INVESTIGATION OF THE EFFECT OF THE GROUND MOTION CHARACTERISTICS ON THE R_y-μ RELATION FOR THE INELASTIC RESPONSE OF SDOF STRUCTURES

I. Taflampas[1] and I.N. Psycharis[2]

[1] *Civil Engineer, Laboratory for Earthquake Engineering , National Technical University of Athens, Greece*
[2] *Assoc. Professor, School of Civil Engineering, National Technical University of Athens, Greece*
Email: taflan@central.ntua.gr, ipsych@central.ntua.gr

ABSTRACT :

In this study, the effect of the ground motion frequency content, especially the period of the directivity pulse in near field records, on the relationship between the reduction factor R_y and the ductility demand μ is investigated. A large sample of strong motion records showing near field characteristics of forward and backward directivity, is examined. The regimes, in which the known assumptions of equal displacement and equal energy are valid, are re-evaluated taking into account the period of the directivity pulse contained in the ground motion. The results show that, for near field records with directivity effects, the thresholds of these regions depend on the period of the directivity pulse, which is related to the magnitude of the earthquake. This observation is valid for ground motions with intense and weak forward directivity and backward directivity effects. This response implies that the equal displacement rule (R_y=μ), which is incorporated in most earthquake design regulations for structures with a period larger than T_c, with T_c being the threshold between the constant acceleration and the constant velocity regions on the response spectrum, might underestimate significantly the ductility demand for structures with long periods. In this analysis, a new method for deriving the pulse duration in case of weak forward and for backward directivities is proposed and a new limit for the regime, in which the equal displacement assumption applies, is defined.

KEYWORDS: Near field, ductility, inelastic response, reduction factor, behaviour factor.

1. INTRODUCTION

According to the Force Based Seismic Design (FBSD), which is adopted in most seismic codes, the seismic load that is applied to a Single-Degree-Of-Freedom (SDOF) structure is derived by dividing the elastic spectral acceleration by a reduction factor, *R*. In Eurocode 8 (EC8), this factor is referred to as behaviour factor and is denoted by *q*. The code reduction factor *R* takes into account the energy dissipation due to the inelastic response and the so-called overstrength, denoted by R_s. It can be defined as $R=R_y \cdot R_s$, in which the term R_y (yield or ductility reduction factor) is related to the energy dissipation and is defined as the ratio of the inertia force that would be developed to the equivalent elastic structure over the yield strength of the inelastic system.

For an effective seismic design, the ductility demand for the design earthquake should be within the limits of the collapse prevention performance criterion. The ductility demand, μ, is defined as the ratio of the maximum displacement over the yield displacement. Since the seismic loads are determined by dividing the elastic ones with the reduction factor, *R*, an adequate relationship between R_y and μ is necessary**.** This issue has been addressed for many years by the research community and various approximate R_y-μ-T_n relationships between the reduction factor, R_y, the ductility, μ and the period T_n of the structure have been proposed.

Most seismic codes adopt the Newmark and Hall (1982) approach, which is based on the R_y-μ-T_n relationships proposed by Veletsos and Newmark (1982), given by the following expressions:

$$R_y = \begin{cases} 1 & \text{for} \quad T_n < T_a \\ \sqrt{2\mu - 1} & \text{for} \quad T_b < T_n < T_{c'} \\ \mu & \text{for} \quad T_n > T_c \end{cases} \tag{1.1}$$

where T_a, T_b and T_c are characteristic periods separating spectral regions and $T_{c'}$ is defined by the relation:

$$T_{c'} = \frac{\sqrt{2\mu - 1}}{\mu} \cdot T_c \tag{1.2}$$

In the regions between T_a and T_b and $T_{c'}$ and T_c, linear interpretation is considered. The periods T_b and T_c define the constant acceleration region in the elastic response spectrum, while the periods T_b and $T_{c'}$ define the corresponding region in the inelastic spectrum. Typical values for the characteristic periods are: T_a=0.03 sec, T_b=0.125 sec and T_c=0.50 sec (Veletsos and Newmark, 1982).

In the constant acceleration region, it is usually assumed that the same energy is absorbed in the elastic and the inelastic systems. This assumption results in a larger maximum displacement for the inelastic system than the elastic one. In the contrary, in the region $T_n > T_c$, the assumption of equal displacements between the elastic and the inelastic systems is made, which results in $R_y = \mu$.

More recent studies (Krawinkler and Nassar, 1992, Vidic et al, 1994), Miranda and Bertero, 1994) present alternative R_y-μ-T_n relationships, based on statistical results for large samples of earthquake records. For example, Krawinkler and Nassar (1992) propose the relationship:

$$R_y = \left[c(\mu - 1) + 1\right]^{1/c} \tag{1.3}$$

where c is a function of the period of the structure, T_n, given by the following expression:

$$c(T_n) = \frac{T_n^a}{1 + T_n^a} + \frac{b}{T_n} \tag{1.4}$$

For elastic-perfectly plastic systems, the coefficients a and b obtain the values: a=1 and b=0.42. It is interesting to note that this relation contains only the period of the elastic system and is independent of the frequency content of the ground motion.

Conversely, Chopra and Chintanapakdee (2001) emphasize the different characteristics between near and far field earthquake records. For this reason, they consider different values for T_a, T_b and T_c in each case. Based on statistical results for a selected sample of ground motions, they propose the values: T_a=0.025 sec, T_b=0.22 sec and T_c=0.42 sec for far field earthquakes and T_a=0.04 sec, T_b=0.35 sec and T_c=0.79 sec for near field records. It should be noted that, for the records considered in their study, the orientation of the near field ones was normal to the fault trace with prominent forward directivity phenomena. In such cases, response spectra show a wide region of constant spectral acceleration and a narrow region of constant spectral velocity. The period T_c defines the threshold between these regions and, according to the study, obtains values less than 1.0 sec, in general.

In a recent study on near field records, Mavroeidis et al (2004) associate the R_y-μ-T_n relationships with the duration of the directivity pulse, T_p, i.e. the pulse period. This period is related to the earthquake magnitude and the source mechanism. The authors propose the following modification of the relationships of Veletsos and Newmark (Eqn. 1.1) in order to take under consideration the pulse duration, T_p:

$$R_y = \begin{cases} 1 & \text{for} \quad T_n/T_p < \left(T_n/T_p\right)_a \\ \sqrt{2\mu - 1} & \text{for} \quad \left(T_n/T_p\right)_b < T_n/T_p < \left(T_n/T_p\right)_{c'} \\ \mu & \text{for} \quad T_n/T_p > \left(T_n/T_p\right)_c \end{cases} \tag{1.5}$$

where, similarly to Eqn. 1.2,

$$\left(T_n/T_p\right)_{c'} = \frac{\sqrt{2\mu - 1}}{\mu} \cdot \left(T_n/T_p\right)_c \tag{1.6}$$

The values of the normalized periods $(T_n/T_p)_a$, $(T_n/T_p)_b$, $(T_n/T_p)_c$ are related to the earthquake magnitude. The parameter $(T_n/T_p)_b$ is set equal to 0.35 for events of medium magnitude and equal to 0.20 for medium to large earthquakes, while $(T_n/T_p)_c$=0.75.

For the application of the above relationships, the value of the pulse period T_p is needed. Empirical expressions, relating the value of T_p to the moment magnitude, M_w, of the earthquake, are given by several researchers. Such a relation was proposed by Rodriguez-Marec (2000), based on a large sample of earthquakes. The mean value is given by the expression:

$$\ln T_p = -8.33 + 1.33 \cdot M_w \tag{1.7}$$

and the standard deviation is 0.54.

In the present paper, a sample of earthquake records is analyzed, in order to evaluate the above-mentioned R_y-μ-T_n relationships for near field records containing directivity effects. A new threshold for the equal displacement regime is proposed, associated with the period of the pulse, T_p and the near field characteristics of the ground motion.

2. GROUND MOTIONS CONSIDERED IN THE ANALYSES

The two horizontal components of 27 earthquake recordings (54 records in total) were used in this study. They were selected from the COSMOS and PEER databases with moment magnitudes ranging from 6.2 to 7.3, with the exception of the San Salvador, 1986 earthquake that was of magnitude M_w=5.6. Most of the records were recorded on sites characterized as soil and alluvia, three recordings were obtained on soft rock and another three on hard rock deposits. The distance from the fault trace varies between 1.2 and 13.7 km. The data sample includes well-known earthquakes such as the Northridge, USA, 1994, the Kobe, Japan, 1995 and the Chi-Chi, Taiwan, 1999 events.

Most records show forward directivity effects with the exception of the earthquakes: Imperial Valley (El Centro station), USA, 1940, Morgan Hill (Halls Valley station), USA, 1984 and Landers (Joshua Tree station), USA, 1992 that are associated with backward directivity phenomena (Mavroeidis and Papageorgiou, 2003). In most recordings, the normal to the fault components show strong directivity effects (e.g. component TAB-074 of the Tabas, Iran, 1978 earthquake, records E02-230 and E04-230 of the El Centro Array, Imperial Valley, USA, 1979 earthquake), while the parallel to the fault components are associated with weak directivity effects. In some cases, prominent directivity effects are evident in both components, as in Erzincan, Turkey, 1992 earthquake, in which the orientation, with respect to the fault trace, of both instruments was close to 45 degrees.

For each record, the velocity and the displacement response spectra for 5% damping were derived and the predominant period associated with each spectrum was determined from the corresponding peak spectral value, as shown in Figures 1a-b, 2a-d and 3a-d. These are characteristic periods for near field earthquakes and are denoted by T_{SV} and T_{SD} respectively. Also, inelastic analyses were performed for SDOF structures with varying period, T_n, and for several values of R_y. However, in the following only the case of R_y=3 since this is a common value for RC structures. From these analyses, the ductility demand was calculated and the corresponding ductility spectrum for each record was drawn (Figures 1c, 3e-f).

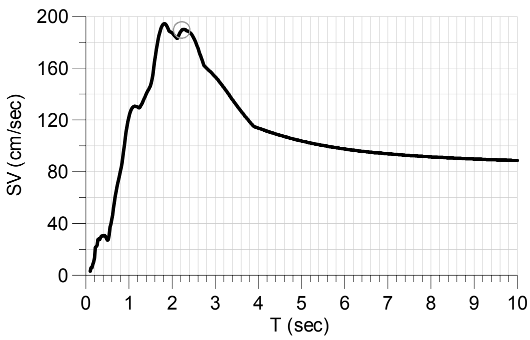
3. PRESENTATION OF THE RESULTS

3.1. Records with prominent forward directivity

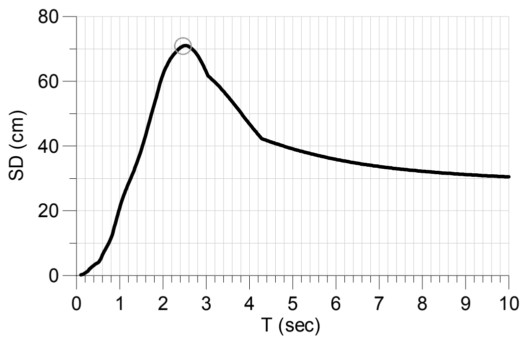
According to Mavroeidis et al (2004), the response spectra of near field records with prominent forward directivity show a wide region of constant spectral acceleration and a narrow region of constant spectral velocity. As a result of the latter, the velocity and the displacement response spectra have similar predominant periods. Also, the predominant period of the velocity spectrum, T_{SV}, can be used to determine the period T_p of the directivity pulse (Alavi and Krawinkler, 2000).

An example of a record with prominent forward directivity is the component ERZ-000 of the Erzincan, Turkey, 1992, earthquake (M_w=6.6). In Figures 1a and 1b, the velocity and the displacement spectra for 5% damping are presented, while in Figure 1c, the ductility demand spectrum for R_y=3 is shown. The predominant period of the velocity spectrum is T_{SV}=2.3 sec (Figure 1a) and the predominant period of the displacement spectrum is T_{SD}=2.5 sec (Figure 1b). For M_w=6.6, Eqn 1.7 gives T_p=1.57 sec and if the standard deviation is added, T_p=2.3 sec, which is equal to T_{SV}.

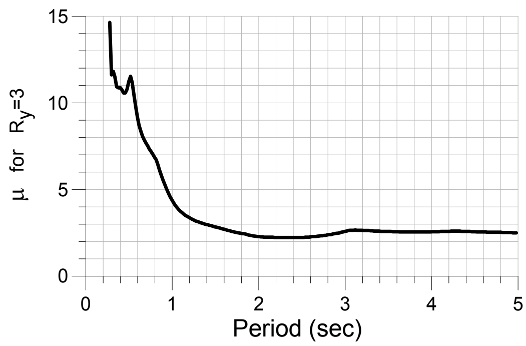
From Figure 1c it can be observed that for structures with period larger than approximately 1.3 sec the equal displacement rule is valid and $R_y \approx \mu$. For structures with a smaller period, the ductility demand increases exponentially as the period decreases. According to Mavroeidis et al (2004), $R_y=\mu$ for $T_n/T_p>(T_n/T_p)_c$ (see third of Eqns. 1.5). For the ERZ-000 record, T_p=2.3 sec and for $(T_n/T_p)_c$=0.75, the third of Eqns. 1.5 gives $R_y=\mu$ for T_n>1.7



(a)



(b)



(c)

Figure 1. ERZ-000 component of Erzincan, Turkey, 1992 earthquake: (a) velocity spectrum for ζ=5%; (b) displacement spectrum for ζ=5%; (c) ductility demand spectrum for R_y=3.

sec, which is consistent with Figure 1c. For smaller periods, Mavroeidis et al (2004) propose the relationship $R_y = \sqrt{2\mu - 1}$ for $(T_n/T_p)_b < T_n/T_p < (T_n/T_p)_{c'}$ (see second of Eqns. 1.5). In this region, the relation between R_y and μ can be written as $\mu = (1 + R_y^2)/2$ and thus, Eqn. 1.6 gives:

$$\left(T_n/T_p\right)_{c'} = \frac{2R_y}{1+R_y^2} \cdot \left(T_n/T_p\right)_c \tag{3.1}$$

For $(T_n/T_p)_c$=0.75 and R_y=3, Eqn. (3.1) results in $(T_n/T_p)_{c'}$=0.45. Taking $(T_n/T_p)_b$=0.20 (medium to large earthquake) and T_p=2.3 sec, the above relation gives μ=5 for 0.5 sec < T_n < 1.0 sec. In Figure 1c, this value of ductility is confirmed only for structures with a period around T_n=1.0 sec. For smaller periods, the ductility demand increases exponentially and attains values significantly larger than μ=5.

In general, for the analyzed records with prominent forward directivity the results show a varying ductility demand in the region $(T_n/T_p)_b < T_n/T_p < (T_n/T_p)_{c'}$. For many ground motions, the ductility demand was smaller than that predicted by the second of Eqns. 1.5, but there were cases, as the ERZ-000 record, for which the ductility demand was larger than the predicted. In most cases, however, Eqns. 1.5 were in the safety side.

3.2.Records with weak forward directivity

The forward directivity effect is also present in ground motion components with orientation different than the vertical to the fault trace. According to Baker (2007), in these records the directivity pulse is less prominent in comparison to the vertical to the fault component. The velocity and the displacement response spectra have diverging predominant periods. The predominant period of the velocity spectrum, T_{SV}, is affected by the high frequency content of the record and, thus, is smaller than the period of the directivity pulse, which is associated with lower frequencies and coincides with a local peak in the long period region of the velocity spectrum. As a consequence, T_{SV} cannot be used to define T_p, as in the case of records with prominent forward directivity.

The different characteristics of records with prominent and weak forward directivities are shown in Figure 2, in which the velocity and the displacement spectra of the two horizontal components (TAB-074 and TAB-344) of the Tabas, Iran, 1978 (M_w=7.1) earthquake are shown. The component TAB-074 was recorded in a direction almost normal to the fault trace and shows all the characteristics of prominent forward directivity: T_{SV} and T_{SD} are similar (see Figures 2a and 2c), with values equal to 4.7 sec and 5.3 sec, respectively. Considering that $T_p \approx T_{SV}$ (Alavi and Krawinkler, 2000), one can set $T_p \approx 5.0$ sec, which is close to the value of 5.2 sec that is obtained using Eqn. 1.7 plus a standard deviation. Conversely, the component TAB-344 shows weak directivity phenomena: the predominant period of the velocity spectrum is close to 1.0 sec, quite smaller than the period of the pulse, T_p.

It is important to note that, for both components, the displacement spectra have similar predominant periods equal to $T_{SD} \approx 5.0$ sec (see Figures 2c and 2d), a value close to T_p. Also note that the velocity spectrum of the weak directivity component TAB-344 shows a local peak at T_n=5.7 sec, i.e. close to the predominant period of the displacement spectrum.

As mentioned above, Eqns. 1.5 were derived using records with prominent forward directivity. Here, we extend these relations to records with weak forward directivity components. In such cases, however, the period T_p should be set equal to T_{SD}, the predominant period of the displacement response spectrum, and not equal to T_{SV}. Thus, the equal displacement rule (R_y=μ) can be applied for T_n>$0.75 \cdot T_p$. For smaller periods, the ductility demand, μ, increases and becomes larger than R_y, although it reduces again, in general, in the region close to the predominant period of the velocity spectrum, T_{SV}. Thus, the assumption of equal energy absorption $\left(R_y = \sqrt{2\mu - 1}\right)$ can be applied in the range $(T_n/T_p)_b < T_n/T_p < (T_n/T_p)_c$. Note that the right threshold is transferred from $(T_n/T_p)_{c'}$ to $(T_n/T_p)_c$ (compare with the second of Eqns. 1.5).

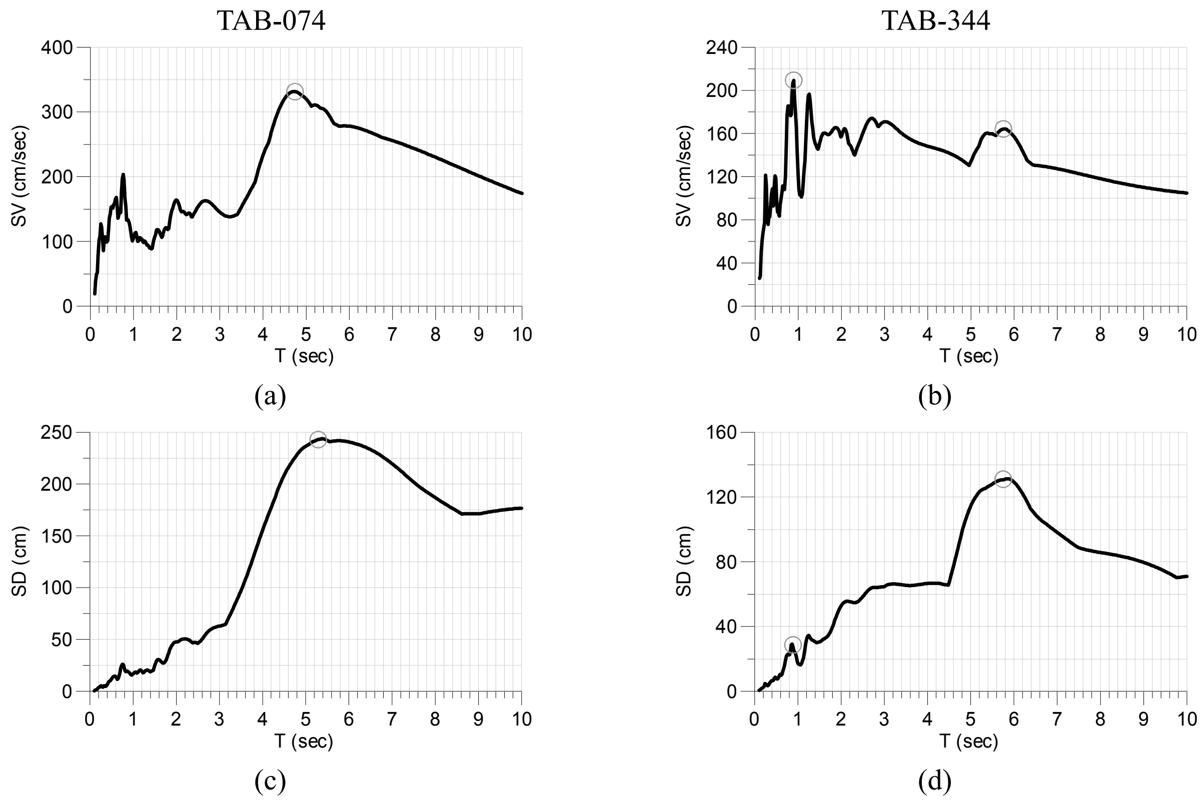


Figure 2. TAB-074 (left column) and TAB-344 (right column) components of Tabas, Iran, 1978 earthquake: (a) and (b) velocity spectra for ζ=5%; (c) and (d) displacement spectra for ζ=5%.

An example of this behaviour is shown in the left column of Fig. 3, in which the velocity and the displacement response spectra and the ductility demand for R_y=3 are shown for the component KAR-000 of the Gazli, USSR, 1976 earthquake (M_w=6.8). The predominant periods of the spectra are T_{SV}=1.0 sec and T_{SD}=4.4 sec, thus, it can be set T_p=4.4 sec. Note that Eqn. 1.7 gives T_p=3.5 sec (a standard deviation is added), i.e. a smaller value. For R_y=3 and a medium to large event, the normalized periods in Eqn. 1.5 are: $(T_n/T_p)_b$=0.2, $(T_n/T_p)_c$=0.75 and $(T_n/T_p)_{c'}$=0.45. Then, for T_p=4.4 sec it comes out: μ=3.0 for T_n>3.3 sec (equal displacement region) and μ=5.0 for 0.9 sec < T_n < 2.0 sec (equal energy region). Figure 3e shows that the equal displacement regime (μ=3.0) can indeed be assigned to T_n>3.3 sec, but the equal energy region (μ=5.0) must be extended to the right, up to T_n=3.3 sec, as proposed above.

3.3. Records with backward directivity

The records with backward directivity are characterized by the time sequence of many pulses of the ground velocity, associated with the fault rupture sub-events. Thus, the strong motion duration is quite long, but the ground velocity is smaller compared to the forward directivity. The long duration results in large spectral amplifications in both velocity and displacement spectra. The predominant periods of these spectra, T_{SV} and T_{SD}, are quite different, as in the case of weak forward directivity. Again, the period of the pulse, T_p, is associated with the predominant period of the displacement response spectrum, T_{SD}. Thus, Eqns. 1.5 can be applied with the modifications proposed above for records with weak forward directivity, namely: T_p should be derived from the predominant period of the displacement response spectrum, T_{SD}, and the right threshold of the equal energy regime should be moved to $(T_n/T_p)_c$ instead of $(T_n/T_p)_{c'}$.

An example is presented in the right column of Figure 4 for the ELC-180 component of the Imperial Valley, USA, 1940 earthquake (M_w=6.2-6.4). The record contains a large number of velocity cycles with small amplitude and Mavroeidis and Papageorgiou (2003) acknowledge it as a case of backward directivity. The predominant periods

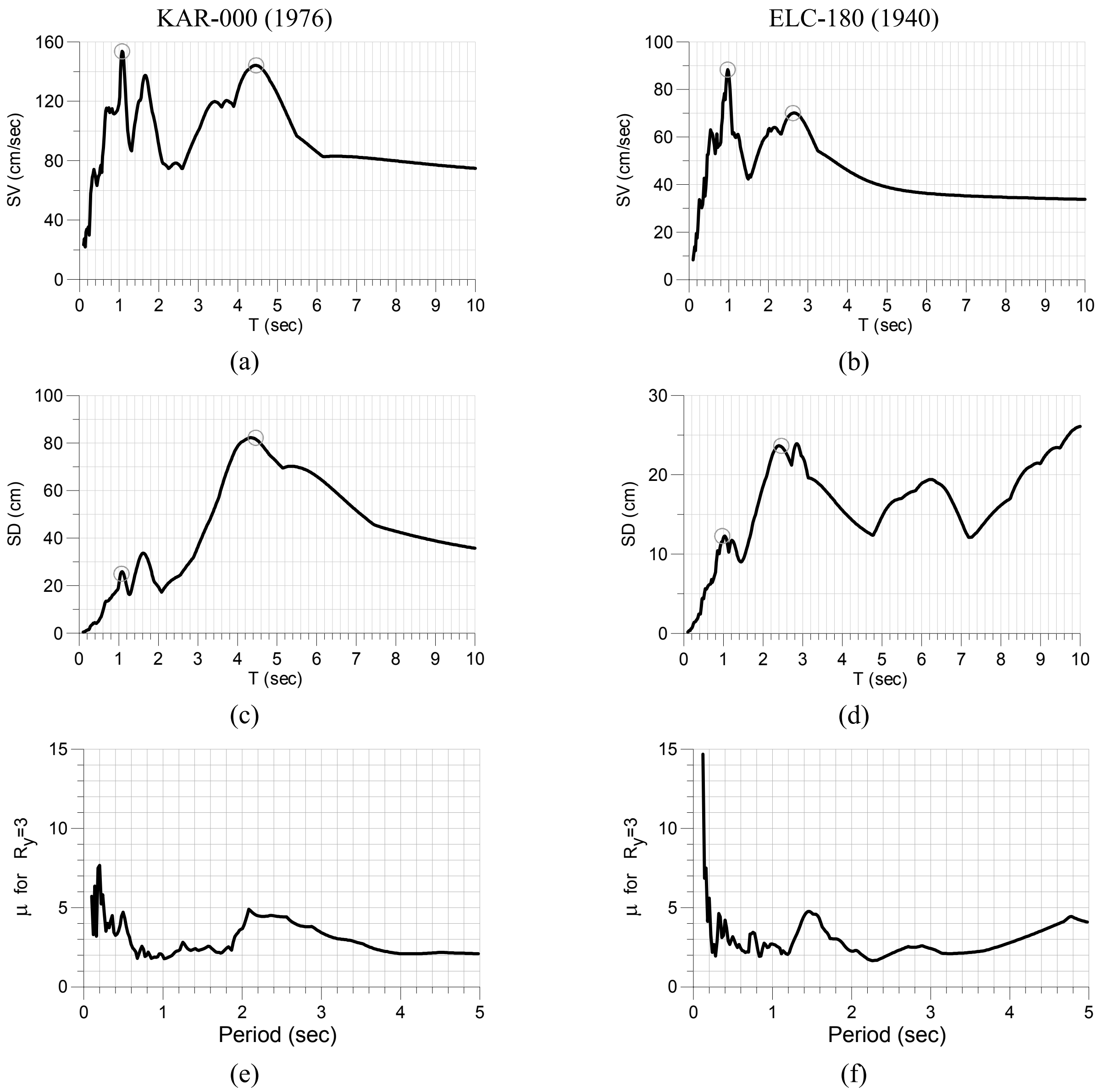


Figure 3. KAR-000 component of Gazli, USSR, 1976 earthquake (left column) and ELC-180 component of the Imperial Valley, USA, 1940 earthquake (right column): (a) and (b) velocity spectra for ζ=5%; (c) and (d) displacement spectra for ζ=5%; (e) and (f) ductility demand spectra for R_y=3.

of the velocity and the displacement spectra are T_{SV}=1.0 sec and T_{SD}=2.4 sec. Thus, the period of the pulse can be set equal to T_p=2.4 sec. Eqn. 1.7 gives T_p=2.05 sec with the standard deviation added, i.e. again a smaller value. The normalized periods in Eqns. 1.5 are: $(T_n/T_p)_b$=0.2, $(T_n/T_p)_c$=0.75 and $(T_n/T_p)_{c'}$=0.45. Consequently, application of Eqns. 1.5 for T_p=2.4 sec results in μ=3.0 for T_n > 2.0 sec (equal displacement region) and μ=5.0 for 0.5 sec < T_n < 1.2 sec (equal energy region). As shown in Figure 3f, the prediction is quite good for the equal displacement regime (T_n>2.0 sec), but the equal energy assumption (μ=5.0) should be extended approximately up to 1.6 sec. The proposed transfer of the right threshold of the equal energy regime to $(T_n/T_p)_c$ corrects this error.

4. CONCLUSIONS

In this analysis, the regions of validity of the R_y-μ-T_n relationships (Eqns. 1.5) proposed by Mavroeidis et al (2004) are examined for near field records with various types of directivity. The results show that for records with prominent forward directivity, these relations give reasonable results. In this case, the period T_p can be derived from the predominant period of the velocity response spectrum, T_{SV}. For records with weak forward directivity and records with backward directivity, these relations should be applied with two modifications: the pulse duration, T_p, should be derived from the predominant period of the displacement response spectrum, T_{SD}, and the right threshold of the equal energy regime should be extended from $(T_n/T_p)_{c'}$ to $(T_n/T_p)_c$.

It is evident from the analysis that, for near field records with directivity effects, the region in which the ductility demand is larger than R_y (equal energy regime) depends on the period, T_p, of the directivity pulse. For records with weak or backward directivity, the right threshold of this region is close to $0.75{\cdot}T_p$, which means that the equal displacement rule might not be valid even for long-period structures, if T_p is long. This effect might be suppressed if a statistical process is applied to the results of a large sample of records. For example, the relationships proposed by Chopra and Chintanapakdee (2001), although they consider different thresholds for near field and far field earthquakes, do not take under consideration the pulse period T_p, and, thus, might underestimate the ductility demand in the long period regime. The same observation holds for the conclusion drawn by Garcia and Miranda (2006) that the ductility demand is not affected by the magnitude of the seismic event. The results presented here show that the R_y-μ-T_n relationships depend on the duration of the directivity pulse, which is associated with the magnitude of the earthquake (Eqn. 1.7).

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