A MEASURE FOR ELIMINATING INSTABILITY OF THE MULTI-TRANSMITTING BOUNDARY 1)

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ABSTRACT : The multi-transmitting boundary is a local artificial boundary used for numerical simulation of the near-field wave motion in infinite media. It has the advantages of clear physical conception, wide application and high precision. But like other local artificial boundaries, the multi-transmitting boundary encounters the instability of numerical simulation. One type of the numerical instabilities, the drift instability, is studied in this paper and results in a simple measure for eliminating this type of instability based on the combination of the multi-transmitting boundary with the viscous-spring boundary. Validity of the suggested measure is tested by numerical experiment of the anti-plane motion in wedge-shaped medium.

KEYWORDS: multi-transmitting boundary, viscous-spring boundary, drift instability

1. INTRODUCTION

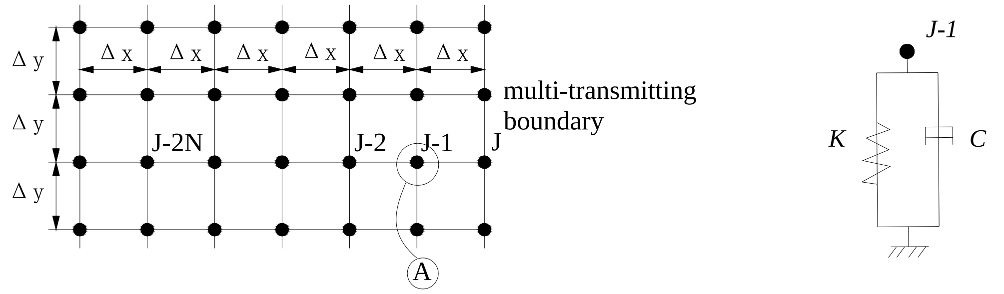
The multi-transmitting boundary is a local artificial boundary with controllable high precision. But there are problems on computational stability needed further research. The mechanism of the high-frequency oscillation was elucidated and the measure for inhibiting this type of instability was proposed by Liao ZP and Liu JB (1992a, 1992b). The other type of the instabilities, the drift instability, was studied by Li Xiaojun and Liao Zhenpeng (1996) for the first time, in which the mechanism of the drift instability was discussed and a reducing-order measure was suggested to eliminate the drift instability. The second scheme for stable implementation of the transmitting boundary is suggested by Zhou Zhenghua and Liao Zhengpeng (2001, 2004), that is to add modified operator γB_0^0 to the multi-transmitting formula. In this paper, the mechanism of the drift instability is studied based on the physical meaning of the calculation model. A scheme is proposed to eliminate the drift instability by combining the transmitting boundary and the viscous-spring artificial boundary. In the scheme, the springs or dampers are emplaced on the computational nodes next to the artificial boundary in the computational region.

2. MECHANISM AND SUPPRESSION METHOD OF THE DRIFT INSTABILITY

In numerical calculation using the multi-transmitting boundary, the motion of boundary point is associated with the motion of interior point by the character of wave propagation. But the relation between the boundary point and the exterior point is not established. In other words, the whole calculation region can't be restricted by the outside parts of boundary. The overestimated displacement on the boundary points will lead to overestimate the displacement of the whole calculation region, and the overestimated displacement can not be effectively restricted by the multi-transmitting boundary, which will cause drift of whole calculation region. This can explain the occurrence of the drift instability from the physical meaning.

Based on this explanation, we suggested that springs or dampers are emplaced on the 2N points inside the boundary as show in Figure 1(a). In this paper, N expresses transmission order. The details of points from the J-1 row to the J-2N row are same as show in Figure 1(b). The dynamic equation of these 2N interior points in numerical calculation changes to:

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=\{F(t)\}+\{F_k(t)\}+\{F_c(t)\} \qquad (1)$$



(a) Discrete model of multi-transmitting boundary (b) Detail schematic diagram of A

Figure 1 Model of multi-transmitting boundary and detail schematic diagram

Among them, $\{F_k(t)\}$ expresses the force from added spring and is the function of $u(t)$, and $\{F_c(t)\}$ expresses the force from added damper and is the function of $\dot{u}(t)$. Because of the extremely small values of spring coefficient and damper coefficient, the effect of the small force from spring or damper on calculation accuracy of the whole calculation region will be slight when the displacement of boundary is rather small, and the restriction effect from spring or damper will prevent the unsuited big displacement from sostenuto developing when this unsuited displacement happens on the boundary. In another word, the accuracy of the whole calculation region is still controlled by the multi-transmitting boundary and the advantage of the multi-transmitting boundary, high precision, isn't affected. The main role of spring or damper is to control the big displacement which possibly occurs continually on the boundary. Thus the drift instability can be inhibited. Then this conclusion is tested by numerical experiments.

3. NUMERICAL EXPERIMENT

The wedge site, such as Figure2, is calculated in this study. The input displacement pulse is shown in Figure3. The coordinate of observation point 1and 2 is respectively （953 m, -500 m） and（693 m, -350 m）. The incident angle of SH wave is 0° and the wave velocity is 2000 m/s. The size of discrete grid is as follows: $\Delta x = 8.66m$,

$\triangle y = 5m$. The displacement time histories of observation point 1 and 2 are shown respectively in Figure 4 and 5. MTB represents multi-transmitting boundary in the figure 4 and 5 and following figures. From these two figures, we can see that the drift instability can be inhibited effectively by spring or damper.

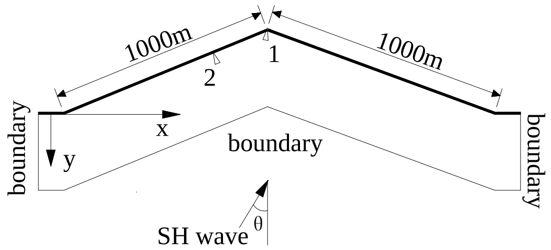


Figure 2 Calculation model of wedge site

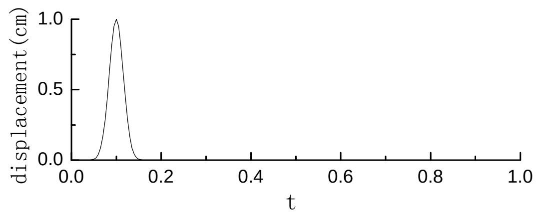


Figure 3 Displacement pulse

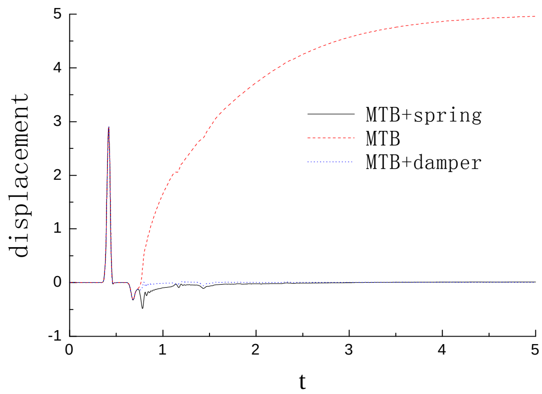


Figure 4 Displacement time history at observation point 1

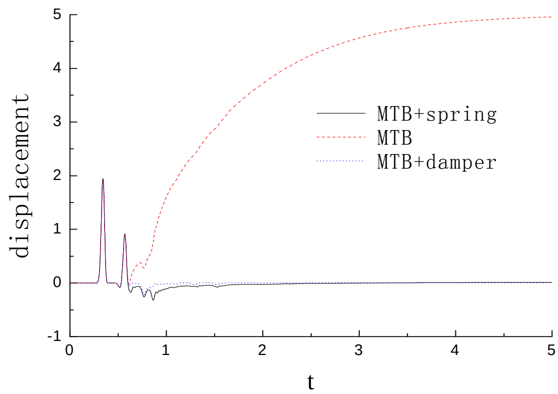
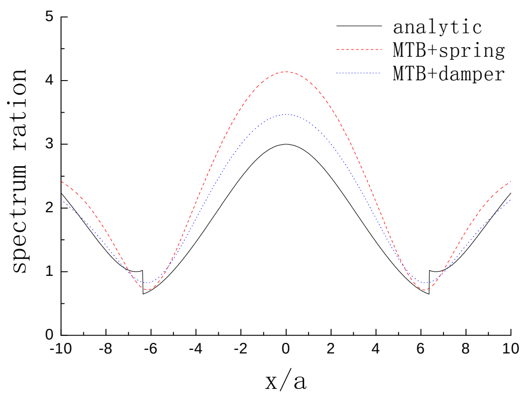
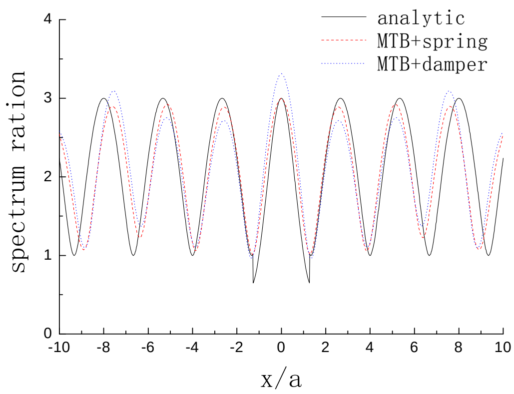


Figure 5 Displacement time history at observation point 2

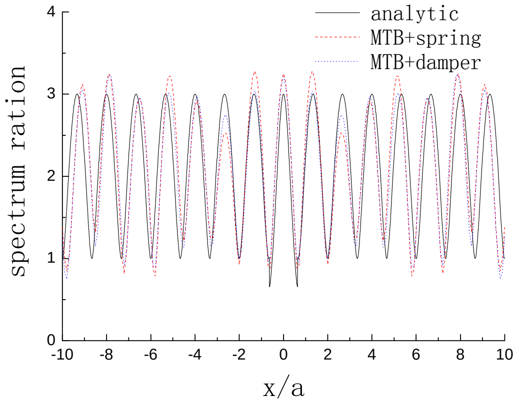
According to the existing research results (Sanchez-Sesma and F. J. 1985, V.W. Lee and R.I. Sherif 1996), approximate analytic solution of wedge site to incident SH waves is obtained. The comparison results of approximate analytic solution and Fourier spectrum rations which are calculated in the case of three kinds of frequency, f=1 Hz, f=5 Hz and f=10 Hz, are shown in Figure 6. The direction of abscissa in Figure 6(a, b, c) is chosen along the wedge site, such as Figure 6(d). Here, a=100 m. We can see that the results of adding spring or damper are close to approximate analytic solution. This implies that the accuracy of numerical calculation isn't affected. Furthermore, there are mutations on approximate analytic solution curve because of calculation accuracy, but no mutation on numerical solution curve. From this point, numerical solution is better.



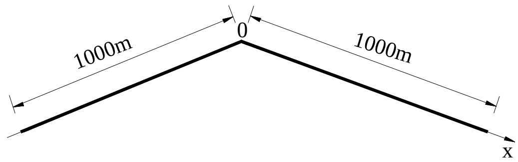
(a) f=1 Hz



(b) f=5 Hz



(c) f=10 Hz



(d) Schematic diagram of coordinate

Figure 6 Fourier spectrum ration

4. CONCLUSIONS

Based on the calculation model, we think the reason for drift instability is no restriction from exterior point to calculation region to lead to the calculated displacements of the whole calculation region sostenuto developing. The suggestion of adding spring or damper on 2N interior points beside boundary to constraint calculation region is put forward and tested to be effective by numerical examples. Some results on how to take value of spring coefficient or damper coefficient and which method of adding spring or damper is more effective will be shown in another paper by further study.

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1) Project 90715038 supported by NSFC