ON EARTHQUAKE RECURRENCE TIME DISTRIBUTION AND ITS SCALING LAW

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ABSTRACT :

The robustness of earthquake recurrence time distribution scaling in a given space-time window is studied in details, using earthquake catalogs from different part of the world (Southern California, Japan and Turkey). The quality of the available catalogs is examined taking into account the completeness of the magnitude and the effective starting time of aftershock sequences. The adjustment to a doubly power law of the distribution scaled with the mean recurrence time reveals that short and long time range power law exponents obey a simple equation, with parameters linked to the recurrence time probability at the distribution tails. The derived equation shows that, big events occurring after seismically quiescent periods and foreshocks preceding large events are linked in a single quantified cause-effect structure.

KEYWORDS: Recurrence times, Scaling law, Universality, Power law, Earthquake catalog

1. INTRODUCTION

Earthquake recurrence time distribution *D* has been found to obey approximate power laws (PLs) with two regimes for short and large recurrence times respectively (Bak et al. 2002, Christensen et al. 2002, Corral 2003, 2004, 2007). These PL asymptotic had been intensively studied to check their consistency and to look for new insights they provide for modeling earthquake occurrence process (Davidsen and Goltz, 2004; Carbone et al., 2005; Lindman et al., 2005, 2006; Molchan, 2005; Corral and Christensen, 2006; Hainzl et al., 2006; Saichev and Sornette, 2006, 2007; Molchan and Kronrod, 2007).

In this study, we aim to test the universality of these PL scaling for broad areas and different magnitudes thresholds, especially when additional attention is given to data quality and catalog parameters estimation as completeness magnitudes and effective starting times of aftershock sequences. Additional attention is also given to sampling of recurrence times, in particular using nonparametric techniques.

First, catalog data for Southern California, Japan and Turkey are collected and prepared for processing. Their quality is enhanced by the selection of completeness periods and magnitudes and the effective starting time of aftershock sequences included in the data. Secondly, recurrence times are intensively sampled using a network of target disks covering the studied area. Finally, the obtained samples are mixed to estimate the distribution and its parameterization under the hypothesis of two PL asymptotic. The results show high fluctuations in the PL parameters obtained with different magnitude cutoffs and from different regions. Finally, we establish a balance equation formalizing equilibrium between short and long time range earthquake processes.

2. DATA AND METHODS

Our Analysis used data from Southern California, Japan and Turkey. Southern California catalog files were provided by Southern California National Network via the link http://www.data.scec.org/ftp/catalogs/SCSN/. Files covering the time period 1932-2005 were compiled together with Kagan catalog available at http://moho.ess.ucla.edu/~kagan/s_cal.dat (Kagan et al., 2006). The space window 32-37º N latitude and 122-114º W longitude has been considered. For Japan, the JMA catalog covering the period 1923-2005 was obtained from the Seismological and Volcanological Bulletin of Japan for November 2005. It was compiled for the period 679-1922 using Utsu catalog (www5b.biglobe.ne.jp/~t-kamada/CBuilder/eqlist.htm). The resulting catalog covers 679-2005 and spans in the space window 24-50º N latitude and 122-152º E longitude. Turkey data were downloaded from Kandilli Observatory website at the link http://www.koeri.boun.edu.tr/sismo/veri_bank/mainw.htm. The obtained catalog covers the space window 34º N-44º N latitude and 24º E-46º E longitude.

Different completeness periods together with the corresponding completeness magnitudes were selected using the maximum curvature method MAXC (Wiemer and Wyss 2000). Then, recurrence times were sampled accordingly using the earthquake random sampling ERS algorithm (Talbi and Yamazaki 2007, 2008). Table 1 below lists the sampling schemes used,

Table 1 Parameters of sampling schemes used in this study. m_c^l, *N*, *R* denotes the completeness magnitude, the number of events and the sampling radius, respectively

Scheme l	Time period	m_c^l	$N = N\left(M \geq m_c^l\right)$	*R* [km]
Southern California				
1	1990-2005	2.5	21257	50
2	1947-2005	3.5	6006	50
3	1932-2005	4.7	556	50
Japan				
1	1990-2005	3.5	37352	50
2	1975-2005	4.5	11406	50
3	1923-2005	5.5	3664	100
4	1890-2005	6.5	590	200
Turkey				
1	1988-2004	3.5	4472	50
2	1921-2004	4.5	2185	100
3	1900-2004	5.5	488	200

Since many events are unreported just after big events (Kagan 2004), it is important to consider recurrence times above the effective starting time τ_c of aftershock sequences present in our data. The following empirical relation linking τ_c to the main shock magnitude m_0 and the magnitude cutoff m_c was reported by Helmstetter et al (2006).

$$\log_{10}\left(\tau_c\right) = \frac{m_0 - m_c - 4.5}{0.75} \tag{1}$$

where τ_c is the recurrence time cutoff, m_0 the mainshock magnitude and m_c the threshold magnitude.

The recurrence time cutoff τ_c = 0.2 days we adopted, corresponds to m_0 = 7.5 and m_c = 3.5 in the former equation. Recurrence times below this level were discarded from the analysis.

Next, recurrence times are scaled by the inverse of their mean $1/\bar{\tau}$, according to the following scaling relation,

$$\bar{\tau}\, D\left(\tau\right) \approx \phi\left(\tau/\bar{\tau}\right) \tag{2}$$

We propose here to test the null hypothesis H_0: "*D* fits to a doubly PL", that is *D* scale as in Eqn. (2) with,

$$\phi(t) = \begin{cases} c_1/t^{p_1} & t < \bar{\tau} \\ c_2/t^{p_2} & t \geq \bar{\tau} \end{cases} \tag{3}$$

where c_1, c_2, p_1, p_2>0.

For $\tau > \bar{\tau}$, It follows from Eqn. (2) and the proprieties of the distribution D that (Talbi and Yamazaki 2008),

$$1 - F_\tau(t) \approx c_2 \bar{\tau}^{p_2 - 1} \frac{t^{1-p_2}}{p_2 - 1} \tag{4}$$

Where F_τ is the cumulative distribution function associated to D. The constant c_2 is obtained for $t_1 = \bar{\tau}\, e^{\frac{\log(p_2 - 1)}{1-p_2}} = \bar{\tau}(p_2 - 1)^{\frac{1}{1-p_2}}$ with p_2>1, which corresponds to 80-85% the mean waiting time (MWT) for the data listed in Table 1. c_2 is simply a rough approximation of the proportion of recurrence times exceeding the level t_1. For $t = \bar{\tau}$, Eqn. (4) gives the probability for recurrence times to exceed the mean value $\bar{\tau}$,

$$P(\tau > \bar{\tau}) = 1 - F_\tau(\bar{\tau}) \approx \frac{c_2}{p_2 - 1} \tag{5}$$

where p_2>1.

Similar relations are obtained for $\tau \leq \bar{\tau}$ (Talbi and Yamazaki 2008),

$$F_\tau(t) \approx c_1 \bar{\tau}^{p_1 - 1} \frac{t^{1-p_1}}{1 - p_1} \tag{6}$$

The constant c_1 is obtained for $t_0 = \bar{\tau}\, e^{\frac{\log(1-p_1)}{1-p_1}} = \bar{\tau}(1 - p_1)^{\frac{1}{1-p_1}}$ with p_1<1, which corresponds to less than 3% the MWT for the data listed in Table 1. c_1 is the estimated proportion of events below the level t_0. For $t = \bar{\tau}$, the last equation gives the probability that a given recurrence time do not exceed the mean value $\bar{\tau}$,

$$P(\tau < \bar{\tau}) = F_\tau(\bar{\tau}) \approx \frac{c_1}{1 - p_1} \tag{7}$$

Finally, combining equations (5) and (7) shows,

$$\varphi(\underline{c}, \underline{p}) = \frac{c_1}{1 - p_1} + \frac{c_2}{p_2 - 1} \approx 1 \tag{8}$$

with $c = (c_1, c_2)$ and $p = (p_1, p_2)$.

The former equation describes the link between the PL behavior at short and long time ranges.

3. RESULTS

Former PL parameters were estimated using the stack of the distributions corresponding to the schemes listed in

Table 1(Figure 1a-c). The obtained results are plotted in Figure 1d. The first PL is clearly instable, whereas the fluctuations of the second PL parameters estimates do not allow significant conclusions. The corresponding estimates of φ (not shown here) depart significantly from one at small magnitudes, showing that the scaling is broken; whereas it decreases to one with increasing magnitudes (Talbi and Yamazaki 2008).

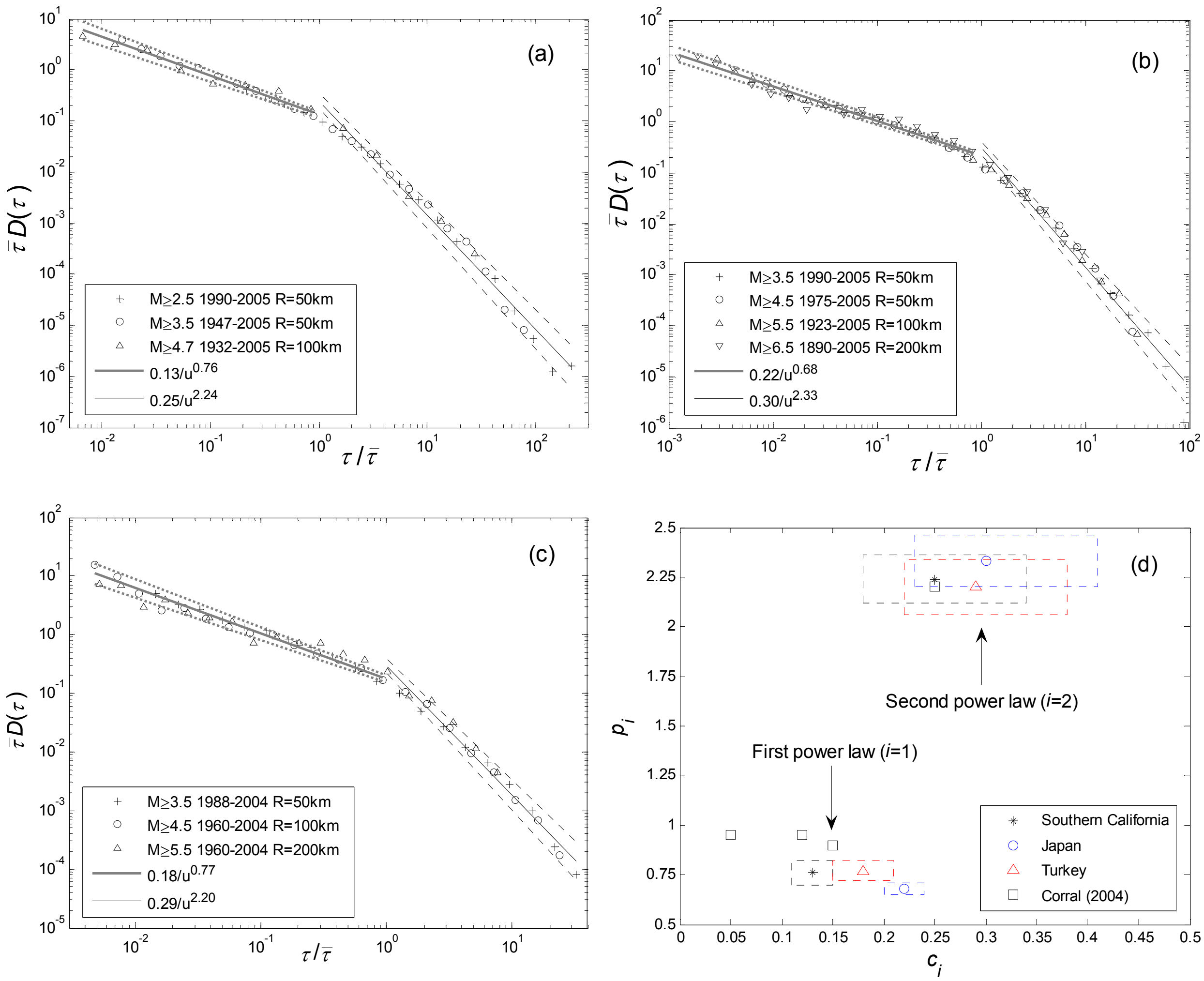

Figure 1. Scaled distributions of interevent times with different magnitudes and different sampling radiuses, for (a) Southern California, (b) Japan and (c) Turkey. *Solid lines* are the linear regressions whereas *dashed lines* are 95% confidence limits. (d) Plot of the PL exponent parameter estimates obtained in our study and those by Corral (2004). *Dashed rectangles* are 95% confidence limits

In the case of Japan and Southern California, Talbi and Yamazaki (2008) showed that c_2 and p_2 takes quite stable values, whereas c_1 is increasing at large magnitudes and p_1 decreasing with magnitudes. However, because of the poor data on the distribution tails, c_1 and c_2 cannot be estimated accurately. For short and moderate magnitudes, c_1 and c_2 can be considered rather stable (Talbi and Yamazaki 2008), so that p_1 and p_2 are decreasing (p_2 is decreasing because p_1 decreases and the equilibrium (8) holds). In other words, slowing the decrease at long time ranges results in slowing the decrease at short ranges $p_2{\downarrow}\Rightarrow p_1{\downarrow}$ and the opposite.

Let us consider the evolution of seismicity in time for a given area. Roughly speaking, if large recurrence times ($\tau > \bar{\tau}$) increases, the long time range decrease is slower ($p_2\downarrow$ according to Eqn. (5)) and the same situation should be observed at short time ranges ($p_1\downarrow$). Eventually, a series of correlated events close in time are predicted by the equilibrium in Eqn. (8). Typical situation is described by big events occurring after seismically quiescent period, and followed by aftershocks. The opposite situation (p_1↓⇒ p_2↓) occurs when one or more foreshocks precede a big event. Both processes described by short and long recurrence times are explained inversely as cause-effect.

4. CONCLUSIONS

Power law (PL) estimates of earthquake recurrence time distributions were refined by introducing first, a preliminary study of completeness magnitudes and effective starting times of aftershock sequences included in the catalogs; and secondly by adopting an intensive sampling strategy of recurrence times. Ten sampling schemes corresponding to different regions, magnitudes and sampling parameters were selected and analyzed. Results show that the first PL behavior occurs with different exponents within each region, whereas significance of the second PL behavior is limited by poor data and high sampling fluctuations.

The obtained power law scaling reveals a simple correlation between short and long time ranges. We established an equilibrium equation describing this correlation and explaining the causality foreshocks-mainshock -aftershocks. The analysis shows potential applications in seismic hazard and earthquake prediction.

REFERENCES

Bak, P., Christensen, K., Danon, L. and Scanlon, T. (2002). Unified scaling law for earthquakes. *Phys Rev Lett* **88**, DOI10.1103/PhysRevLett.88.178501.

Carbone, V., Sorriso-Valvo, L., Harabaglia, P. and Guerra, I. (2005). Unified scaling law for waiting times between seismic events. *Europhys Lett:* **71**, 1036–1042, DOI 10.1209/epl/i2005-10185-0.

Christensen, K., Danon, L., Scanlon, T. and Bak, P. (2002). Unified scaling law for earthquakes. *Proc Natl Acad Sci USA* **99**, 2509–2513.

Corral, A. (2003). Local distributions and rate fluctuations in a unified scaling law for earthquakes. *Phys Rev E* **68**, DOI 10.1103/PhysRevE.68.035102.

Corral, A. (2004). Universal local versus unified global scaling laws in the statistics of seismicity. *Physica A* **340**, 590–597. DOI 10.1016/j.physa.2004.05.010.

Corral, A. (2007). Statistical features of earthquake temporal occurrence. *Lecture notes in physics* **705**, 191–221, DOI 10.1007/3-540-35375-5_8, Springer, Berlin Heidelberg.

Corral, A. and Christensen, K. (2006). Comment on "Earthquakes descaled: on waiting time distributions and scaling laws". *Phys Rev Lett* **96**, DOI 10.1103/ PhysRevLett.96. 109801.

Davidsen, J. and Goltz, C. (2004). Are seismic waiting time distributions universal? *Geophys Res Lett* **31:21:**L21612, DOI 10.1029/2004GL020892.

Hainzl, S., Scherbaum, F. and Beauval, C. (2006) Estimating background activity based on interevent-time distribution. *Bull Seismol Soc Am* **96:1**, 313–320.

Helmstetter, A., Kagan, Y.Y. and Jackson, D.D. (2006) Comparison of short-term and time-independent earthquake forecast models for Southern California. *Bull Seismol Soc Am* **96:1**, 90–106.

Kagan, Y.Y. (2004) Short-term proprieties of earthquake catalogs and models of earthquake source. *Bull Seismol Soc Am* **94:4**, 1207–1228.

Kagan, Y.Y., Jackson, D.D. and Rong, Y.F. (2006) A new catalog of Southern California earthquakes, 1800-2005. *Seismol Res Lett* **77:1**, 30–38.

Lindman, M., Jonsdottir, K., Roberts, R., Lund, B., Bodvarsson, R. (2005) Earthquakes descaled: on waiting time distributions and scaling laws. *Phys Rev Lett* **94**, DOI 10.1103/ PhysRevLett.94.108501.

Lindman, M., Jonsdottir, K, Roberts, R., Lund, B., Bodvarsson, R. (2006) Earthquakes descaled: on waiting time distributions and scaling laws (Reply). *Phys Rev Lett* **96**, DOI 10.1103/PhysRevLett.96.109802.

Molchan, G. (2005) Interevent time distribution in seismicity: a theoretical approach. *Pure Appl Geophys* **162**, 1135–1150.

Molchan, G. and Kronrod, T. (2007) Seismic interevent time: a spatial scaling and multifractality. *Pure Appl Geophys* **164**, 75–96, DOI 10.1007/s00024-006-0150-y.

Saichev, A. and Sornette, D. (2006) “Universal” distribution of interearthquake times explained. *Phys Rev Lett* **97**, DOI 10.1103/PhysRevLett.97.078501.

Saichev, A. and Sornette, D. (2007) Theory of earthquake recurrence times. *J Geophys Res* **112:**B04313, DOI 10.1029/2006JB004536.

Talbi, A. and Yamazaki, F. (2007) Earthquake waiting time distribution: modeling and scaling law. In: Abstracts of *Japan Geosciences Union meeting*, Chiba, Japan, 19–24 May 2007.

Talbi, A. and Yamazaki, F. (2008) Sensitivity analysis of the parameters of earthquake recurrence time power law scaling. *Journal of Seismology* DOI 10.1007/s10950-008-9115-1.

Wiemer, S. and Wyss, M. (2000) Minimum magnitude of completeness in earthquake catalogs: examples from Alaska, the western Unites States, and Japan. *Bull Seismol Soc Am* **90:4**, 859–869.