DEVELOPMENT OF SPONTANEOUS RUPTURE PROCESS OF A SHEAR CRACK USING SLIP-WEAKENING MODEL

Juin-Fu Chai[1] and Tsung-Jen Teng[1]

[1] *Research Fellow, Center for Research on Earthquake Engineering (NCREE), Taipei, Chinese Taiwan*
Email: chai@ncree.org.tw

ABSTRACT :

Incorporated with the hyper-singular boundary element method (HBEM), the slip-weakening model was adopted to develop the spontaneous rupture process of a shear crack. The shear crack was considered to be embedded in an infinite space (anti-plane problem), and further, the slip-weakening model was adopted to define the constitutive law by the linear relationship between the shear stress and slip within the cohesive zones. On the other hand, based on the HBEM with the developed regularization technique, the crack opening under a specified stress drop can be determined in time domain. In this study, the zero frequency approximation was considered first to solve the opening for a shear crack under the slip-weakening model. Based on the dynamic stress intensity factor, both the stress drop and the crack opening within the cohesive zones can be determined by the proposed iteration process. In addition, for the dynamic slip process, the equal rupture interval of the crack tips was defined for each rupture step, and the time that the crack tips arrive at the definite locations, as well as the associated crack opening and the stress drop, can be determined subsequently on the basis of the developed iteration process. Therefore, based on the discrete points of crack tips and associated arrival times, the dynamic slip function can be achieved piecewisely.

KEYWORDS: dynamic slip function, HBEM, slip-weakening model, stress intensity factor

1. INTRODUCTION

Near-fault ground motions, which have created severe damages in recent disastrous earthquakes, are characterized by a short-duration impulsive motion that will transmit large energy into the structures at the beginning of the earthquake. The ground velocity pulse is resulted from the slip pulse on a fault plane owing to the healing process. The detail of possible mechanism to cause the slip pulse can be found in Zheng and Rice (1998) and Nielsen and Carlson (2000). Based on the source mechanism combined with the asperity or barrier model as well as the associated parameters, the rupture processes of nuclear, its propagation, and then healing can be defined well.

The crack rupture models can be classified into self-similar and spontaneous models by the rupture velocities of crack's tips. The rupture velocity of a crack's tip is one of the specified source parameters for the self-similar model. However, for the spontaneous rupture model, the rupture velocities are not specified initially but should be determined by the rupture criteria during the rupture process. The rupture model of a crack with cohesive zones can be recognized as a spontaneous rupture model. Based on the specified constitutive laws for the cohesive zones, the stress is dependent on the crack opening or the slip velocity in the cohesive zones and the stress at the crack tip is finite and is equal to the shear strength. In addition, the width of the cohesive zone is not constant but it changes when the crack tips propagate during the rupture process.

In this study, a shear crack was considered to be embedded in an infinite space (anti-plane problem), and, furthermore, the slip-weakening model (Andrews, 1976) was adopted to define the constitutive law such that the shear stress within the cohesive zone is dependent linearly on the slip. On the other hand, the hyper-singular boundary element method (HBEM) in the time domain was used to determine the crack opening under a specified stress drop. Likely, the iteration process which is based on the dynamic stress intensity factor was developed and the spontaneous rupture process can be achieved to determine the dynamic slip function of cracks.

2. SLIP-WEAKENING MODEL

As shown in Fig. 1, the crack with width of 2a was considered in an infinite elastic medium, and the initial stress before slip is τ_i. After the crack slipped, the central area with slip larger than D_c ($w>D_c$) was taken for the complete opening and the final stress at this area was defined by the dynamic friction stress τ_d. The final stress at the crack tips with zero opening was defined by the finite static friction stress τ_s that is coincidently equal to the shear strength. On the other hand, the area with $w\le D_c$ is recognized as the cohesive zone, and according to the slip-wakening model, the stress drop $\Delta\tau$ in cohesive zone can be defined by

$$\frac{\Delta\tau}{\tau_s-\tau_d}=1-\gamma-\frac{w}{D_c} \tag{1}$$

where D_c is the slip at the boundary of cohesive zone with $x=\pm x_c$, and the parameter γ is defined by

$$\gamma=\frac{\tau_i-\tau_d}{\tau_s-\tau_d} \tag{2}$$

Figure 1 shows the example with γ=0.4 where μ is the shear modulus of the elastic medium.

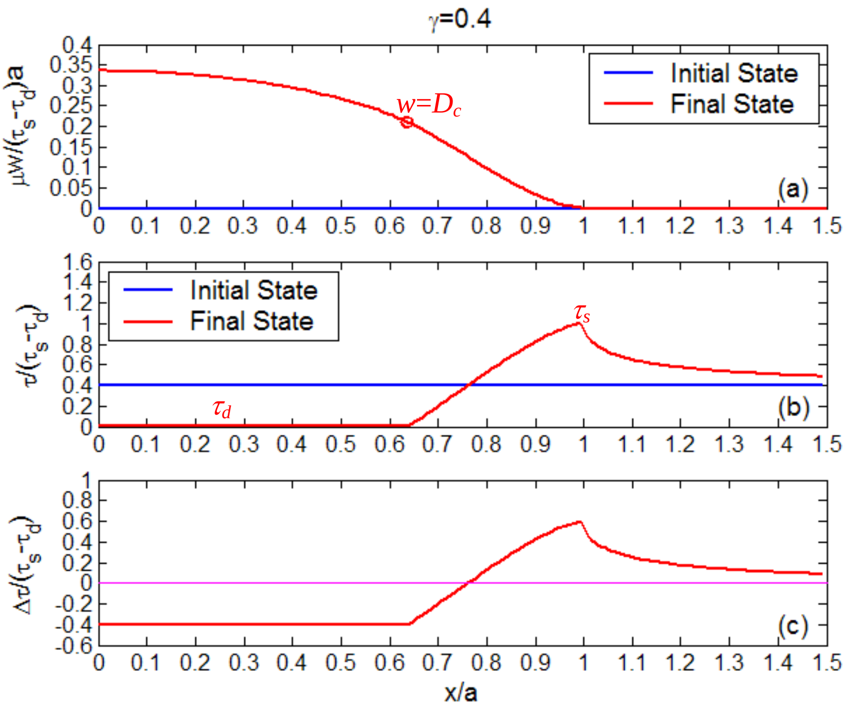

Figure 1 The (a) crack opening, (b) stress distribution, and (c) stress drop for the slip-weakening model.

3. CRACK OPENING UNDER ZERO FREQUENCY APPROXIMATION

The HBEM has been developed in the frequency domain to determine the dynamic slip function for a shear crack under constant stress drop (Chai and Teng, 2006), and owing to the zero frequency approximation, the

HBEM can be adopted to solve the static solution of a crack under a specified stress drop. Then, together with the slip-weakening model, the crack opening and the stress drop will be interacted with each other. The crack opening was determined by the HBEM under specified stress drop, and further, the stress drop was defined according to the constitutive law stated in Eq. (1) for the cohesive zone.

In this study, the zero frequency approximation was considered to solve the opening for a shear crack under the slip-weakening model. However, in order to apply the HBEM to determine the crack opening, all of the stress drop at the collocation points within the slip area ($|x|\le a$) should be positive or negative. Hence, as shown in Fig. 2, the stress drop within the slip area should be separated into two terms and be expressed by $\Delta\tau = \Delta\tau_A + \Delta\tau_B$, while $\Delta\tau_A$ and $\Delta\tau_B$ being defined by:

$$\begin{aligned} &\frac{\Delta\tau_A}{\tau_s-\tau_d} = -\gamma \quad ; \quad (|x|\le a) \\ &\frac{\Delta\tau_B}{\tau_s-\tau_d} = \begin{cases} 0 & ; \quad (|x| < x_c) \\ 1 - w/D_c & ; \quad (x_c \le |x| \le a) \end{cases} \end{aligned} \tag{3}$$

It can be found that $\Delta\tau_A$ is a constant stress drop, and then, straightforwardly, the associated crack opening w_A and the stress intensity factor K_A^{III} can be determined by HBEM. However, the stress drop $\Delta\tau_B$ is dependent on the boundary of cohesive zone and the crack opening within the area. In this study, based on the determined w_A and K_A^{III}, the iteration process as shown in Fig. 3 was developed to determine both the stress drop $\Delta\tau_B$ and the crack opening w_B.

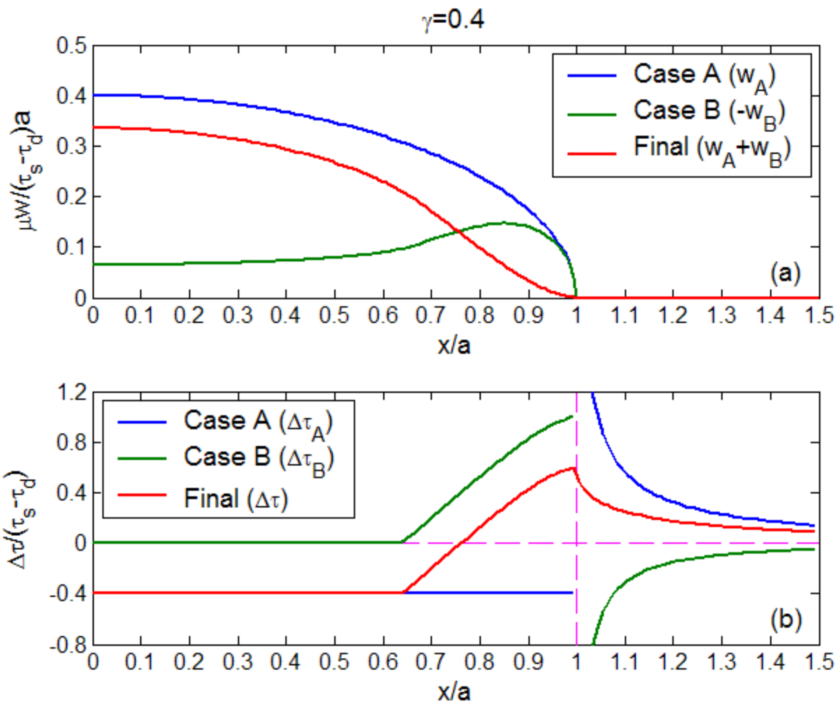

Figure 2 The (a) crack opening and (b) stress drop for the separated stress drops of $\Delta\tau_A$ and $\Delta\tau_B$

The iteration process will stop if the determined boundary of the cohesive zone at one iteration step is the same as that determined in the previous step, and the final crack opening can be defined by $w=w_A+w_B$. Subsequently, based on the similar matrix form as expressed in the HBEM to solve the crack opening under specified stress drop, both the stress drops corresponding to w_A and w_B near the crack tips with $|x_p|>a$ can be solved respectively, and the summation results in the total stress drop. Figure 2 shows the results and it can be observed that the crack opening w_A due to the constant stress drop will cause an infinite stress drop near the crack's tip ($|x_p|>a$). However, based on the additional stress drop caused by w_B under the slip-weakening model, the finite total stress drop can be obtained near the crack's tip while the total stress is continuous at this area.

4. FINAL SLIP DUE TO RUPTURE OF CRACK TIPS

A crack with initial width of $2a_0$ was assumed to rupture and the final width becomes $2a$. Then, based on the slip-weakening model, the final slip can be determined using the HBEM as well as the separation of the stress drop within the cohesive zone. For an example with γ=0.4 and a_0/a=0.83, the crack opening w_0 and stress drop $\Delta\tau_0$ at the initial state can be solved initially as shown in Fig. 2. Similarly, the stress drop from the initial state to the final state can be separated into two terms, namely, $\Delta\tau_A$ and $\Delta\tau_B$ (cohesive zone), and they are defined by the following:

$$\begin{aligned} \Delta\tau_A/(\tau_s-\tau_d) &= -\gamma-\Delta\tau_0/(\tau_s-\tau_d) \\ \Delta\tau_B/(\tau_s-\tau_d) &= 1-w/D_c \end{aligned} \tag{4}$$

It can be found from Eq. (4) that the combination of $\Delta\tau_A$ and the initial stress drop $\Delta\tau_0$ will result in the same effect of a constant stress drop. Similarly, the case of stress drop $\Delta\tau_A$ was considered in the beginning and the associated crack opening w_a and the stress intensity factor K_A^{III} were determined by the HBEM. Afterward, the stress drop $\Delta\tau_B$ and the crack opening w_B were obtained using the same iteration process as shown in Fig. 3. The determined crack opening and the stress drop for the individual state are shown in Fig. 4.

5. DYNAMIC SLIP FUNCTION

In this study, the fixed rupture interval of the crack tips was defined for each rupture step, and the time that the crack tips arrive at the definite locations, as well as the associated crack opening and the stress drop, can be determined subsequently. Therefore, by the connection of the determined discrete time points, the dynamic slip function was achieved piecewisely.

Considering an existing crack with width of $2a_0$, there were N_0 collocation points located on the crack surface with an interval of $\Delta x=2a_0/N_0$, and the crack opening w_0, critical opening of cohesive zone D_c, and the stress drop $\Delta\tau_0$ was determined for the case of γ=0.5. The crack was assumed to release the stress at time T_0 and begin to rupture until the crack tips arrived at $x=\pm a_1$. There will be N_1 collocation points ($N_1=N_0+2$) with $a_1/a_0=N_1/N_0$ to keep the same interval Δx. The initial guess of the total stress drop $\Delta\tau_1$ as the crack tips rupture from $\pm a_0$ to $\pm a_1$ is defined from $\Delta\tau_0$ following the rule that $\Delta\tau_0$ is shifted toward both tips by a collocation point to keep the same shape and width of cohesive zone, and the stress drop of $-\gamma$ was defined for the central points. Based on the Laplace transformation, the stress drop $\Delta\bar{\tau}_1$ in the frequency domain can be separated into two terms, and they are defined as follows:

$$\begin{aligned} \frac{\Delta\bar{\tau}_{A1}}{\tau_s-\tau_d} &= \left(-\gamma-\frac{\Delta\tau_0}{\tau_s-\tau_d}\right)\cdot\frac{\exp[-(A+i\omega)T_0]}{A+i\omega} \\ \frac{\Delta\bar{\tau}_{B1}}{\tau_s-\tau_d} &= \left(\Delta\tau_1+\gamma+\frac{\Delta\tau_0}{\tau_s-\tau_d}\right)\cdot\frac{\exp[-(A+i\omega)T_0]}{A+i\omega} \end{aligned} \tag{5}$$

Therefore, based on the HBEM, the associated crack opening can be determined in the frequency domain, and then be transferred to time domain to achieve the time histories of crack opening $w_1(t)$ and the stress intensity factor $K^{III}(t)$. However, only the condition that the stress intensity factor vanishes can satisfy the slip-weakening model, and hence the time when K^{III} vanishes can be taken for the exact time T_1 that the crack tips arrive at

$x=\pm a_1$ under the initial guess of $\Delta\tau_1$. Then, based on the crack opening w_1 at T_1, as well as the critical opening D_c for the cohesive zone, the more accurate stress drop $\Delta\tau_1$ can be defined by the constitutive laws as given in Eq. (1). After that, the stress drop $\Delta\tau_1$ can be substituted into Eq. (5) and the associated time histories of $w_1(t)$ and $K^{\mathrm{III}}(t)$ can be determined. Then, the revised T_1 can be obtained by the time that K^{III} vanishes, thus, the more accurate stress drop $\Delta\tau_1$ can be expressed again by w_1 at T_1. Finally, based on the proposed iteration process, the time T_1 that the crack ruptures to $x=\pm a_1$ can be solved and the associated crack opening w_1, total stress drop $\Delta\tau_1$, and the boundary of cohesive zone x_{c1} can then be derived.

Initial Guess (linear stress drop)

1. for a trial boundary of cohesive zone x_c
2. $\Delta\tau_B/(\tau_s-\tau_d)=(x-x_c)/(a-x_c)$
3. determine w_B by HBEM
4. determine K_B^{III} $\Rightarrow$ check $K_A^{III}+K_B^{III}=0$? (No → back to 1)
5. determine crack opening $w_1=w_A+w_B$

The i^{th} iteration ($i\geq1$)

1. for a trial boundary of cohesive zone x_c
2. based on $w_i \Rightarrow$ determine $D_c=w_i(x_c)$
3. $\Delta\tau_B/(\tau_s-\tau_d)=1-w_i/D_c$
4. determine w_B by HBEM
5. determine K_B^{III} $\Rightarrow$ check $K_A^{III}+K_B^{III}=0$? (No → back to 1)
6. crack opening $w_1=w_A+w_B$

Figure 3 The iteration process to determine the stress drop $\Delta\tau_B$ and the crack opening w_B.

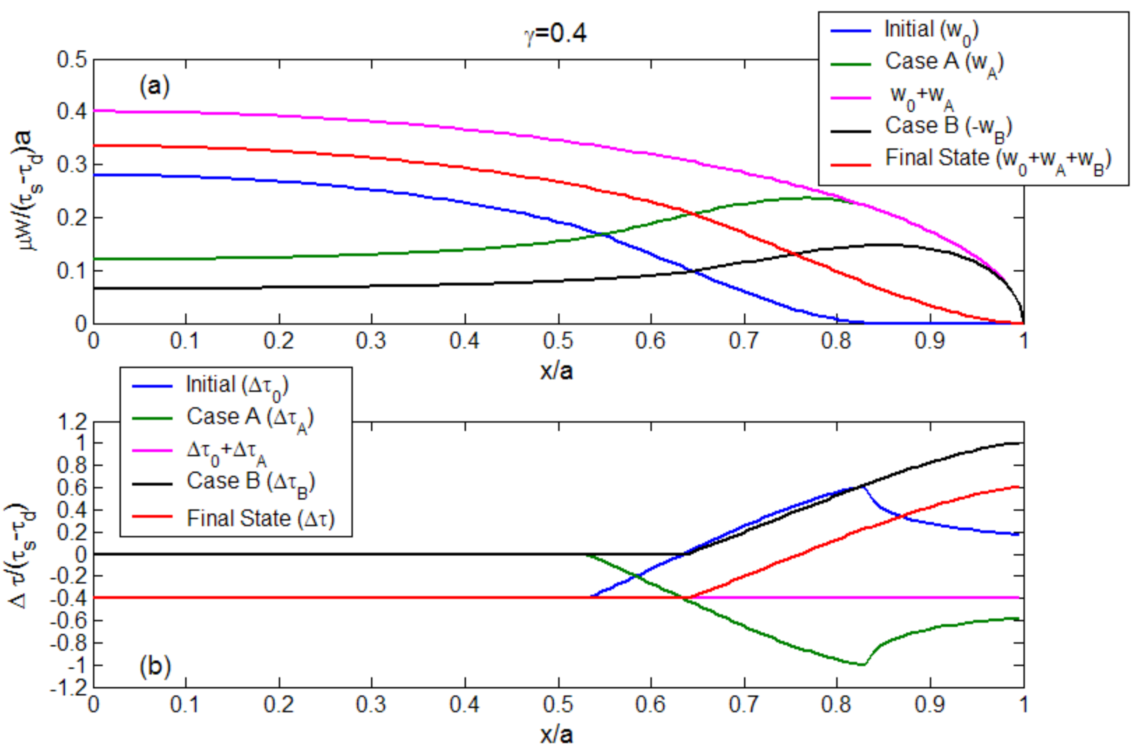

Figure 4 The (a) crack opening and (b) stress drop for the individual state as the crack ruptures with width from $2a_0$ to $2a$.

When the crack tips rupture to $x=\pm a_1$ at the time T_1, the crack releases the stress immediately and begins to rupture to $x=\pm a_2$. Similarly, there will be N_2 collocation points ($N_1=N_0+4$) with $a_2/a_0=N_2/N_0$ to keep the same interval Δx. The initial guess of the total stress drop $\Delta\tau_2$ as the crack tips rupture from $x=\pm a_1$ to $x=\pm a_2$ can be defined from $\Delta\tau_1$ by the same above rule. Therefore, the stress drop $\Delta\bar{\tau}_2$ in the frequency domain can be separated into two terms, and can be expressed mathematically as,

$$\begin{aligned}\frac{\Delta\bar{\tau}_{A2}}{\tau_s-\tau_d}&=\frac{\Delta\bar{\tau}_{A1}}{\tau_s-\tau_d}+\left(-\gamma-\frac{\Delta\tau_1}{\tau_s-\tau_d}\right)\cdot\frac{\exp[-(A+i\omega)T_1]}{A+i\omega}\\ \frac{\Delta\bar{\tau}_{B2}}{\tau_s-\tau_d}&=\frac{\Delta\bar{\tau}_{B1}}{\tau_s-\tau_d}+\left(\Delta\tau_2+\gamma+\frac{\Delta\tau_1}{\tau_s-\tau_d}\right)\cdot\frac{\exp[-(A+i\omega)T_1]}{A+i\omega}\end{aligned} \tag{6}$$

Based on the aforementioned iteration process, the time T_2 that the crack tips ruptures to $x=\pm a_2$ can be determined, and also the associated crack opening w_2, total stress drop $\Delta\tau_2$ and the boundary of cohesive zone x_{c2}.

Subsequently, the crack tips will rupture to $x=\pm a_n$ with N_n collocation points ($N_n=N_0+2n$) to keep the same interval Δx. Based on the same iteration process, the time T_n that the crack tips arrive at $x=\pm a_n$ can be solved, and the associated crack opening w_n, total stress drop $\Delta\tau_n$, and the boundary of cohesive zone x_{cn} will follow. The propagation of crack tips, the boundary and width of the cohesive zone, and the rupture velocity during the rupture process are shown in Fig. 5, and Figure 6 shows the spontaneous dynamic slip function under the consideration of slip-weakening model.

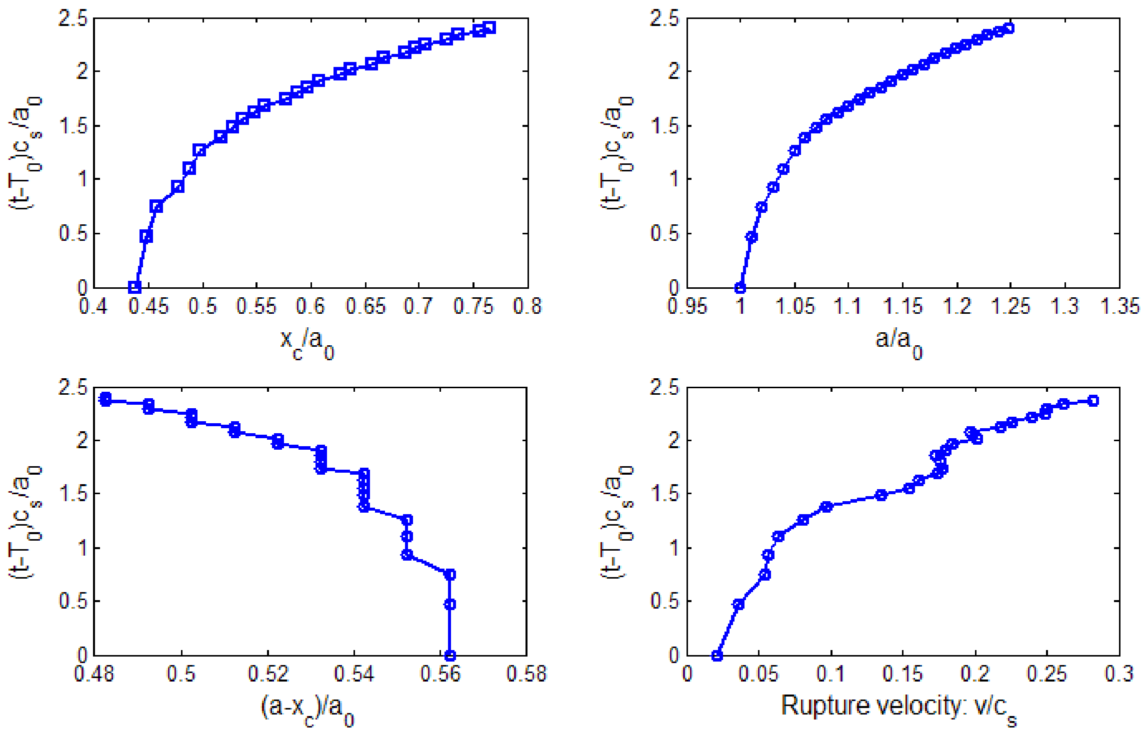

Figure 5 The propagation of crack tips, boundary and width of the cohesive zone and the rupture velocity during the rupture process

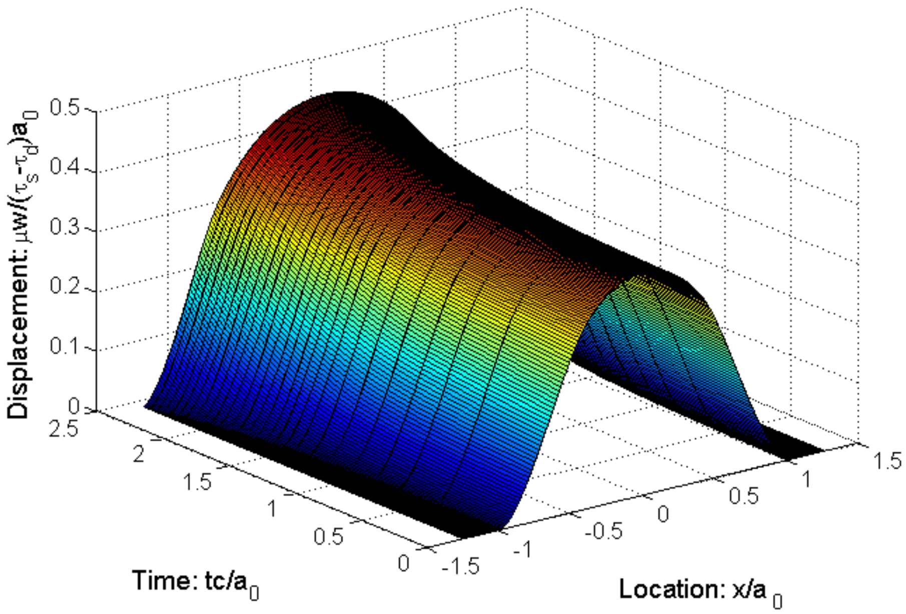

Figure 6 The spontaneous slip function under the consideration of slip-weakening model

6. CONCLUSIONS

In this study, the slip-weakening model as well as the HBEM was adopted to develop the spontaneous slip function of a shear crack. The time that the crack tips arrive at the specified locations can be determined using the proposed iteration process, and, based on the connection of the determined discrete time points, the dynamic slip function was achieved piecewisely.

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