = i\mu\left(\frac{\partial W}{\partial z} - \frac{\partial W}{\partial \overline{z}}\right) \tag{2.5}$$

In the polar coordinate, the corresponding stresses are given by

$$\tau_{rz} = \mu\left(\frac{\partial W}{\partial z}e^{i\theta} + \frac{\partial W}{\partial \overline{z}}e^{-i\theta}\right), \tau_{\theta z} = i\mu\left(\frac{\partial W}{\partial z}e^{i\theta} - \frac{\partial W}{\partial \overline{z}}e^{-i\theta}\right) \tag{2.6}$$

### *2.2. The Scattering Wave around the Circular Lining Structure*

*2.2.1 The scattering wave in the district Ⅰ*

Now, according to the symmetry of the scattering wave and multi-polar coordinates, the scattering wave in the medium $G_{\mathrm{I}}^{(s)}$ around the circular lining structure can be districted, which should satisfy the Eqn. (2.2) and the radiation condition beside the stress free condition on the horizontal interface.
In the complex plane $(z,\bar{z})$, $G_{\mathrm{I}}^{(s)}$ can be expressed as:

$$G_{\mathrm{I}}^{(s)}=\sum_{n=-\infty}^{\infty}A_n\left\{H_n^{(1)}\left(k_1|z|\right)\left[\frac{z}{|z|}\right]^n+H_n^{(1)}\left(k_1|z-2h_1i|\right)\left[\frac{z-2h_1i}{|z-2h_1i|}\right]^{-n}\right\} \tag{2.7}$$

in which, $A_n$ are the unknown coefficients, which are determined by the boundary condition of the circular lining structure.
In the polar coordinate, the corresponding stress can be expressed as:

$$\tau_{rz,\mathrm{I}}^{(s)}=\frac{k_1\mu_1}{2}\sum_{n=-\infty}^{\infty}A_n\left\{H_{n-1}^{(1)}\left(k_1|z|\right)\left[\frac{z}{|z|}\right]^{n-1}e^{i\theta}-H_{n+1}^{(1)}\left(k_1|z|\right)\left[\frac{z}{|z|}\right]^{n+1}e^{-i\theta}-H_{n+1}^{(1)}\left(k_1|z-2h_1i|\right)\right.$$
$$\left.\left[\frac{z-2h_1i}{|z-2h_1i|}\right]^{-(n+1)}e^{i\theta}+H_{n-1}^{(1)}\left(k_1|z-2h_1i|\right)\left[\frac{z-2h_1i}{|z-2h_1i|}\right]^{-(n-1)}e^{-i\theta}\right\} \tag{2.8}$$

$$\tau_{\theta z,\mathrm{I}}^{(s)}=i\frac{k_1\mu_1}{2}\sum_{n=-\infty}^{\infty}A_n\left\{H_{n-1}^{(1)}\left(k_1|z|\right)\left[\frac{z}{|z|}\right]^{n-1}e^{i\theta}+H_{n+1}^{(1)}\left(k_1|z|\right)\left[\frac{z}{|z|}\right]^{n+1}e^{-i\theta}-H_{n+1}^{(1)}\left(k_1|z-2h_1i|\right)\right.$$
$$\left.\left[\frac{z-2h_1i}{|z-2h_1i|}\right]^{-(n+1)}e^{i\theta}-H_{n-1}^{(1)}\left(k_1|z-2h_1i|\right)\left[\frac{z-2h_1i}{|z-2h_1i|}\right]^{-(n-1)}e^{-i\theta}\right\} \tag{2.9}$$

*2.2.2 The scattering wave in the district Ⅱ*

In the complex plane $(z,\bar{z})$, the scattering wave inside of circular lining structure can be expressed as:

$$G_{\mathrm{II}}^{(s)}=\sum_{n=-\infty}^{\infty}B_n\left\{H_n^{(2)}(k_2|z|)\left(\frac{z}{|z|}\right)^n\right\}+\sum_{n=-\infty}^{\infty}C_n\left\{H_n^{(1)}(k_2|z|)\left(\frac{z}{|z|}\right)^n\right\} \tag{2.10}$$

in which, $B_n, C_n$ are the unknown coefficients, which are determined by the boundary condition of the circular lining structure.
Substituting these expressions into Eqn. (2.6), in the polar coordinate, the relevant stress can be obtained:

$$\tau_{rz,\mathrm{II}}^{(s)}=\frac{k_2\mu_2}{2}\sum_{n=-\infty}^{\infty}B_n\left\{H_{n-1}^{(2)}\left(k_2|z|\right)\left[\frac{z}{|z|}\right]^{n-1}e^{i\theta}-H_{n+1}^{(2)}\left(k_2|z|\right)\left[\frac{z}{|z|}\right]^{n+1}e^{-i\theta}\right\}$$

$$+\frac{k_2\mu_2}{2}\sum_{n=-\infty}^{\infty}C_n\left\{H_{n-1}^{(1)}\left(k_2|z|\right)\left[\frac{z}{|z|}\right]^{n-1}e^{i\theta}-H_{n+1}^{(1)}\left(k_2|z|\right)\left[\frac{z}{|z|}\right]^{n+1}e^{-i\theta}\right\} \tag{2.11}$$

$$\tau_{\theta z,\mathrm{II}}^{(s)}=i\frac{k_2\mu_2}{2}\sum_{n=-\infty}^{\infty}B_n\left\{H_{n-1}^{(2)}\left(k_2|z|\right)\left[\frac{z}{|z|}\right]^{n-1}e^{i\theta}+H_{n+1}^{(2)}\left(k_2|z|\right)\left[\frac{z}{|z|}\right]^{n+1}e^{-i\theta}\right\}$$
$$+i\frac{k_2\mu_2}{2}\sum_{n=-\infty}^{\infty}C_n\left\{H_{n-1}^{(1)}\left(k_2|z|\right)\left[\frac{z}{|z|}\right]^{n-1}e^{i\theta}+H_{n+1}^{(1)}\left(k_2|z|\right)\left[\frac{z}{|z|}\right]^{n+1}e^{-i\theta}\right\} \tag{2.12}$$

### *2.3. Derivation of Green's Function*

The Green's function in this paper is the solution of the displacement field for an elastic half space surface with an elastic shallow-embedded circular lining structure impacted by a time harmonic out-of-plane line source loading at an arbitrary point which is in the half space. The displacement is expressed as $e^{-i\omega t}$ and the displacement function $G$ satisfies the governing equation (2.2).
In a full elastic space, the wave field excited by the out-of-plane line source loading $\delta(\vec{r}-\vec{r}_0)$ is $G^{(i)}$, which can be described as a known incident wave, it takes the form

$$G^{(i)}=\frac{i}{4\mu_1}H_0^{(1)}\left(k_1|r''-r_0''|\right)e^{-i\omega t} \tag{2.13}$$

where $H_0^{(1)}(\,\cdot\,)$ expresses the first kind Hankel function with zero order. According to the addition theorem of the Bessel function, and time-harmonic factor $e^{-i\omega t}$ has been omitted, Eqn.(2.13) can be writen as(Pao and Mow 1973):

$$G^{(i)}=\frac{i}{4\mu_1}\sum_{m=0}^{\infty}\varepsilon_m\cos m(\theta''-\theta_0'')\begin{cases}J_m\left(k_1r_0''\right)H_m^{(1)}\left(k_1r''\right) & r''>r_0''\\ J_m\left(k_1r''\right)H_m^{(1)}\left(k_1r_0''\right) & r''<r_0''\end{cases} \tag{2.14}$$

when $m=0,\varepsilon_m=1$; $m\geq 1,\varepsilon_m=2$.
According to Eqn.(2.1), Eqn.(2.13) can be written as

$$G^{(i)}=\frac{i}{4\mu_1}H_0^{(1)}\left(k_1|z''-z_0''|\right) \tag{2.15}$$

According to the 'symmetry theory'(Lee *et al* 1999), the reflection wave reflected by a horizontal surface can be expressed as

$$G^{(r)}=\frac{i}{4\mu_1}H_0^{(1)}\left(k_1|z''-\overline{z}_0''|\right) \tag{2.16}$$

Substitution of Eqn.(2.15) and (2.16) into Eqn.(2.6), the stresses from incident wave $G^{(i)}$ and reflected wave $G^{(r)}$ can be obtained

$$\tau_{rz}^{(i)}=-\frac{ik_1}{8}\left\{\left[H_1^{(1)}\left(k_1\left|z-z_0\right|\right)\frac{\left|z-z_0\right|}{z-z_0}\right]e^{i\theta}+\left[H_1^{(1)}\left(k_1\left|z-z_0\right|\right)\frac{z-z_0}{\left|z-z_0\right|}\right]e^{-i\theta}\right\} \tag{2.17}$$

$$\tau_{\theta z}^{(i)}=\frac{k_1}{8}\left\{\left[H_1^{(1)}\left(k_1\left|z-z_0\right|\right)\frac{\left|z-z_0\right|}{z-z_0}\right]e^{i\theta}-\left[H_1^{(1)}\left(k_1\left|z-z_0\right|\right)\frac{z-z_0}{\left|z-z_0\right|}\right]e^{-i\theta}\right\} \tag{2.18}$$

$$\tau_{rz}^{(r)}=-\frac{ik_1}{8}\left\{\left[H_1^{(1)}\left(k_1\left|z-\overline{z}_0-2h_1i\right|\right)\frac{\left|z-\overline{z}_0-2h_1i\right|}{z-\overline{z}_0-2h_1i}\right]e^{i\theta}+\left[H_1^{(1)}\left(k_1\left|z-\overline{z}_0-2h_1i\right|\right)\frac{z-\overline{z}_0-2h_1i}{\left|z-\overline{z}_0-2h_1i\right|}\right]e^{-i\theta}\right\} \tag{2.19}$$

$$\tau_{\theta z}^{(r)}=\frac{k_1}{8}\left\{\left[H_1^{(1)}\left(k_1\left|z-\overline{z}_0-2h_1i\right|\right)\frac{\left|z-\overline{z}_0-2h_1i\right|}{z-\overline{z}_0-2h_1i}\right]e^{i\theta}-\left[H_1^{(1)}\left(k_1\left|z-\overline{z}_0-2h_1i\right|\right)\frac{z-\overline{z}_0-2h_1i}{\left|z-\overline{z}_0-2h_1i\right|}\right]e^{-i\theta}\right\} \tag{2.20}$$

The total wave field is

$$G^{(t)}=G^{(i)}+G^{(r)}+G_{\mathrm{I}}^{(s)} \tag{2.21}$$

In the complex plane $(z,\overline{z})$, the boundary condition can be expressed as:

$$\begin{cases} G^{(i)}+G_{\mathrm{I}}^{(s)}+G^{(r)}=G_{\mathrm{II}}^{(s)}, \quad r=R_1 \\ \tau_{rz}^{(i)}+\tau_{rz}^{(r)}+\tau_{rz,\mathrm{I}}^{(s)}=\tau_{rz,\mathrm{II}}^{(s)}, r=R_1 \\ \tau_{rz,\mathrm{II}}^{(s)}=0, r=R_2 \end{cases} \tag{2.22}$$

Substituting the expression of displacements into the boundary conditions (2.22), and multiplying both sides by $\mathrm{e}^{-im\theta}$ and integrating over the interval $(-\pi,\pi)$, a set of infinite algebraic equations to determine the coefficients $A_n, B_n, C_n$ can be obtained.

$$\sum_{n=-\infty}^{n=+\infty}\begin{bmatrix}\zeta_{mn}^{(11)} & \zeta_{mn}^{(12)} & \zeta_{mn}^{(13)} \\ \zeta_{mn}^{(21)} & \zeta_{mn}^{(22)} & \zeta_{mn}^{(23)} \\ \zeta_{mn}^{(31)} & \zeta_{mn}^{(32)} & \zeta_{mn}^{(33)}\end{bmatrix}\begin{bmatrix}A_n \\ B_n \\ C_n\end{bmatrix}=\begin{bmatrix}\zeta_m^{(1)} \\ \zeta_m^{(2)} \\ \zeta_m^{(3)}\end{bmatrix} \qquad \text{m=0,±1, ±2, …} \tag{2.23}$$

## 3 SCATTERING OF SH-WAVES BY THE CIRCULAR LINING STRUCTURE AND THE CRACK

As shown in Figure.1, the steady-state SH-wave $W^{(i)}$ would be reflected from the interface, and the reflected wave $W^{(r)}$ is also SH-wave. In the complex plane, $W^{(i)}$ and $W^{(r)}$ can be given by

$$W^{(i)}=W_0\exp\{\frac{ik_1}{2}[\left(z-ih_1\right)e^{-i\alpha}+\left(\overline{z}+ih_1\right)e^{i\alpha}]\} \tag{3.1}$$

$$W^{(r)}=W_0\exp\{\frac{ik_1}{2}[\left(z-ih_1\right)e^{i\alpha}+\left(\overline{z}+ih_1\right)e^{-i\alpha}]\} \tag{3.2}$$

where $W_0$ is amplitude of incident wave, $\alpha$ is incident angle.
The relevant stress:

$$\tau_{\theta z}^{(i)} = -i\tau_0 \sin(\theta-\alpha)\exp\{\frac{ik_1}{2}[(z-ih_1)e^{-i\alpha}+(\overline{z}+ih_1)e^{i\alpha}]\} \tag{3.3}$$

$$\tau_{\theta z}^{(r)} = -i\tau_0 \sin(\theta+\alpha)\exp\{\frac{ik_1}{2}[(z-ih_1)e^{i\alpha}+(\overline{z}+ih_1)e^{-i\alpha}]\} \tag{3.4}$$

$$\tau_{rz}^{(i)} = i\tau_0 \cos(\theta-\alpha)\exp\{\frac{ik_1}{2}[(z-ih_1)e^{-i\alpha}+(\overline{z}+ih_1)e^{i\alpha}]\} \tag{3.5}$$

$$\tau_{rz}^{(r)} = i\tau_0 \cos(\theta+\alpha)\exp\{\frac{ik_1}{2}[(z-ih_1)e^{i\alpha}+(\overline{z}+ih_1)e^{-i\alpha}]\} \tag{3.6}$$

The scattering waves $W_{\mathrm{I}}^{(s)}$ and $W_{\mathrm{II}}^{(s)}$ excited by the elastic circular lining structure in half space can be described by Eqn. (2.7)and (2.10), respectively. In which $A_n$, $B_n$ and $C_n$ are also unknown coefficients. And the process of solving $A_n$, $B_n$ and $C_n$ is the same as that of the Green's function discussed in this paper.
The displacement field of the elastic half space with a circular lining structure can be written as

$$W_{\mathrm{I}} = W^{(i)} + W^{(r)} + W^{(s)} \tag{3.7}$$

For an arbitrary point in the elastic half space, the total stresses from the incident wave $W^{(i)}$, the reflected wave $W^{(r)}$ and the scattered wave $W^{(s)}$ can be solved. If the additional stresses with the same magnitude and opposite direction to the total stresses are applied on the same point, the ultimate stresses of this point are zero. Therefore, when a pair of forces with the same magnitude and opposite direction is loaded along the region where the crack wants to be set, the resultant forces on the region are zero, which can be thought as a crack.
Then the total wave field in domain I is

$$W_{\mathrm{I}}^{(t)} = W^{(i)} + W^{(r)} + W^{(s)} - \int_{(b,-h_3)}^{(2a+b,-h_3)} \tau_{\theta z,\mathrm{I}} G^{(t)} dz' \tag{3.8}$$

in which $G^{(t)}$ is the expression of Green's function (2.21).

## 4. RESULTS AND DISCUSSION

In this part, some numerical examples are examined based on the above theoretical derivation. The specific results of scattering of SH-wave by the elastic circular lining structure and the crack in half space, the ground motion above the inclusion is provided. For various parameters, which include the shear modulus ratio of the media and the inclusion $\mu_2/\mu_1$, the wave number ratio of the inclusion and the media $k_1/k_2$, the wave number of incident wave $k_1R_2$, the incident angle $\alpha$, the length of crack 2a, the position angle of crack $\beta_0$, the ratio of the distance of the center of elastic circular lining structure to the horizontal interface to the radius of elastic circular lining structure $h_1/R_2$ and the ratio of the distance of the center of elastic circular lining structure inclusion to the crack tip to the radius of elastic circular inclusion $h_2/R_2$, the effects of them on the ground motion are discussed. Figures 2-9show the three-dimensional graph of the ground motion.
(1)Figures 2-4 show the variations of ground motion when the crack is just below the lining structure with 2a increasing from 0-10, and SH-wave incident with different angles. From each figure, it can be seen that the

maximum value of ground motion always appears on the position just above the center of the lining structure, and when 2a=10, the value of ground motion is always bigger than 2a with other values. The maximum value of ground motion appears when $\alpha = 0$ and the value is about 4.35.

(2)Figure 5 and figure 6 show the variation of ground motion with increasing $h_1 / R_2$ and $h_2 / R_2$ respectively by normally incident SH-waves,. It can be seen that the change in the value of ground motion is more obvious with $h_1 / R_2$ increasing than with $h_2 / R_2$ increasing. In Figure 5, it also can be seen that the value of ground motion decrease with increasing $h_1 / R_2$. In Figure 6, when $h_2 > 6a$, the value of ground motion tends to be stable.

(3) In Figure 7, when $k_1 / k_2 = 4$,the maximum value of ground motion is about 9.16. In Figure 8, $\mu_2/\mu_1 < 1$ is the condition that the inclusion is softer than the media, $\mu_2/\mu_1 > 1$ is the condition that the inclusion is harder than the media.It can be seen that the much harder the inclusion is, the more intense the ground motion is. Figure 9 shows the condition when the position angle of crack $\beta_0 = 45^0$ the variations of ground motion with increasing $k_1 R_2$. It can be seen that the value of ground motion decrease with increasing wave number $ka$, the graph is not symmetrical.

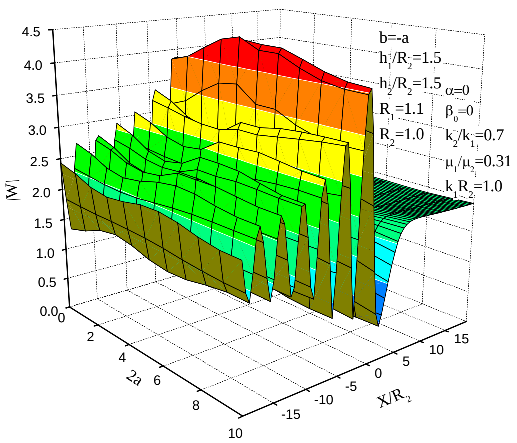


Figure 2 The variation of ground motion with increasing 2a when SH-wave incident with $\alpha = 0$

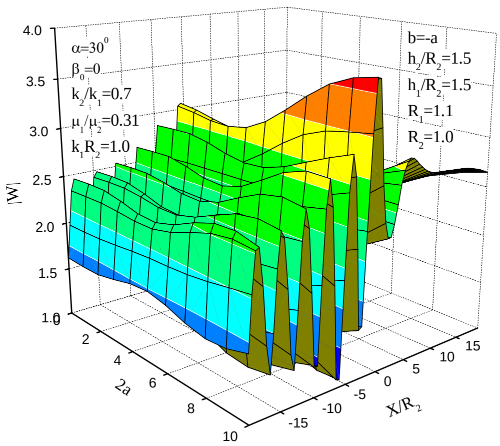


Figure 3 The variation of ground motion with increasing 2a when SH-wave incident with $\alpha = 30^0$

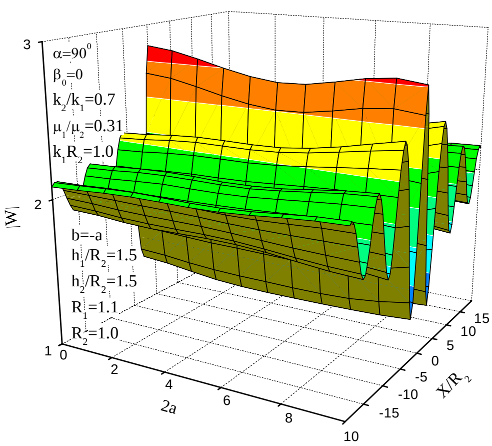


Figure 4 The variation of ground motion with increasing 2a when SH-wave incident with $\alpha = 90^0$

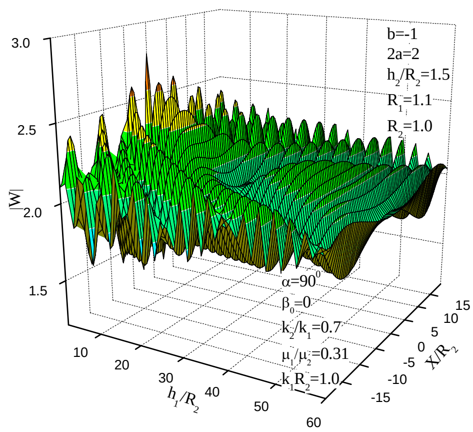


Figure 5 The variation of ground motion with increasing $h_1 / R_2$

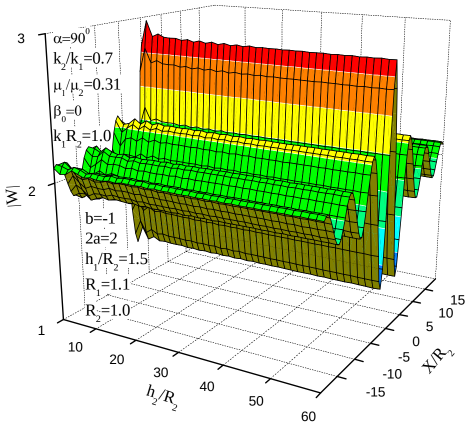


Figure 6 The variation of ground motion with increasing $h_2 / R_2$

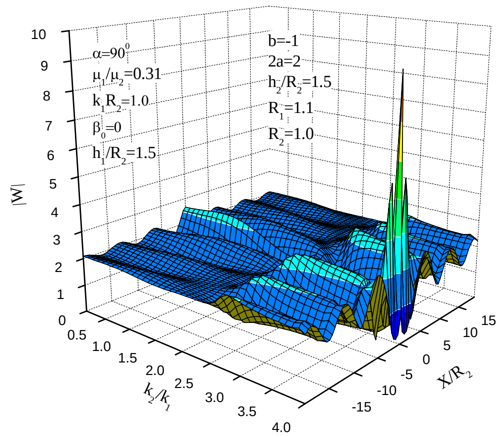


Figure 7 The variation of ground motion with increasing $k_1 / k_2$

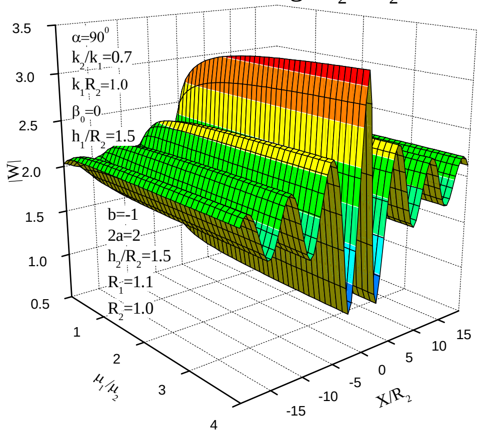


Figure 8 The variation of ground motion with increasing $\mu_2/\mu_1$

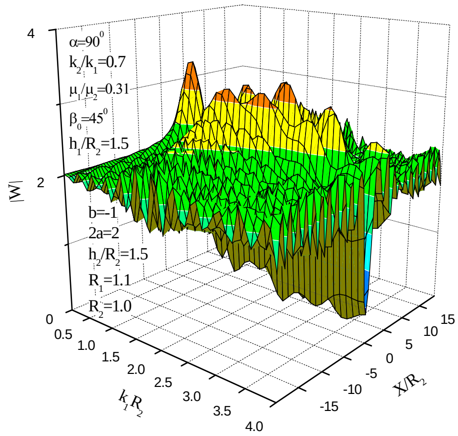


Figure 9 The variation of ground motion with increasing $k_1 R_2$ when $\beta_0 = 45^0$

## REFERENCES


Qi Hui, Wang Yan, Liu Diankui. (2003). Dynamic analysis of shallow-embedded lining structure by incident SH-wave. *Earthquake engineering and engineering vibration* **23:3**, 41-46.

Li Hongliang, Liu Diankui. (2004). Interaction of SH-wave by cracks with a circular inclusion. *Journal of Harbin Engineering University* **25:5**, 618-622.

Liu Diankui, Lin Hong. (2004). Scattering of SH-waves by an Interface Linear Crack and a Circular Cavity near Bimaterial Interface. *ACTA Mechanica Sinica* **20:3**. 317-326.

Yang Zailin, Liu Diankui, Lv Xiaotang. (2006). Dynamics of a mode III crack impacted by elastic wave in half space with a removable rigid cylindrical inclusion . *5th International Conference on Fracture and Damage Mechanics*, 679-682.

Pao, Y.H. and Mow, C.C. (1973). Diffraction and elastic waves and dynamic stress concentrations. *Crane and Russak, New York*, 114-304

Lee, V.W., Chen, S. and Hsu, I.R. (1999). Antiplane diffraction from canyon above subsurface unlined tunnel. *Journal of Engineering Mechanics* **125:10**, 1106-1114.