PROBABILISTIC SEISMIC HAZARD ANALYSIS TO CONTENTS AND NON STRUCTURAL ELEMENTS: CORRELATION BETWEEN PEAK GROUND ACCELERATION AND VELOCITY

Miguel A. Jaimes[1], Cesar Arredondo[2], Eduardo Reinoso[3] and Mario Ordaz[4]

[1] *Assistant Researcher, Institute of Engineering, UNAM. Mexico*
[2] *Assistant Research, Institute of Engineering, UNAM, Mexico*
[3] *Researcher, Institute of Engineering, UNAM, Mexico*
[4] *Researcher, Institute of Engineering, UNAM, Mexico*
Email: mjaimest@iingen.unam.mx, carredondov@iingen.unam.mx, ere@pumas.iingen.unam.mx, mors@pumas.iingen.unam.mx

ABSTRACT :

We propose an equation that correlates the seismic parameters of peak ground motion acceleration (A_{max}) and velocity (V_{max}) commonly used to estimate damage to pipelines, building contents and non structural elements. This equation allows us to estimate peak velocity V_{max} in terms of peak acceleration A_{max}. This means that, from an existing attenuation relation for A_{max} and a source model, it is possible to carry out a bivariated probabilistic seismic hazard analysis. An example of the probabilistic hazard assessment is presented for two sites at firm zone.

KEYWORDS: acceleration, velocity, seismic, hazard, contents

1. INTRODUCTION

The probabilistic seismic hazard assessments are frequently carried out by characterizing the seismic intensity with a single parameter, which empirically correlates better the intensity of the earthquake with the damage. However, it is very difficult to describe accurately a complex phenomenon with a single parameter, since a great deal of information is inevitably lost (Housner and Jennings, 1982). In buildings, for example, the spectral acceleration is mainly used due to the relation between the inertial forces imposed to the structure and the displacement; however, the damages in pipelines, contents and non structural elements are not only correlated with these parameters of intensity. The peak acceleration (A_{max}), velocity (V_{max}) and displacement (D_{max}) of the motion are the most accepted parameters in the characterization of the demand for contents (Ishiyama 1982 and 1984; Choi and Tung, 2002; Konstantinidis and Makris, 2003 and 2006; Abadi *et al.*, 2006) and non structural systems like pipelines (e.g. Katayama *et al.*, 1975; O'Rourke and Ayala, 1993), the relation of fragility are expressed in terms of V_{max}. However, the isolated use of these parameters can result in an inadequate description of the damage (Riddell, 2007) because, while A_{max} is an useful parameter for rigid systems, V_{max} is better for systems of intermediate period, and D_{max} for flexible systems. In the last years combined relations of A_{max} and V_{max} that reproduce more adequately the estimate of damages in contents and non structural elements have been proposed. Some of these are:

- The parameters V^2_{max} and A_{max} are correlated with the damage produced by the sliding displacement of an unanchored body subjected to earthquake excitation (Choi and Tung, 2002, Konstantinidis and Makris, 2003).
- The A_{max}/V_{max} ratio is adopted by Tso *et al.* (1992) as a characteristic measure of the frequency content of ground motion.
- The V^2_{max}/A_{max} index has been used as an indicator of the damage in pipelines given the relation of this parameter with the peak ground displacement (Pineda, 2007).
- The V^2_{max}/A_{max} and $(V_{max}/A_{max})^2$ parameters are correlated with overturning of rigid free-standing objects (Arredondo and Reinoso, 2008). The first parameter is used because the occurrence of overturning is directly related to the maximum displacement of the centre of gravity of the object as opposed to the

instantaneous acceleration experienced by the object or the floor supporting the object (Al Abadi *et al*., 2006). The second parameter is related to the vibration frequency of the motion; it has been observed that this is very important in the dynamic response of contents, because they could be more vulnerable before motion of low frequency (Psycharis *et al*., 2002; Arredondo and Reinoso, 2008).

In view of the information above, it is necessary to carry out a bivariated probabilistic seismic hazard analysis (associated to two seismic parameters) to know the joint probability distribution of A_{max} and V_{max}, therefore, it is necessary to have attenuation relations that correspond to both seismic parameters and their possible correlation. Unfortunately, it is not common to obtain attenuation relations where, for a given magnitude and distance, the values of A_{max} and V_{max} are obtained in a joint way, neither it is a common to obtain the correlation between them. In general, the seismic attenuation laws are built to estimate A_{max} mainly as a function of magnitude and distance. This implies the calculation of a set of coefficients through regressions methods (Ordaz *et al*., 1989; Ambraseys, 2006); in contrast, few attenuation laws of V_{max} have been built (e.g. Akkar and Bommer, 2007).

In this work an applicable expression for sites in Mexico that correlates the seismic parameters of peak ground acceleration (A_{max}) and velocity (V_{max}) is presented. This expression allows us to estimate the value of the parameter V_{max} in terms of A_{max}. Through this equation, from an existing attenuation relation for A_{max} and a source model, it is possible to carry out a bivariated probabilistic seismic hazard analysis to be used in the evaluation of losses in contents and non structural elements. Finally, an example of the obtaining of the bivariared seismic hazard is presented for two sites at firm zone.

2. SITES AND EVENTS

For the calculation of an expression correlating peak ground acceleration and velocity (implicitly the estimate of V_{max} in terms of A_{max}), ground motion records inside and outside the valley of Mexico were revised and selected. The considered records correspond to subduction and intermediate depth earthquakes. Only horizontal components were taken into account since vertical motion is, especially for Mexico City and distant stations, less important from the engineering point of view. The selected earthquakes have magnitudes between 5.0 and 8.1. The value of A_{max} was directly read from ground motion records. To obtain V_{max} from the records, the acceleration time histories are integrated after applying a base line correction and a filter between 0.1 and 10 Hz (Oppenheim and Schafer, 1975).

3. THE PARAMETER ω

If the ground motion were harmonic, then A_{max} and V_{max} would be perfectly correlated and

$$V_{max} = \frac{A_{max}}{\omega} \tag{3.1}$$

where ω is the natural circular frequency of the motion. Notice that an empiric deduction of ω would lead us to a simple estimate of the correlation between both parameters. Some previous works present ω values for different types of soil calculated as average values obtained from recorded motions. It is the case of studies like that of Newmark and Hall (1982) where they recommend for rock and soil conditions in California values of ω=10.8 and 8.0 rad/sec, respectively. Santa-Cruz *et al*. (2000) found average values for Acapulco city of 28 rad/sec, and values of 3.5 and 3.0 rad/sec for hard and soft soil of Mexico city, respectively. As it will be explained later, these average values can be calculated in a more precise way by considering the geotechnical conditions and seismic intensities that are present in the site.

3.1 Calculation of the parameter ω for a ground motion

For this work we examine several ways to estimate ϖ, among them: the frequency associated to the peak value of the Fourier amplitude spectrum, the frequency associated to the peak of the velocity and acceleration spectra, the expression of Fajfar *et al.* (1992), and the frequency corresponding to the intersection point between the spectral regions of acceleration and constant velocity in a velocity spectra. It was found that the following equation (Rathje *et al.*, 2004) provides the parameter that better represents the frequency content of the motion for the correlation between A_{max} and V_{max},

$$\varpi = \frac{\int |A(w)|^2 \, dw}{\int \frac{|A(w)|}{w} \, dw} \tag{3.2}$$

where $A(\varpi)$ is the Fourier amplitude spectra and ϖ is the frequency. Figure 1 shows the Fourier amplitude spectra (FAS) corresponding to the September 19, 1985 earthquake for four sites, as well as the value from the Eqn. (3.2). Notice that the value of ϖ differs from FAS peak value and it could even be in a valley of the FAS; nevertheless, as it will be presented later, ϖ is an appropriate parameter of correlation between A_{max} and V_{max}.

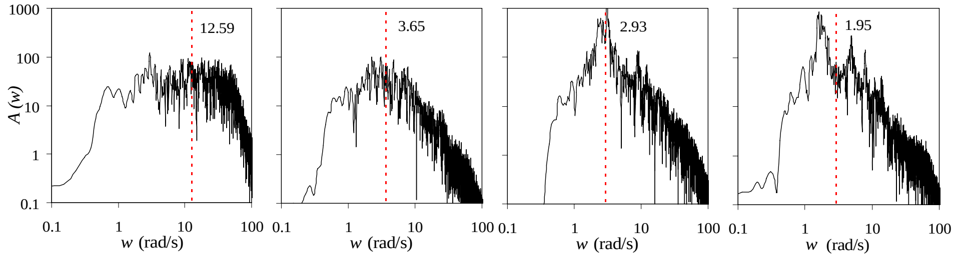


Figure 1 Fourier amplitude spectra of the September 19, 1985 earthquake for sites in Mexico. The value of the parameter of the equation (2) is indicated.

4. ESTIMATE OF AN EXPRESSION OF ϖ FOR THE VALLEY OF MEXICO

The peak intensities (A_{max} and V_{max}) and the parameter ϖ were obtained for sites in the valley of Mexico. Sets of 452, 504, and 2802 ground motions were employed for hard, transition, and lake soil sites, respectively. From those, 2332 correspond to subduction earthquakes and 1426 correspond to intermediate depth earthquakes. The distinction of ground motions according to the origin of the earthquake is because intermediate depth earthquakes are motions less harmonic than that of subduction earthquakes and differences were observed in the results.

4.1 ϖ in terms of the intensity

Important differences in the frequency content for ground motions with approximately the same epicentral distances but with different seismic magnitude have been observed (Reinoso and Ordaz, 1999); therefore, it is possible to expect a clear dependence of the parameter ϖ with the magnitude and consequently with the seismic intensity. Figure 2 shows the relation between A_{max} and ϖ obtained for three sites (CU, SCT and CD) subjected to subduction (top) and intermediate depth (bottom) earthquakes. Their respective fittings by means of a continuous line are also shown. It is possible to observe how ϖ decreases inversely to the intensity (A_{max}) in all sites and for both types of earthquakes due to large events (implicitly larger A_{max}) which generate motions of low frequency (Reinoso and Ordaz, 1999).

4.2 ϖ in terms of the site period

Due to the nature of the geotechnical composition of the valley of Mexico, it is reasonable to expect that the parameter ω is characterized almost exclusively by site effects, making it independent of the frequency content of the seismic source and attenuation. In sites in Mexico city considered as firm soil, it is still possible to expect ground motion of relatively narrow band, since an important difference exists between the rigidities of the deep basements and the rock in surface. This causes anomalous amplifications in frequencies around 0.5 Hz (Ordaz and Singh, 1992). Figure 3 shows the parameter ω for the three sites previously mentioned, subjected to subduction (squares) and intermediate depth (triangles) earthquakes and with their fittings shown as a discontinuous and a continuous thick lines, respectively. The values corresponding to the soil dominant frequency (ω_s=2 π/T_s) are also shown with a gray thin line. It has been observed (Reinoso and Ordaz, 1999) that the value of T_s=0.5 sec is the limit between the transition zone of the valley of Mexico and the soil classifies as firm. Conventionally, it is accepted to assign a value of T_s=0.5 sec to hill zone sites. In other words, every site with a clear dominant period larger than 0.5 sec is considered lakebed. It is observed that differences exist with respect to the origin of the earthquake (subduction or intermediate depth) in the behavior pattern of the parameter.

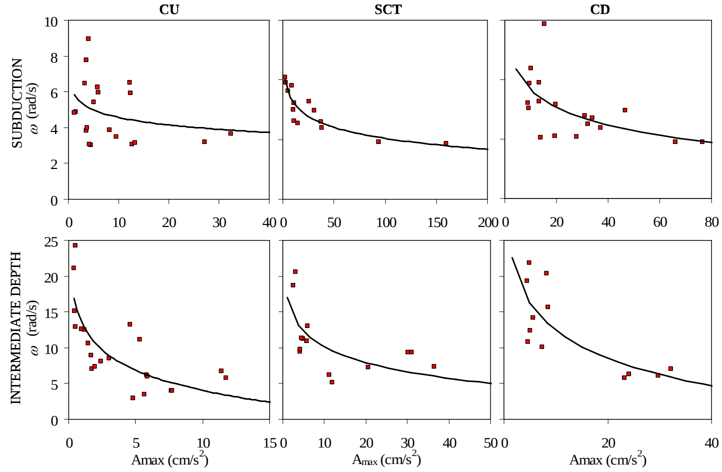


Figure 2 ω in terms of A_{max} for sites CU, SCT and CD during subduction and intermediate depth earthquakes

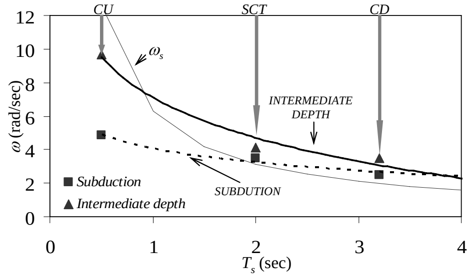


Figure 3 ω for subduction (squares) and intermediate depth (triangles) earthquakes and their fittings by means of a discontinuous and a continuous line, respectively; with gray line the relation obtained for the soil dominant frequency ω_s=2π /T_s is also shown

Figure 4 shows the relation of ω respect to the soil dominant period T_s and to the peak ground acceleration A_{max}, for subduction and intermediate depth earthquakes (right and left, respectively) for sites from the valley of Mexico. The surface that better fits the relations is shown as well. It was found that the parameter of interest for subduction earthquakes can be calculated as

$$\ln\omega = 1.8349 - 0.4043\ln T_s - .1146\ln A_{\max} \tag{4.1}$$

In the previous equation, T_s must equal 0.5 sec for sites with T_s < 0.5 sec; i.e. rock and firm sites. For intermediate depth earthquakes, it was determined as

$$\ln \omega = 1.9628 - 0.5508 \ln T_s - .0834 \ln A_{max} \tag{4.2}$$

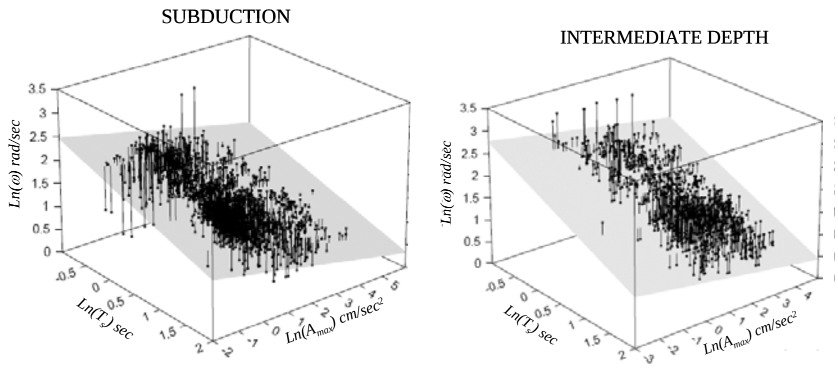


Figure 4 Parameter ω respect to the soil dominant period (T_s) and A_{max} for subduction (left) and intermediate depth (right) earthquakes in sites from the valley of Mexico

Using expressions like those above allow us to estimate V_{max} for sites in the valley of Mexico in terms of A_{max} and ω. By obtaining logarithm in both sides of the Eqn. (3.1), aimed to substitute the term of Eqns. (4.1) or (4.2), we have that

$$\ln V_{max} = \ln A_{max} - \ln \omega \tag{4.3}$$

Substituting the Eqn. (4.1) in Eqn. (4.3) the following is obtained

$$\ln V_{max} = 1.8349 + 0.8854 \ln A_{max} - 0.4043 \ln T_s \tag{4.4}$$

where V_{max} is function of A_{max} for subduction earthquakes.

Substituting the Eqn. (4.2) in Eqn. (4.3) the equivalent one is obtained for intermediate depth earthquakes

$$\ln V_{max} = 1.9628 + 0.9166 \ln A_{max} - 0.5508 \ln T_s \tag{4.5}$$

In the previous equations, T_s must equal 0.5 sec for sites with $T_s < 0.5$ sec (i.e., rock and firm sites).

By employing Eqns. (4.4) or (4.5) together with an existing attenuation relation of A_{max} and a source model, it is possible to carry out a probabilistic seismic hazard analysis. Additionally to Eqns. (4.4) or (4.5) it is still necessary to have statistical parameters as the standard deviation of the natural logarithm defined as σ and the correlation coefficient defined as ρ. These two parameters were calculated comparing the observed and estimated velocity (Eqns. 4.4 and 4.5). Values of σ=0.30 and 0.34 and correlation coefficients of ρ=0.965 and 0.957 were obtained for subduction and intermediate depth earthquakes, respectively. Due to the high correlation (values near to one), the error in the estimation of the parameter V_{max} given A_{max} in the valley of Mexico is considered acceptable. Figure 5 shows the relation between estimated and observed V_{max} (Eqns. 4.4 or 4.5, for subduction and intermediate depth earthquakes, respectively) in sites of the valley of Mexico (square points). It is possible to appreciate a good estimation of V_{max} in terms of A_{max} due to the excellent correlation values.

5. ESTIMATE OF ω FOR SITES OUTSIDE THE VALLE DE MÉXICO

For sites outside the valley of Mexico 54 stations were analyzed in hill zone using 1612 ground motions recorded in directions north-south and east-west, where 1186 and 426 records correspond to subduction and intermediate depth earthquakes, respectively. Proceeding in a similar way as was shown for sites in the valley of Mexico, peak intensities (A_{max} and V_{max}) and the parameter ω were obtained. We were able to establish a correlation of the parameter ω with the intensity of the earthquake, as was previously done for sites in the valley of Mexico; however, we found a great variability of ω with A_{max}. Due to this, an average value of the parameter for sites located outside the valley of Mexico was obtained, like in previous studies (Newmark and Hall, 1982; Santa-Cruz *et al.*, 2000), but considering the different origin of the earthquake. The calculated values of the logarithm of ω were 3.27 and 3.07 for subduction and intermediate depth earthquakes, respectively. Substituting these values in Eqn. (4.3), the expression of V_{max} in term of A_{max} for subduction events is

$$\ln V_{\max} = \ln A_{\max} - 3.27 \tag{5.1}$$

and for intermediate depth events is

$$\ln V_{\max} = \ln A_{\max} - 3.07 \tag{5.2}$$

Finally, we also obtained the error comparing the observed and estimated velocity. The average errors for all events were 0.63 and 0.88, and the correlation coefficients were 0.891 and 0.829 for each case, being the first value of each statistical parameter for subduction earthquakes and the second one for intermediate depth events. Note that the error in sites outside the valley of Mexico is larger in comparison with the obtained for sites in the valley of Mexico.

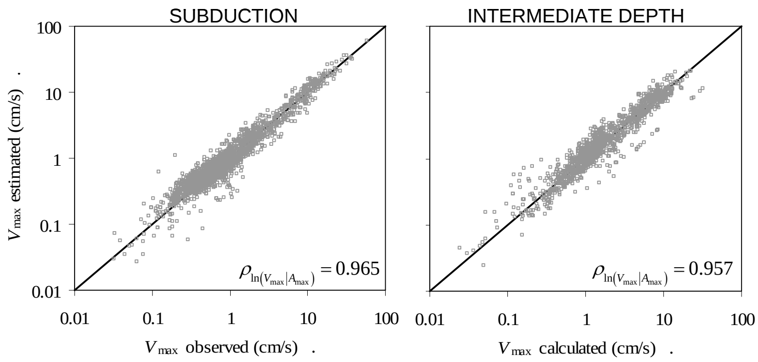


Figure 5 Comparison of observed and estimated V_{max} by means of the equations (4.4) and (4.5) (subduction earthquakes and intermediate depth earthquakes, respectively) for sites inside the valley of Mexico

6. EXAMPLE OF APPLICATION: PROBABILISTIC SEISMIC HAZARD ASSESMENT APPLIED TO CONTENTS AND NON STRUCTURAL ELEMENTS

Esteva (1967) presents the well-established technique to obtain the exceedance rate of peak ground acceleration, defined as

$$\nu(a) = \sum_{i=1}^{N} \int_{M_O}^{M_U} \lambda_o p(M) \cdot \Pr(A > a | M, R) dM \tag{6.1}$$

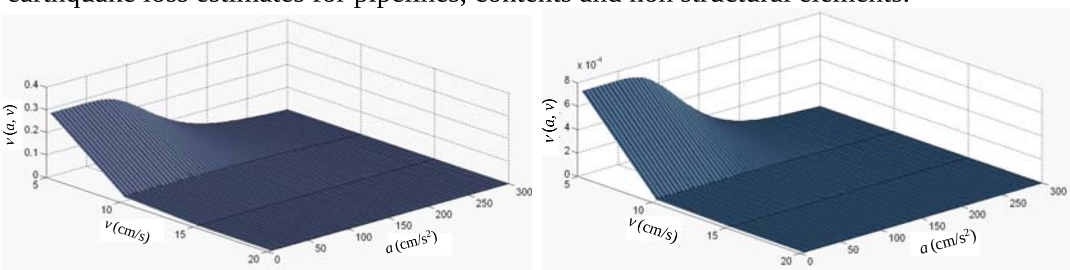
In Eqn. (6.1), N is the total number of seismic sources, M_O and M_U are the minimum and maximum magnitude that can be generated in the seismic source; λ_o is the magnitude exceedance rate for $M = M_O$, $p(M)$ is the probability density function of magnitude, $\Pr(A > a| M, R)$ is the probability that the peak ground acceleration A exceeds the value of a in the site given that at distance R an earthquake of magnitude M took place. However, in the case of the seismic estimation associated to two correlated parameters, the exceedance rate can be calculated according to that proposed by Santa Cruz *et al.* (2000) as

$$\nu(a,v) = \nu(a) + \nu(v) - \sum_{i=1}^{N} \int_{M_O}^{M_U} \lambda_o p(M) \cdot \Pr(A > a \cup V > v | M, R) dM \tag{6.2}$$

where $\nu(a)$ and $\nu(v)$ are the exceedance rates of acceleration and velocity, respectively.

To evaluate the terms $\nu(a)$ and $\nu(v)$ of the Eqn. (6.2), it is required to have not only a source model of the seismic activity and an attenuation law of V_{max}, but also to know the joint probability distribution of A and V. The obtaining of V_{max} in term of A_{max} and the correlation between the random variables A and V is as discussed previously; this will be employed later to calculate the bivariated probabilistic seismic hazard analysis.

We present an example to calculate the bivariated probabilistic seismic hazard for two sites outside the valley of Mexico, which are affected by a point seismic source. It is considered that sites are located at 30 and 100 km, respectively. The considered seismicity pattern to estimate the seismic activity in the source is given by Cornell and Varmacke (1969). In the following, we present an example to obtain the bivariated seismic hazard curve. The parameters of seismicity are: λ_o=1/year, β=2, M_O=2 and M_U=8 (Ordaz, 2004). Likewise the form of the attenuation relation and its respective coefficients that relate the magnitude and distance from the source to site with A_{max} are taken from Ordaz (2004). The standard deviation of the natural logarithm of A_{max} was considered constant equal to σ=0.7. Figure 6 shows the bivariated seismic hazard curves for two sites located in hill zone at 30 km (Figure 6a) and 100 km (Figure 6b) of the seismic source, respectively; each hazard curve is obtained applying Eqn. (6.2). It was considered that subduction earthquakes are produced in the source; therefore, Eqn. (5.1) with its corresponding statistical values were used. From the figure, it can be observed that when the seismic source is far away (R=100 km) from the site of interest, the exceedance rate is smaller and therefore the probability of large number of contents overturning decreases. This type of seismic hazard curve could be used later in the earthquake loss estimates for pipelines, contents and non structural elements.



a) b)

Figure 6 Exceedance rates of intensity a and v, computed with equation (6.2). These rates correspond to certain point source and a probabilistic attenuation relation, and a value of σ=0.7. a) R=30 km and b) 100 km

7. CONCLUSION

An expression that correlates the seismic parameters of peak ground acceleration (A_{max}) and velocity (V_{max}) commonly used to estimate damage to pipelines, building contents and non structural elements is presented. This equation allows us to estimate peak velocity V_{max} in terms of peak acceleration A_{max}. This means that, from an existing attenuation relation for A_{max} and a model source, it is possible to carry out a bivariated probabilistic seismic hazard analysis. Parameters were obtained by comparing the observed and estimated velocity for sites in the valley of Mexico, resulting in standard deviations of the natural logarithm equal to σ=0.30 and 0.34 and correlation coefficients equal to ρ=0.965 and 0.957 for subduction and intermediate depth earthquakes, respectively. On the other hand, for firm sites outside the valley of Mexico, larger values of standard deviation were obtained in relation to sites in the valley of Mexico, with correlation coefficients around 0.8. Finally, the seismic hazard curve expressed in terms of peak ground acceleration and velocity for two sites was presented.

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