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CORRELATIONS BETWEEN THE DISPLACEMENT RESPONSE SPECTRA AND THE PARAMETERS CHARACTERISING THE MAGNITUDE OF THE GROUND MOTION

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ABSTRACT :

Within a Displacement-Based Seismic Design approach, the seismic input is described through displacement design spectra. Until now, these spectra are commonly obtained from the corresponding pseudo-acceleration design spectra, widely used in a Force-Based Seismic Design approach, and are therefore affected by uncertainties regarding the schematization of pseudo-acceleration spectra.
In this paper, the relationships between earthquake ground motion parameters and displacement spectra have been faced starting afresh to provide an updated schematization of the spectra which consciously accounts for the most representative ground motion parameters.
First, in order to identify the fewest independent parameters which best describes the seismic input, for each record, different earthquake ground motion parameters have been computed and compared to each other through correlation analyses. The results indicate that the parameters Peak Ground Acceleration (PGA), Peak Ground velocity (PGV) and Peak Ground Displacement (PGD) are both necessary and sufficient to exhaustively characterize the seismic input.
Second, on the basis of the well-established "tripartite" earthquake elastic response spectrum, analytical relationships have been derived between the PGA, PGV and PGV parameters and the average amplification factors of the structural response.
Finally, simple formulas are proposed for the displacement spectrum which account for the PGA, PGV and PGD. Comparison with common code displacement spectra which accounts for the PGA only show that, to best capture the trend of real earthquake spectra and to avoid both over- and under-estimations, it is fundamental to take into account at least the PGV (if not also the PGD).

KEYWORDS: earthquake ground motion parameters, displacement spectra, linear regression, "tripartite" earthquake response spectrum, peak ground acceleration, peak ground velocity

1. INTRODUCTION

The common practice of structural design does not see the structural system reach its plastic limit. Thus, the usual Force-Based Design (FBD) approach (commonly used for static design), in which demand and capacity are compared in terms of forces. In detail, design forces, F_{design}, are compared to the corresponding ones which lead to the "yielding" of the structure, F_y. The safety of the system can be approximately accounted for through the ratio of the yielding to the design force, as per $FS = F_y / F_{design}$. The system being in "failure conditions" when $FS \le 1$.
For special types of loading (as it is the case of seismic design), the structural systems may reach their plastic limit and exploit their non-linear behavior (which, in most cases are welcomed as beneficial). It is therefore clear that, in these cases, comparisons based upon forces loose any meaning. Thus, the so-called Displacement-Based Design (DBD) approach, in which demand and capacity are compared in terms of displacements. In detail, the design displacement demand δ_{design} is compared to the corresponding one which exhausts the system capacity leading to failure δ_u (ultimate displacement). The safety of the system can be approximately accounted for through the ratio of the ultimate to the design displacement ($FS = \delta_u / \delta_{design}$). The system being in "failure conditions" when $FS \le 1$.
It is thus clear that, for seismic design purposes, the recently proposed direct DBD approach represent a meaningful alternative to the traditional FBD approach which borrows (due to cultural and historical reasons), for seismic design, static design methodologies. In the traditional Force-Based Seismic Design (FBSD), the design demand (actions) are generally estimated by means of (pseudo) acceleration response spectra S_a. In the direct Displacement-Based Seismic Design (DBSD), the design demand (displacements) are generally estimated by means of displacement response spectra S_d. It is thus clear that, in order to establish the new direct DBSD procedure, the definition of displacement response spectra is crucial. Design response spectra encompass a number of piece of information (hazard analysis, social considerations, economic evaluations) which often go beyond the capacity of the scientist; however, the procedure currently used to define the displacement response spectra in the few codes which allows for DBSD seems to raise few issues. The design displacement response spectra are indeed derived from integration ($S_d = S_a / \omega^2$), and this allows to obtain response spectra which are somehow "compatible". However, it is also clear that the approximations and the assumptions (necessary to formulate a "design" spectrum that, with few equations, schematize a probabilistic demand), which are somehow "acceptable" for S_a, may not be equally acceptable for S_d, and may lead to significant "errors" in the identification of the probabilistic demand which thus may be substantially different in the two cases.
For this reason, in this research work, the displacement response spectra are not derived from the acceleration ones. Indeed, acceleration response spectra may assume, as recently proposed by some codes, articulated shapes (somewhat more complicated than those which can be obtained from the schematized tripartite response spectrum) which may be good for the identification of the design acceleration, but not necessarily good for the identification of the design displacement. Instead the design response displacement spectra are obtained directly from the tripartite response spectra with the aim of capturing the essence of the parameters which control its shape.

2. THE SCHEMATIZED TRIPARTITE RESPONSE SPECTRUM

Each seismic record can be represented through its tripartite elastic response spectrum (Fig. 1), which, in turn, can be simplified into the schematized tripartite response spectrum.
From a practical point of view, the design tripartite response spectrum is defined by the following 6 (3+3) "control" parameters:

- *PGA*, *PGV*, and *PGD*, referred to as "peak ground motion" parameters. These parameters are somewhat related to the energy released by the earthquake ground motion, and therefore they represent a subset of the more general set of the "energy" parameters (also referred to as "intensity measures"), and change their values with the record scaling;

- α_A, α_V, and α_D, referred to as "amplification" parameters. These parameters, though they may be somewhat correlated with the intensity measures, are not an expression of the energy released by the earthquake ground motion and therefore do not change with record scaling.

It follows that, even though the 6 above-mentioned parameters collectively control the shape of the schematised tripartite response spectrum, they can be divided in two groups with different physical meaning. For this reason, the peak ground motion parameters and the amplification parameters will be treated separately.

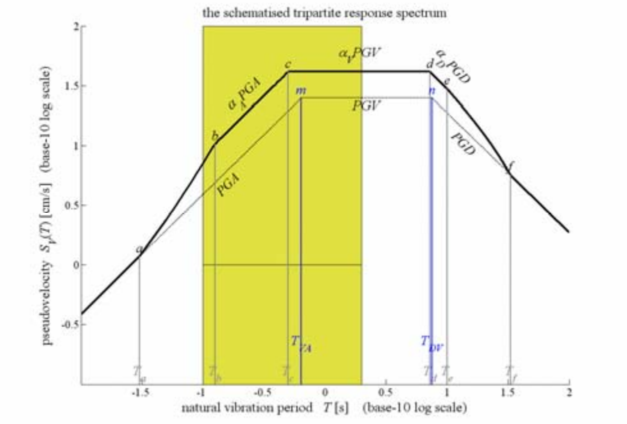
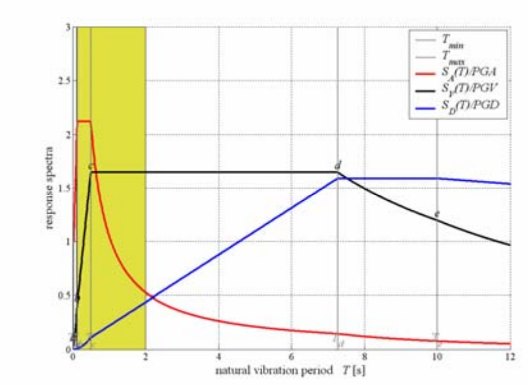
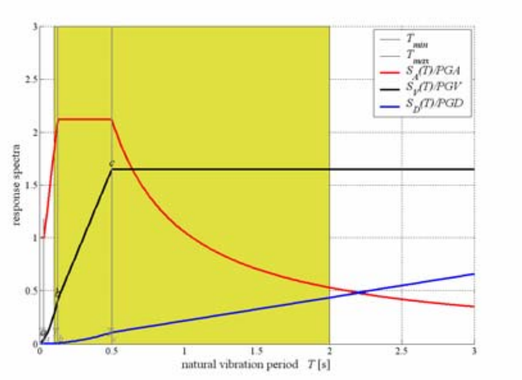


Fig. 1. The schematized tripartite response spectrum.

Fig. 2 represents the schematizations of the pseudoacceleration, pseudovelocity and displacement response spectra, as derived from the schematized tripartite response spectra of Fig. 1. Note that, as a direct consequence of the schematized tripartite spectrum, in each one of these three spectra, it is possible to identify an area characterized by a linear variation of the response. Fig. 2 also highlights (in yellow color) the range of structural periods ($0.1\text{s} \le T \le 2.0\text{s}$) of major interest for seismic design. Note that, for this range of period values, the linear schematization well captures the essence of the displacement dynamic response.



(a)



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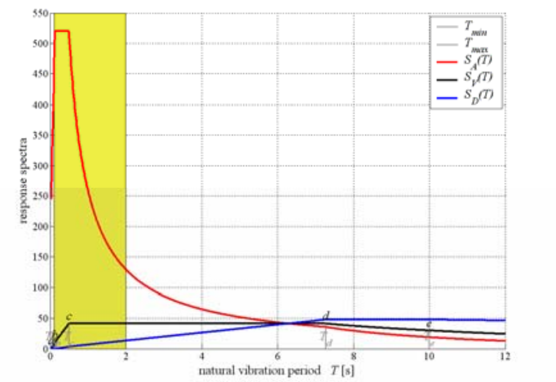


(c)

Fig. 2. (a) Pseudoacceleration, (b) pseudovelocity and (c) displacement response spectra, as derived from the schematised tripartite spectrum.

3. THE PEAK GROUND MOTION PARAMETERS

Each seismic record may be characterized through a number of different energy parameters or intensity measures (*IM*), as defined by various authors, widely in excess of the three (*PGA*, *PGV* and *PGD*) used to "control" the schematization of the tripartite response spectrum recalled above. Nonetheless, the three peak ground motion parameters may be either insufficient, essential or redundant in capturing the substantial energy characteristics of each seismic record. For this reason, it has been investigated haw the different energy parameters relate to the three peak ground motion parameters used in the schematized tripartite spectrum. After a careful review of the technical literature (Kramer 1996) about 15 energy parameters have been identified (i.e. *PGA*, *PGV*, *PGD*, *PGV* to *PGA* ratio T_{VA}, *PGD* to *PGV* ratio T_{DV}, total duration, total intensity, rms Acceleration, Arias Intensity, Characteristic Intensity, Cumulative Absolute Velocity, Acceleration Spectrum Intensity, Acceleration Spectrum Intensity, Housner's coefficient, Epicentral distance, Magnitude). In order to identify whether the three peak ground motion parameters are the essential ones to capture the energy of the earthquake ground motion, a comprehensive correlation analysis is performed with reference to 344 historical time histories all recorded on soil with similar characteristics ($360 \le v_{S30} \le 750$ m/s). The results indicate clearly (i) how all three control parameters *PGA*, *PGV* and *PGD* are fairly independent, or better how they are somehow positively correlated (with a correlation coefficient of about 0.60÷0.70) with the other physical quantities directly obtainable with single integration or derivation, while they are weakly correlated (almost independent) with the other physical quantities obtainable with double integration or derivation (Table 1); (ii) all of the considered energy parameters are strongly correlated (correlation coefficient in excess of about 0.60) with the *PGA*; (iii) only the total duration of the record (which may play a fundamental role in the response of building structures, especially when non-linear behavior of the system is considered) is basically independent from the *PGA*, while it is more closely correlated to the *PGV*; (iv) the *PGV*, which is not particularly correlated to any energy parameter, seems to represent, on the other hand, a good compromise among the three peak motion parameters (Fig. 3).

Table 1. Correlation coefficients between the three peak ground motion parameters.

ρ	*PGA*	*PGV*	*PGD*
PGA	1	0.65	0.11
PGV	0.65	1	0.69
PGD	0.11	0.69	1

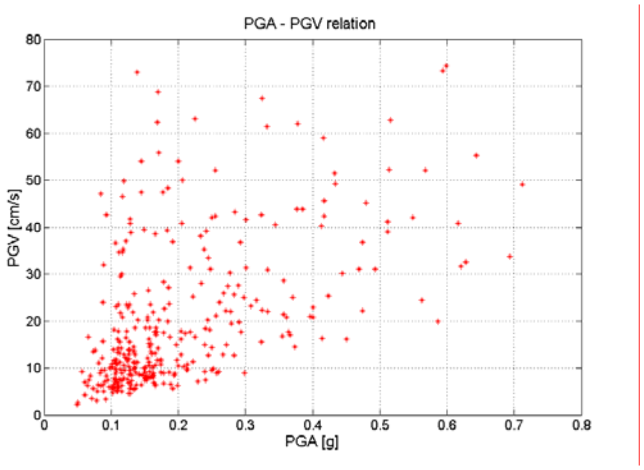


Fig. 3. Visual result of the *PGA*-*PGV* correlation analysis.

4. THE AMPLIFICATION PARAMETERS

A wide investigation campaign is carried out in order to obtain updated estimation of the "amplification" parameters α_A, α_V, and α_D, through a direct and an indirect methods.
With the direct method an estimation $\alpha_A = S_A(\bar{\xi})/PGA$, where $S_A(\bar{\xi})$ indicates the spectral acceleration, can be numerically obtained on the basis of N seismic records and their pseudoacceleration response as computed at M periods T_j, ($T_b \leq T_j \leq T_c$, with T_b and T_c, being the two periods which bounds the region of constant acceleration in the schematized tripartite spectrum); this investigation proved to be highly variable, depending upon the choice of the bounding periods T_b and T_c (a small change in the bounding periods may lead to substantial variation in the estimated $\alpha_{A,i}$ values). Similar considerations, not reported here for sake of conciseness, can be made regarding the other two amplification coefficients α_V and α_D.
Using the indirect method, for each *i*-th seismic record, an estimation of the amplification factors $\alpha_{A,i}$ and $\alpha_{V,i}$ can be obtained as $\alpha_{A,i} = 2\pi \cdot \varphi_{V,i} / PGA_i$ and $\alpha_{V,i} = 2\pi \cdot \varphi_{D,i} / PGV_i$, where $\varphi_{V,i}$ and $\varphi_{D,i}$ represent the angles of the line segment of the *i*-th pseudovelocity and displacement response spectra, respectively, while PGA_i and PGV_i represent the peak ground acceleration and velocity of the *i*-th seismic record, respectively. The angles $\varphi_{V,i}$ and $\varphi_{D,i}$ can be estimated with a high degree of reliability through a linear regression of the data of the pseudovelocity and displacement response spectra. Moreover, this estimation of the angles $\varphi_{V,i}$ and $\varphi_{D,i}$ is much less sensitive to the choice of the bounding periods rather than the direct estimation of $\alpha_{A,i}$ and $\alpha_{V,i}$, and the values of these bounding periods (which correspond to the limit where the displacement spectra change from a linear to a constant schematization) can be estimated with quite a good degree of confidence.
The indirect method leads to the most reliable estimations of the amplification factors, and, therefore, it has been used for the estimations of α_A and α_V (due to its intrinsic characteristics it cannot be used to identify α_D), this latter has been then estimated using the direct method.

5. THE DESIGN SPECTRA AND THE PROPOSED DISPLACEMENT DESIGN SPECTRUM

In order to define design spectra, it is of prime importance to take into account the different values that the peak ground motion parameters may assume in accordance with the results of appropriate probabilistic hazard analysis. It is also seen how, in the definition of the displacement design spectra, it is of prime importance to consider the appropriate value of the *PGV* corresponding to the target hazard (given probability of exceedance over prescribed period of observation). Nonetheless, the common suggestion of current codes is to derive the displacement design spectrum from the acceleration one, which, being based upon the *PGA* only, does not take into account any information regarding the *PGV* (which, as also seen before, is not closely correlated to the *PGA*). This may lead to substantial errors, as investigated in detail in the following section.
It is to be noted that acceleration design spectra for a given site draw the design *PGA* value from Seismic Hazard Maps of peak ground acceleration. At the present time, the scientific knowledge allows to produce also Seismic Hazard Maps of peak ground velocity (few peak ground velocity attenuation laws are available in literature, from which the design *PGV* value for a given site can be taken. At the present time, at the knowledge of the authors (even though feasible from a scientific point of view), no displacement attenuation laws are available and consequently no Seismic Hazard Maps of peak ground displacement are available. Therefore, the *PGD* parameter corresponding to given level of hazard necessary to fully identify the design schematized tripartite spectrum and the derived design spectra is not available. Only an estimation of the *PGD* can be obtained via the *PGV* given the fair correlation between these two quantities identified ($\rho_{PGD,PGV} = 0.69$).
On the basis of the investigation campaign developed by the authors, it is here proposed to use as elastic displacement response spectrum the following one:

$$S_D(T) = \varphi_D \cdot T \qquad \text{(for } T_c < T \leq T_d\text{)} \tag{5.1}$$

$$S_D(T) = \alpha_D PGD \qquad (\text{for } T > T_d) \tag{5.2}$$

with parameters specialized (in the case of 5% damped systems and soil characterized by a shear velocity at 30 m of depth between $360 \le v_{S30} \le 750$ m/s) as follows:

$$\varphi_D = 0.33 \cdot PGV \qquad (\text{with } \rho_{\varphi_D, PGV} \cong 0.85),\ \alpha_D = 1.92 \cdot PGD \qquad (\text{with } \rho_{\alpha_D, PGD} \cong 0.85) \tag{5.3}$$

$$T_c = 3.65 \frac{PGV}{PGA} = 3.65 \cdot T_{VA} \qquad (\text{with } \rho_{T_c, T_{VA}} \cong 0.67),\ T_d = 20 \cdot \frac{PGV}{PGA} = 20 \cdot T_{VA} \qquad (\text{with } \rho_{T_d, T_{VA}} \cong 0.48) \tag{5.4}$$

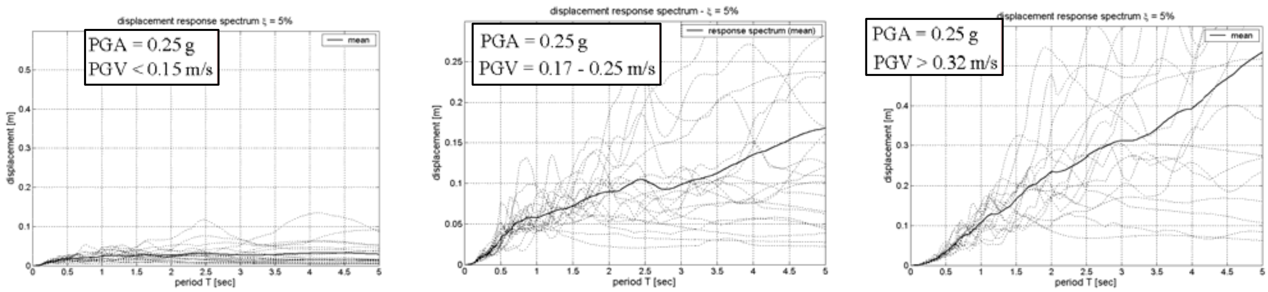
This last expression can be substituted by the more reliable (correlation coefficient of about 0.82 vs. 0.48) following one if the PGD is available:

$$T_d = 6.57 \frac{PGD}{PGV} = 6.57 \cdot T_{DV} \qquad (\text{with } \rho_{T_d, T_{DV}} \cong 0.82) \tag{5.5}$$

where *PGA* is expressed in [m/s^2], PGV is expressed in [m/s], *PGD* is expressed in [m], T_{VA} and T_{DV} are expressed in [s]. The resulting displacement design spectrum is expressed in [m].

6. THE ROLE OF PGA AND PGV FOR THE IDENTIFICATION OF THE DISPLACEMENT DESIGN SPECTRUM

As clearly pointed out in the previous sections, the *PGV* plays a central role in the determination of the displacement response spectrum, as it directly affects both the inclination (φ_D) and the maximum value of the spectrum ($S_{D,\max} = S_D(T_d) = \varphi_D T_d = \alpha_D PGD$).
To investigate how strongly the *PGV* may affect the shape of the displacement response spectrum, three groups of earthquake records have been extracted from the strong motion database. All records are characterized by the same *PGA* (about 0.25 *g*) and no scaling has been applied. The first group (hereafter referred to as G1) is characterized by *PGV* smaller than 0.15 m/s, the second group (hereafter referred to as G2) by *PGV* between 0.17 and 0.25 m/s, while the third group (hereafter referred to as G3) by *PGV* larger than 0.32 m/s.
Figs 4a, 4b and 4c plot the displacement response spectra obtained for each record, together with the mean spectrum) of groups G1, G2 and G3, respectively. It is clear how the maximum displacement response varies significantly with the *PGV*, with the maximum displacement increasing by more than an order of magnitude passing from group G1 to group G3 in the case of structural systems characterized by large periods.



(a) (b) (c)

Fig.4. Displacement response spectra for groups (a) G1, (b) G2 and (c) G3 (characterised by fixed value of the *PGA* and different range values of the *PGV*).

Thus, it is clear that a displacement design spectrum based upon the *PGA* only may lead to substantial over- or under-estimations of the actual response which strongly depends upon the specific value assumed by the *PGV*.
As illustrative example, Fig. 5 plots the displacement response spectrum of 18 records characterized by *PGA* approximately equal to 0.25 *g* and *PGV* approximately equal to 0.21 m/s, together with the displacement response spectrum obtained from Eurocode 8 (which is based upon the *PGA* only) and the here proposed spectrum based upon both the *PGA* and the *PGV*. It is clear how, in this case, the spectrum of Eurocode 8 considerably over-estimates the actual response spectra.

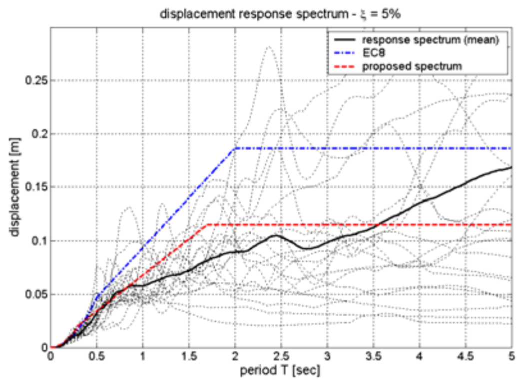
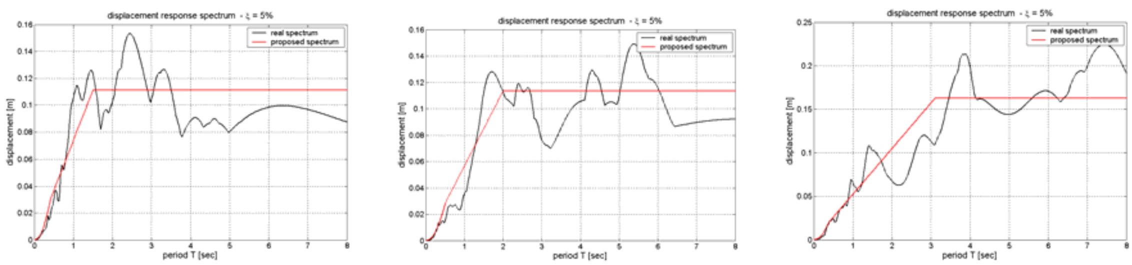


Fig.5. comparison between the displacement response spectrum of 18 records characterized by *PGA* approximately equal to 0.25 *g* and *PGV* approximately equal to 0.21 m/s, and the displacement response spectrum obtained from Eurocode 8

It is thus clear that, in order to correctly identify the seismic displacement demand, it is necessary to account for the design *PGV* at the site.
A comparison between the displacement response spectra obtained from historical records and their schematized counterparts obtained using the formulations of Eqs. (1) and (2) with the recorded *PGA*, *PGV* and *PGD* values (as unique record specific parameters characterizing each schematization) has been carried out with reference to the three selected earthquake records reported in Table 1. The results are displayed in Figs. 6.

Table 1. Earthquake ground motions considered

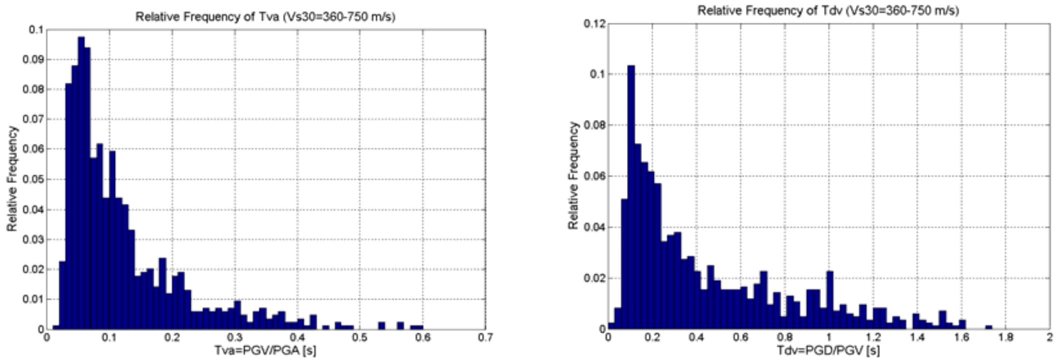
	Earthquake	Station	PGA [g]	PGV [m/s]	PGD [m]
(a)	Northridge	24389 LA-Century City	0.222	0.252	0.057
(b)	Cape Mendocino	89509 Eureka-Myrtle & West	0.154	0.2	0.06
(c)	Chi Chi Taiwan	HWA033	0.167	0.17	0.08



(a) (b) (c)

Fig. 6. Overlapping of proposed spectra on real displacement response spectra relative to the registration of Northridge (a), Cape Mendocino (b) and Chi Chi Taiwan (c).

The above results represent a clear indication that, in order to develop meaningful direct Displacement-Based Seismic Design, the seismic hazard at the site should evolve from site characterization in terms of the *PGA* only (which is more suitable for the traditional Force-Based Seismic Design) toward site characterization in terms of both the *PGA* and the *PGV*, or, as used in geophysics, in terms of the *PGA* and the *PGV/PGA* ratio, previously defined as T_{VA}. It is interesting to point out how T_{VA} corresponds, in modelling the properties of hydraulic shaking tables to the so-called “corner period”, which, together with the maximum force exerted by the actuator (which is somehow correlated to the peak table acceleration) controls the overall table performances, representing, in the performance envelope curve, the intersection between the physical limits imposed by the maximum forces imposed by the actuators and the maximum oil flow in the system. Similarly, T_{DV} corresponds to the other “corner period” (somewhat less significant) at the intersection between the physical limitations due to the oil flow and the piston stroke. It is clear how both T_{VA} and T_{DV} represent fundamental characteristics of each seismic record, which can (and must) be derived from specific Probabilistic Seismic Hazard Analysis. To elucidate how T_{VA} and T_{DV} may vary from record to record, Figs. 7 plot the Probability Density Function (PDF) of the values of T_{VA} and T_{DV}, as computed for the 344 records considered in this study. In the sample population at hand, the values of coefficients of variation estimated through statistical inference for T_{VA} and T_{DV} are large enough to indicate that T_{VA} and T_{DV} must be considered for the identification the seismic hazard.



(a) (b)

Fig.7. Relative frequency of (a) T_{VA} and (b) T_{DV}.

7. CONCLUSIONS

In this paper the authors show how to obtain in a direct way a new and simple formulation for the shape of the displacement design spectrum, starting from the well-established "tripartite" earthquake elastic response spectrum to be used in a Displacement Based Seismic Design approach.

For given soil conditions, 344 historical seismic records have been considered. First, for each record, different and several earthquake ground motion parameters have been computed and compared to each other through correlation analyses: the results indicate that the parameters *PGA*, *PGV* and *PGD* are both necessary (given that they are not correlated to each other) and sufficient (given that each one of all the other considered parameters is well correlated with at least one out of these three parameters) to exhaustively characterise the seismic input. Second, "robust" linear regressions have been performed on a large number of response spectra to derive updated values of the average amplification factors (α_A, α_V, and α_D) which characterize the form of the "tripartite" earthquake response spectrum.

From the analysis of the comparison of the results obtained, a schematization of the displacement design spectrum is proposed, which accounts for the *PGA*, the *PGV* and for the *PGD*, which is resulted to be a good proposed spectrum as if compared to real earthquake spectra. Comparing different displacement design spectra, the results show that the spectra obtained accounting only for the *PGA* values are not correct, so that the concurrent use of *PGA*, *PGV* and (if available) *PGD* values is basic for a careful schematization of the displacement design spectrum, which in turn is fundamental for the development of an accurate seismic input.

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