

A REGRESSION MODEL FOR ESTIMATING POWER SPECTRAL DENSITY FUNCTION OF GROUND ACCELERATION

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ABSTRACT :

The purpose of this study is to develop a new regression model for power spectral density function (PSDF) of ground acceleration by using the strong motion data of the K-NET and the KiK-NET in Japan. The PSDF is simply expressed as the PA model, proposed by authors before, by using three parameters: the mean square value of the ground acceleration, the predominant frequency, and the shape factor. The regression analyses for the PA-model parameters are carried out based on the maximum likelihood method with respect to the explanatory variables: the distance from the fault to the site, the magnitude, the focal depth, and the averaged shear-wave velocity at the site. The obtained PSDF regression model can express well the averaged tendency, in quality and in quantity, of the observed PSDF. Furthermore, the efficiency and the applicability of the proposed model are demonstrated through the estimation of the earthquake input energy based on the random vibration theory.

KEYWORDS:

Power Spectral Density Function, Regression Model, Maximum Likelihood Method, K-NET, KiK-NET

1. INTRODUCTION

For the last several decades, many types of regression models have been proposed for strong-motion parameters such as peak ground motion^{1,2,3}, response spectrum⁴, and so on in reply to the increasing needs for engineering prediction of strong motion characteristics. Especially in this decade, in Japan, dense observation networks such as the K-NET⁵ and the KiK-NET⁶ provide plenty of strong-motion data that enables to advance these kinds of studies⁷. Based on these databases, this study aims to develop a new regression model with engineering applicability for power spectral density function (PSDF) of ground acceleration.

In the field of earthquake engineering, the Kanai-Tajimi model has been widely used to express PSDF of earthquake ground acceleration. However, this model sometimes induces overestimations of the spectral values at low frequencies because of its restriction on the zero-frequency spectral value. To overcome this shortcoming, we proposed a simple PSDF model, named as PA model, which corresponds to the 'Pseudo-Acceleration' response of a single-degree-of-freedom (SDOF) system under a white noise excitation^{8,9}. A PA model is simply described by three parameters: the mean square value of the ground acceleration, the predominant frequency, and the shape factor corresponding to the damping factor of the SDOF system. In this study, regression analyses are carried out for these PA-model parameters with respect to the parameters characterizing the earthquake source, the propagation path, and the local site condition so that a new regression model is developed for the PSDF of ground acceleration. The PA model has a valuable advantage as an input motion model for random response analysis. The applicability of the obtained regression model is demonstrated, therefore, through the estimation of the earthquake input energy based on the random vibration theory.

2. POWER SPECTRAL DENSITY FUNCTION MODEL

The PA model^{8,9} used in this study to express the PSDF $S(\omega)$ of ground acceleration $A(t)$ is given in the form:

$$S(\omega) = \frac{4\zeta_g \omega_g \omega^2}{(\omega^2 - \omega_g^2)^2 + 4\zeta_g^2 \omega_g^2 \omega^2} \sigma_A^2 \quad (2.1)$$

where ω_g is the predominant angular frequency, ζ_g is the shape factor, and σ_A^2 is the mean square value of $A(t)$. $S(\omega)$ has one peak at around $\omega = \omega_g$ and ζ_g controls the sharpness of the peak. The area under the $S(\omega)$ curve equals to $2\pi\sigma_A^2$. Based on the random vibration theory, the earthquake input energy to a SDOF system with a natural angular frequency ω_s and a damping factor ζ_s under an excitation characterized by $S(\omega)$ is analytically derived as

$$E(\omega_s) = \frac{2\gamma(\zeta_g \gamma + \zeta_s) \sigma_A^2 T_d}{F(\zeta)} \frac{\omega_s}{\omega_g}, \gamma = \frac{\omega_s}{\omega_g} \quad (2.2)$$

$$F(\gamma) = \gamma^4 + 4\zeta_s \zeta_g \gamma^3 + 2(2\zeta_s^2 + 2\zeta_g^2 - 1)\gamma^2 + 4\zeta_s \zeta_g \gamma + 1$$

where T_d is the effective duration time of the ground motion.

3. REGRESSION MODEL

3.1. Regression Variables and Database

For regression analysis, four explanatory variables are selected: the magnitude M (Japan Meteorological Agency magnitude), the fault distance R (the hypocentral distance or the shortest distance between the site and the fault if available^{5,6}), the focal depth D , and the averaged shear-wave velocity V of the site (velocity averaged over the top 30 m of the sediments). The response variables are the PA-model parameters: σ_A , ω_g , and ζ_g for the PSDF of the vector combination of two horizontal components, and additionally T_d for the evaluation of the earthquake input energy.

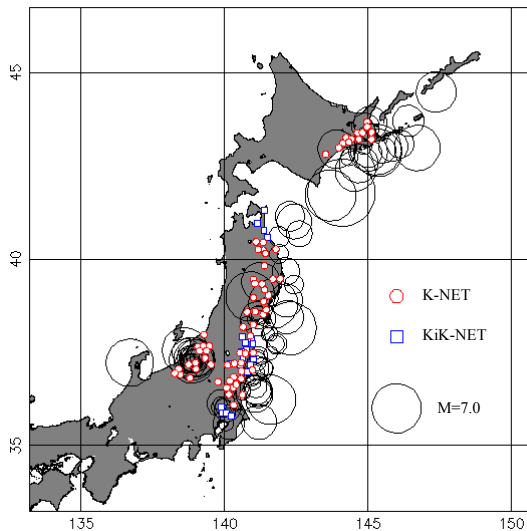


Figure 1. Locations of sample earthquakes and sites

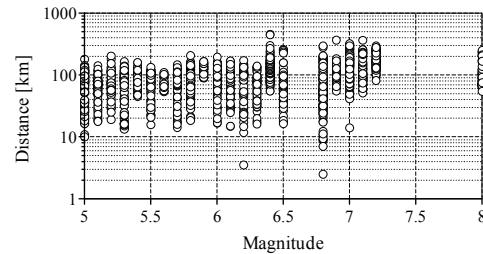


Figure 2. $M - R$ distribution

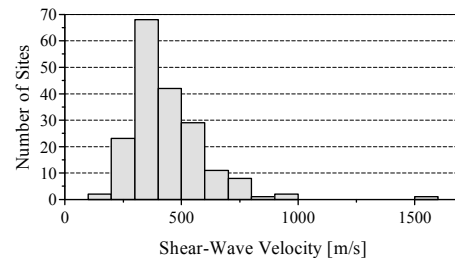


Figure 3. V distribution

The K-NET and the KiK-NET records from 1996 to 2008 are used to make the sample database of the explanatory variables and the response variables. Sample records are selected so as to satisfy the following conditions: (1) all of the explanatory and the response variables are available, (2) $M \geq 5.0$, (3) the maximum horizontal acceleration ≥ 50 cm/s, (4) each of the earthquakes has more than 5 records, and (5)

each of the sites has more than 5 records. The selected 1539 records contain 90 earthquakes and 187 sites. Selected earthquakes and sites locations, the $M - R$ distribution, and the V distribution are shown in Fig. 1 to Fig. 3, respectively.

The effective duration time T_d is determined by the following procedure: from the squared acceleration time history $A^2(t)$ smoothed by moving average with an interval of 2.0 sec, find two points of time on both side of the maximum peak at which the level firstly falls down below the 1 percent of the maximum level, and take the difference between the two time points as shown in Fig. 4. The PSDF and the mean square value σ_A^2 are calculated from this portion of the time history with duration T_d . The PA-model parameters ω_g and ζ_g are determined so as to minimize the log-residual between the PSDF of the observation record and of the PA model. The PSDF of the observation is smoothed along the frequency with 1-dB width, and 20 log-discretized frequencies from 0.1 Hz up to 10 Hz are used to estimate the residual. Fig. 5 shows a sample of PSDFs and the fitted PA model. Fig. 6 shows the distribution of root-mean-square (RMS) residuals of all samples versus σ_A . The 95-percentile of the residuals is about 0.64.

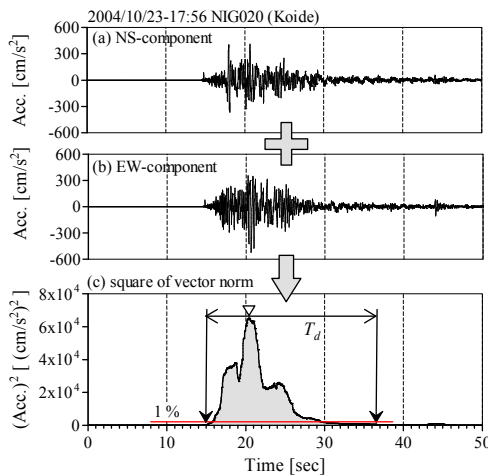


Figure 4. Definition of the effective duration time T_d

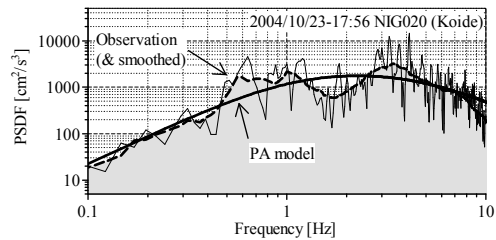


Figure 5. PSDF model

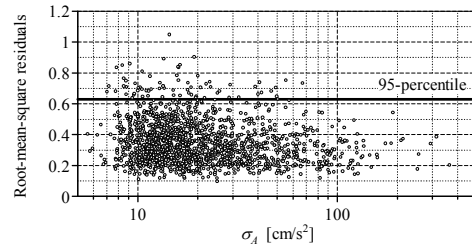


Figure 6. RMS residuals for PA model

3.2. Maximum Likelihood Method

The regression model employed in this study is of the following form:

$$\log_{10} Y_{ij} = a_1 \log_{10} R_{ij} + a_2 R_{ij} + a_3 M_I + a_4 D_I + a_5 \log_{10} V_j + a_6 + \varepsilon_{ij} \quad (3.1)$$

where Y_{ij} represents one of the response variables: σ_A , ω_g , ζ_g , and T_d . The subscripts I and j refer to the earthquake and the site, respectively. ε_{ij} is a random variable with zero mean value corresponding to the residual for the record of the I th earthquake observed at the j th site. The first term on the right-hand-side of Eqn. (3.1) diverges to infinity at $R_{ij} = 0$. To utilize this regression model in the near-fault region, it is needed to insert some near-fault term into the argument of the logarithmic function. As the trial analysis with a near-fault term did not give any improvement of regression as far as for the samples selected in this study, the functional form given by Eqn. (3.1) is used as is. The second term on the right-hand-side is used only for σ_A ; it is not used for ω_g , ζ_g , and T_d in order to avoid the trade-off with the first term.

The covariance between the residual ε_{ij} and $\varepsilon_{k\ell}$ is supposed to have the following form:

$$Cov[\varepsilon_{ij} \cdot \varepsilon_{kl}] = \begin{cases} \sigma_e^2 & , I = K \text{ and } j \neq l \\ \sigma_s^2 & , I \neq K \text{ and } j = l \\ \sigma_e^2 + \sigma_s^2 + \sigma_r^2 = \sigma^2 & , I = K \text{ and } j = l \\ 0 & , I \neq K \text{ and } j \neq l \end{cases} \quad (3.2)$$

where σ_e^2 , σ_s^2 , and σ_r^2 are inherent variances in earthquakes, in sites, and in records, respectively. By the maximum likelihood method¹⁰⁾, these variances are determined so as to maximize the likelihood of the samples.

3.3. Regression Result

The obtained variances and the resultant coefficients a_i with standardized ones are listed in Table 1.

Table 1. The obtained regression coefficients and standard deviations of residuals
 (Values in parenthesis are the standardized regression coefficients.)

	a_1	a_2	a_3	a_4	a_5	a_6	σ	σ_e^2/σ^2	σ_s^2/σ^2	σ_r^2/σ^2
σ_A	-7.520E-01 (-0.73)	-1.072E-03 (-0.21)	3.016E-01 (0.78)	4.935E-03 (0.50)	2.264E-02 (0.01)	6.807E-01	0.211	0.225	0.248	0.527
ω_g	1.504E-01 (0.17)		-1.653E-01 (-0.50)	8.376E-04 (0.10)	5.755E-01 (0.43)	6.644E-01	0.178	0.098	0.534	0.368
ζ_g	-3.284E-02 (-0.02)		3.219E-01 (0.52)	-2.410E-03 (-0.15)	3.301E-01 (0.13)	-2.866E+00	0.399	0.124	0.366	0.510
T_d	2.771E-01 (0.36)		1.344E-01 (0.46)	-7.287E-04 (-0.10)	-1.581E-01 (-0.13)	5.520E-01	0.155	0.216	0.178	0.606

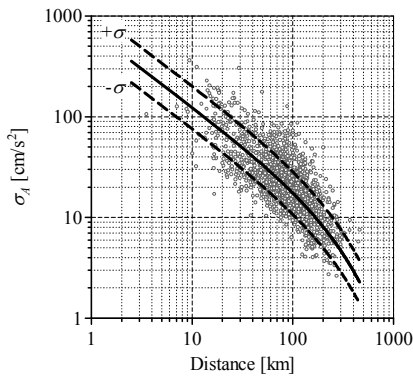


Figure 7. Regression curve for σ_A
 (normalized to $M = 7.0$, $D = 10 \text{ km}$, $V = 300 \text{ m/s}$)

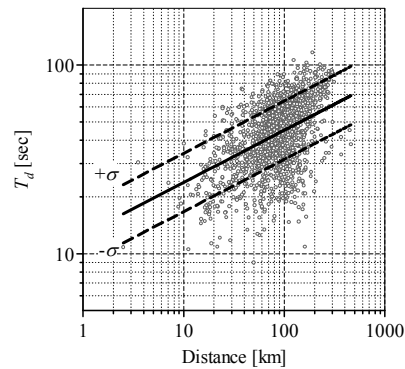


Figure 8. Regression curve for T_d

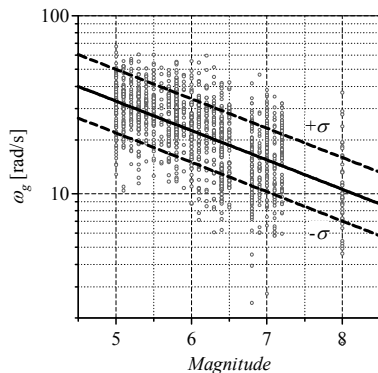


Figure 9. Regression curve for ω_g
 (normalized to $R = 50 \text{ km}$, $D = 10 \text{ km}$, $V = 300 \text{ m/s}$)

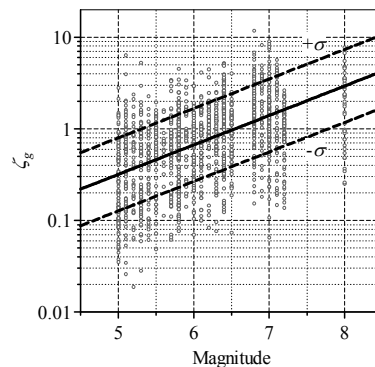


Figure 10. Regression curve for ζ_g

From the Table 1, it is observed that σ_A and T_d depend on the distance R and on the magnitude M in the manner as empirically expected. On the other hand, the predominant angular frequency ω_g depends not only on the shear-wave velocity V but also, more strongly, on M . Although it can also be observed that the shape factor ζ_g has strong dependence on M , its standard deviation is relatively large as compared with other three response variables. The regression curves for σ_A , T_d (versus R), ω_g , and ζ_g (versus M) are shown in Fig. 7 to 10, respectively.

As the error distributions are not strongly distorted, the regression method seems to give appropriate results for the regression models. To examine the resultant fitness of PSDF, the log-residuals are plotted in Fig. 11, in the same manner as Fig. 5. The cumulative deviation of σ_A , ω_g , and ζ_g raise the 95-percentile of the residuals to 0.94, which is larger than that of the sample set shown in Fig. 5. Fig. 12 and 13, for examples, illustrate PSDFs at NIG020 and NIG025 during the 2004 Niigata-ken Chuetsu earthquake ($M = 6.8$). The RMS residuals for NIG020 (Fig. 12) and NIG025 (Fig. 13) are 0.304 and 0.714, respectively. In both cases, the PA-model described by the regressed parameters seems to express well the average tendency, in quality and in quantity, of the observed PSDF.

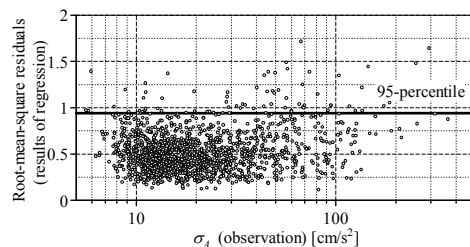


Figure 11. RMS residuals for composed PSDFs

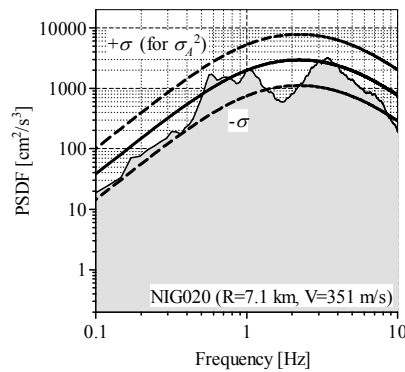


Figure 12. PSDF of NIG020

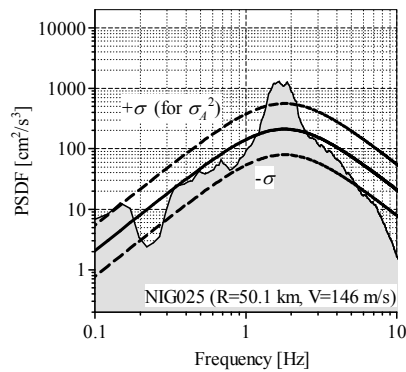


Figure 13. PSDF of NIG025

4. DISCUSSION

The main reason why to use the PA model for PSDF owes to its valuable feature as an input motion model for random response analysis. For example, as mentioned before, the earthquake input energy can be expressed in closed form as given by Eqn. (2.2). The energy-equivalent velocity spectra corresponding to the same cases as in Fig. 12 and 13 are shown in Fig. 14 and 15, respectively.

Furthermore, the qualitative and quantitative relations of the earthquake and site parameters such as the distance, the magnitude, the focal depth, and the shear-wave velocity to the earthquake input energy can be examined directly by using this model as shown in Fig. 16 to 19. These figures show the energy-equivalent velocity spectra for various combinations of the earthquake and site parameters.

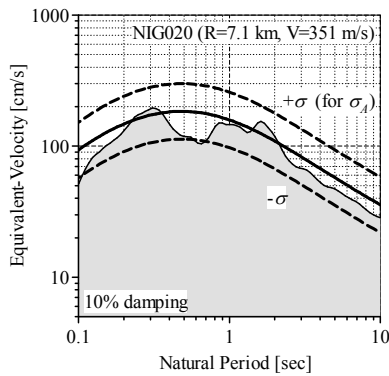


Figure 14. Energy-equivalent velocity for NIG020

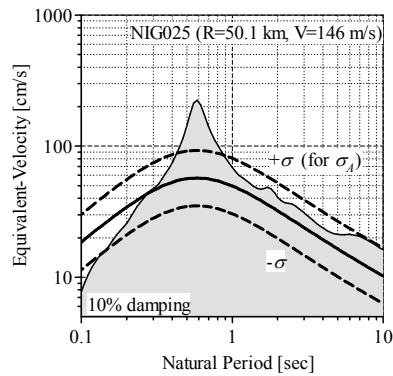


Figure 15. Energy-equivalent velocity for NIG025

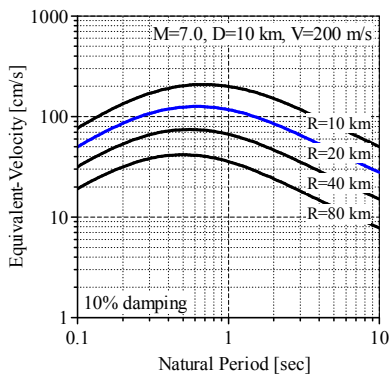


Figure 16. Energy-equivalent velocity spectra for various R

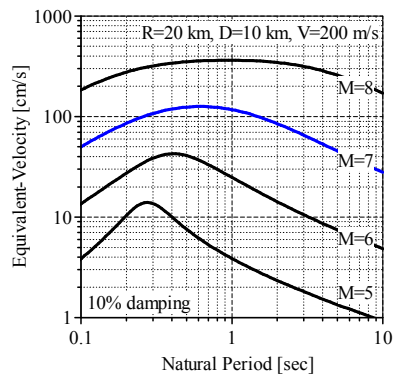


Figure 17. Energy-equivalent velocity spectra for various M

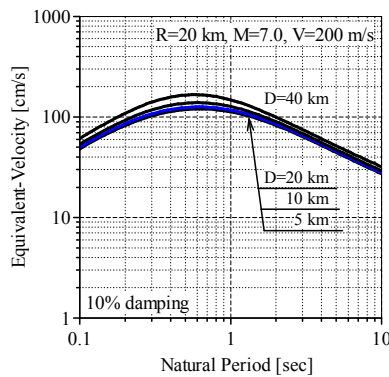


Figure 18. Energy-equivalent velocity spectra for various D

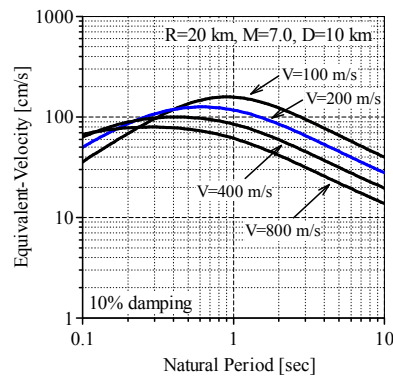


Figure 19. Energy-equivalent velocity spectra for various V

The spectral level increases with the decrease of the distance R , when the spectral shape dose not change significantly. The increase of the magnitude M leads to the increase of the spectral level and the shift of the peak to longer periods. It simultaneously make the spectral shape flatter because of the increase of ζ_g . The decrease of the shear-wave velocity V has the same influences in quality except for the flattening effect. The increase of the focal depth D also leads to the increase of the spectral level, but its influence is relatively small.

5. CONCLUSION

A new regression model for PSDF of ground acceleration has been developed by using the strong-motion

data of the K-NET and the KiK-NET in Japan. The PSDF of ground acceleration is expressed by the PA model, which can be described with three parameters: the mean square value of the ground acceleration σ_A^2 , the predominant frequency ω_g , and the shape factor ζ_g . The regression analyses for these PA-model parameters and the effective duration time T_d have been carried out based on the maximum likelihood method with respect to the explanatory variables: the distance R , the magnitude M , the focal depth D , and the averaged shear-wave velocity V . The obtained regression model can express well the average tendency, in quality and in quantity, of the PSDF of the observed ground motion. Furthermore, the applicability of the proposed regression model has been demonstrated through the estimation of the earthquake input energy based on the random vibration theory. It has been shown that the proposed model can be used efficiently not only in engineering prediction of strong ground motion but also in the direct estimation of the structural response.

ACKNOWLEDGEMENT

This study has made use of the data provided by K-NET and KiK-NET.

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