A GENERAL BOUNDARY CONDITION IN 3D HYBRID WAVE INJECTION MODELING BASED ON ALTERMAN AND KARAL (1968) METHOD

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ABSTRACT:

This paper describes the generalized hybrid approach of wave injection for elastic linear case originated by Alterman and Karal (AK) in 1968 as a domain coupling technical algorithm. The general approach is described by binding two sub-volumes treated per partes by arbitrary wave-propagation methods. The generalized AK two-step procedure possibly combines the source and path effects computed by one arbitrary method and local site effects computed by another (arbitrary) method using the first method's wave field as input. The advantage of the approach arises from the fact that the connection between the methods keeps the formal wave-injection boundary perfectly permeable for any part of the wave field and can be applied to a variety of hybrid formulations. This hybrid approach leads to more effective modelling of combined source, path and site effects (e.g., by multiple second step computations with varying structure, using single first step input), saving computer memory and time. The main innovation of the paper is the generalization of the boundary condition acting between two complementary sub-volumes of AK hybrid wave-propagation methods.

KEYWORDS: Hybrid methods, seismic waves modelling, wave-field injection, source box method, DRM (domain reduction method), parameter studies, complex local structures.

1 INTRODUCTION

Recently, the hybrid modelling in 3D wave-propagation methods has grown in its importance. Combining the methods is often necessary due to limited computational and temporal capacity. Hybrid wave propagation modelling in seismology takes advantage of methods, used per partes, accounting for complete source-path-site effects. The main reasons for such and approach are not only the combination of the advantages of the methods, but also increasing the efficiency of computationally sophisticated, or even un-computable problems (in terms of all-in-one computational demands). This applies also for the cases where lower-order methods are used at a free-surface vicinity (Hestholm, 2006) since the numerical dispersion of Rayleigh waves can be reduced only by finer spatial sampling, not by increasing the order of the method (Xu et al., 1999). A number of various combined approaches is mentioned at, for example; Moczo et al. (2007a,b), Opršal et al. (2002) and Opršal & Zahradník (2002).

Here we concentrate on the wave-field injection approach probably introduced in the pioneering work of Alterman & Karal (1968), where separate computations are done due to a source singularity in finite-difference (FD) computations. The singularity was avoided by computing the exact wave field for the source in a full space and by computation of the wave propagation in the original volume subdivided into a part enveloping the original source sub-volume and its complement. Different wave fields, satisfying the same elastodynamic equation, were computed in each of the complements, with special FD coupling on boundary between them.

The objective of this paper is twofold: (i) to formulate a general condition on the boundary between the two complementary parts of the wave propagation volume and, (ii) using the boundary condition, to discuss various specific implementations.

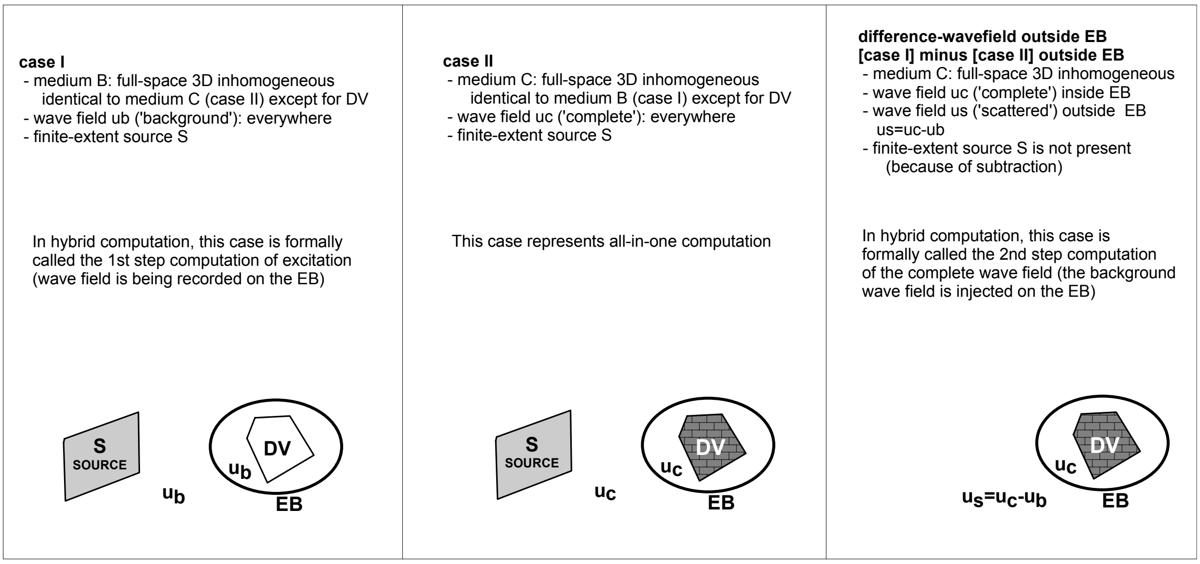

Figure 1: Two wave fields (cases I and II) generated for various media. In case I (left panel), the source S in full-space 3D inhomogeneous medium B generates so-called 'background' displacement u_b. The same source S in full-space 3D inhomogeneous medium C (case II, mid panel) generates so-called 'complete' displacement u_c. The excitation box EB, difference volume DV and sources in cases I and II are identical. The DV is enclosed by the EB and includes all portions of the full space, where medium B may differ from medium C; hence media B and C are identical outside the DV. The source S is placed outside the EB. The hybrid wave field (right panel) is a difference between cases I and II outside of the EB ($\vec{u}_s = \vec{u}_c - \vec{u}_b$) with the source term diminished, while the complete wave field $\vec{u}_c$ of the case II is kept inside the EB; the structure is C (case II) everywhere.

2 THE METHOD

2.1. *General Conditions for Elastic Linear Case*

Let B and C be the full space medium (see Fig. 1). Let $\vec{u}_b$ be displacement wave field (called the background wave field) due to the source S in full-space medium B (case I) and $\vec{u}_c$ be the displacement wave field (called the complete wave field) due to the same source S placed in full-space medium C (case II). Let the excitation box (EB) fully enclose a difference volume (DV); the DV contains all portions of the full space, where medium C may differ from medium B (hence media B and C are identical elsewhere); the sources S, position of the EB and the DV being identical in cases I and II.

Let scattered wave field $\vec{u}_s$ be given by difference between wave fields in case I and II (see Fig. 2):

$$\vec{u}_s = \vec{u}_c - \vec{u}_b \ . \tag{2.1}$$

Let wave fields $\vec{u}_b$ and $\vec{u}_c$ be solutions to the same elastodynamic equation. Then $\vec{u}_s$ is a solution to that elastodynamic equation as well. A very important implication of this fact is that the EB is not a boundary for any of the three wave fields ($\vec{u}_b, \vec{u}_c, \vec{u}_s$), thus these propagate through the EB freely.

Hooke operator, applied to displacement, writes:

$$H(\vec{u}) = \tau = \lambda \nabla \cdot \vec{u} \bar{I} + \mu \left(\nabla \vec{u} + (\nabla \vec{u})^T \right) \ , \tag{2.2}$$

τ denotes the stress tensor, λ, μ Lamé's elastic coefficients, and

$$\tau_i = H(\vec{u}_i) \ , \qquad i \in \{c, s, b\} \ . \tag{2.3}$$

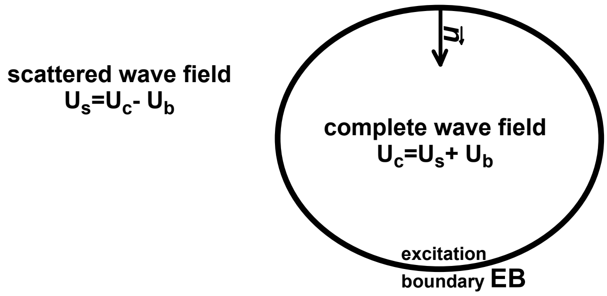

Figure 2: Second step of the hybrid method. The two wave-propagation domains coupled by the generalized AK algorithm divided by the excitation boundary (EB) represented by the ellipse. The background wave field ($\vec{u}_b$, Eqn. 2.4) computed at the first step in whole volume is saved on the EB. In the second step, the EB-saved ($\vec{u}_b$) then fully represents the wave field from the first step. The complete (u_c) and scattered ($\vec{u}_s$) wave fields are both computed in the second step inside and outside the EB, respectively (see also right panel of Figure 1).

The background wave field at the EB can be expressed as displacement discontinuity $[\vec{u}]$ (after using Eqn. (2.1)) is:

$$[\vec{u}] = \vec{u}_c^{EB} - \vec{u}_s^{EB} = \vec{u}_b^{EB} \, , \tag{2.4}$$

where the "positive side" is inside the EB, $\vec{u}_i^{EB}$, τ_i^{EB}, $i \in \{c, s, b\}$ are displacements and stress tensors at EB. The traction discontinuity $[\vec{T}]$ at the EB then writes:

$$[\vec{T}] = [\tau \cdot \vec{n}] = (\tau_c^{EB} - \tau_s^{EB}) \cdot \vec{n} = \vec{T}_c - \vec{T}_s = \vec{T}_b \, , \tag{2.5}$$

after inserting (2.3) and (2.4) into (2.5) we get:

$$[\vec{T}] = (H(\vec{u}_c^{EB} - \vec{u}_s^{EB})) \cdot \vec{n} = H(\vec{u}_b^{EB}) \cdot \vec{n} = H([\vec{u}]) \cdot \vec{n} \, , \tag{2.6}$$

where $\vec{n}$ is the normal vector to the EB boundary pointing into the interior of the EB.

After using (2.2) in (2.6) we get the traction discontinuity at the EB:

$$[\vec{T}] = \lambda \nabla \cdot \vec{u}_b^{EB} \bar{I} \cdot \vec{n} + \mu \left(\nabla \vec{u}_b^{EB} + (\nabla \vec{u}_b^{EB})^T \right) \cdot \vec{n} \, . \tag{2.7}$$

Equations (2.4), (2.5), and (2.7) express the EB boundary conditions (here for isotropic elastic medium). In principle, there is no difference between the use of $\vec{u}$, $\dot{\vec{u}}$, $\ddot{\vec{u}}$ in Eqn. (2.4). Neither is in Eqns. (2.5) to (2.7), where $\vec{T}$ would have a meaning of $\dot{\vec{T}}$, and $\ddot{\vec{T}}$, respectively.

Note that in a case of an interface intersecting the EB, the space is subdivided into 4 subsets (Fig. 3), the boundary conditions (2.4), (2.5), and (2.7) remain unchanged also in the point of intersection because we require continuity of displacement and traction in each point of the interface outside the EB boundary. Thus the condition on interface intersecting the EB is a trivial extension of e.g., (2.7) and writes:

$$\begin{aligned} H(\vec{u}_s^+, \vec{r}^+) &= H(\vec{u}_c^+ - \vec{u}_b^+, \vec{r}^+) = H(\vec{u}_c^+, \vec{r}^+) - H(\vec{u}_b^+, \vec{r}^+) \, , \\ H(\vec{u}_s^-, \vec{r}^-) &= H(\vec{u}_c^- - \vec{u}_b^-, \vec{r}^-) = H(\vec{u}_c^-, \vec{r}^-) - H(\vec{u}_b^-, \vec{r}^-) \, , \end{aligned} \tag{2.8}$$

where $\vec{r}$ is vector inside the interface plane. The traction discontinuities (2.6) $[\vec{T^+}]$, and $[\vec{T^-}]$ at '+', and '$-$' sides of the EB, respectively, then writes:

$$\begin{aligned} [\vec{T^+}] &= H(\vec{u}_c^+, \vec{r}^+) \cdot \vec{n} - (H(\vec{u}_c^+, \vec{r}^+) - H(\vec{u}_b^+, \vec{r}^+)) \cdot \vec{n} = H(\vec{u}_b^+, \vec{r}^+) \cdot \vec{n} \\ [\vec{T^-}] &= H(\vec{u}_c^-, \vec{r}^-) \cdot \vec{n} - (H(\vec{u}_c^-, \vec{r}^-) - H(\vec{u}_b^-, \vec{r}^-)) \cdot \vec{n} = H(\vec{u}_b^-, \vec{r}^-) \cdot \vec{n} \, . \end{aligned} \tag{2.9}$$

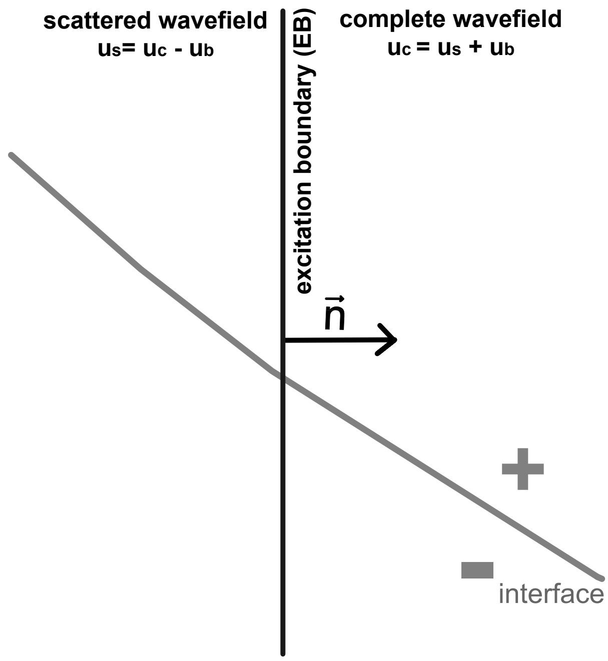

Figure 3: A detail of the interface (of the 1st step regional structure) intersecting the EB. This structure is present in both steps of the hybrid computation outside and on the EB. It may be changed inside the EB for second step computation. See also Fig. 2

Thus the traction discontinuities (2.6) $[\vec{T^{+}}]$, and $[\vec{T^{-}}]$ at the 'EB'-'interface' intersection:

$$
\begin{aligned}
[\vec{T^{+}}] &= H(\vec{u}_c^{EB+}, \vec{r}^{EB+}) \cdot \vec{n} - (H(\vec{u}_c^{EB+}, \vec{r}^{EB+}) - H(\vec{u}_b^{EB+}, \vec{r}^{EB+})) \cdot \vec{n} = \\
&= H(\vec{u}_b^{EB+}, \vec{r}^{EB+}) \cdot \vec{n} \\
[\vec{T^{-}}] &= H(\vec{u}_c^{EB-}, \vec{r}^{EB-}) \cdot \vec{n} - (H(\vec{u}_c^{EB-}, \vec{r}^{EB-}) - H(\vec{u}_b^{EB-}, \vec{r}^{EB-})) \cdot \vec{n} = \\
&= H(\vec{u}_b^{EB-}, \vec{r}^{EB-}) \cdot \vec{n}\ .
\end{aligned} \tag{2.10}
$$

Equations 2.10 then define the jump in discontinuity of traction $[\vec{T}]$ at the point of the 'EB'-'interface' intersection:

$$
[\vec{\tilde{T}}] = [\vec{T^{+}}] - [\vec{T^{-}}] = H(\vec{u}_b^{EB+}, \vec{r}^{EB+})) \cdot \vec{n} - H(\vec{u}_b^{EB-}, \vec{r}^{EB-})) \cdot \vec{n};. \tag{2.11}
$$

As a special case and for checking purposes, let also the media in cases I and II be identical. Then complete wave field:

$$
\vec{u}_c = \vec{u}_b\ , \tag{2.12}
$$

and scattered wave field:

$$
\vec{u}_s = 0\ . \tag{2.13}
$$

The case II wave fields $\vec{u}_c$ and $\vec{u}_s$ for structure (and source) identical with case I are called replication wave fields.

2.2. *Application of the Principle in Hybrid Approach*

So far, the independent wave fields in case I and II (Fig. 1) were considered separately, while Eqns. (2.4), (2.5), and (2.7) express relationship between them. Knowing $\vec{u}_b$ (from case I) allows then to express the complete wave field $\vec{u}_c$ (case II) inside the EB.

Practically the computation is subdivided into two steps: In the first step $\vec{u}_b$ is computed and its values on the EB are saved. After that the second-step (case II) complete wave field $\vec{u}_c$ is computed in the interior of the EB and $\vec{u}_s$ outside the EB. The local area of interest (site conditions) is enveloped by the EB. The advantage is that the volume of the second step can be truncated into a finite volume containing the EB and its close vicinity, typically without a strong impact on the complete wave field. Hence the computational domain for this second step may be significantly reduced.

The AK-type approaches allow for 'perfect' connection between the background and complete wave field domains, but still, the methods use the principle in a way tailored to an applied numerical approach without showing any boundary condition in general. However, the approaches may be unified by the general boundary condition on the EB (Eqns. (2.4) to (2.7)). The thought of a rigorously described boundary condition thus came as a need to simply unify the many applied combinations by boundary conditions applied at the EB. The reasons are not only scientific but also educational - the tailored implementation may be more difficult to understand than the general description that does not require a detailed knowledge of methods used in a particular combination, especially in cases where the wave-field injection is not fully theoretically justified, but implemented in a purely technical way.

Computation of replication wave fields ($\vec{u}_c$, $\vec{u}_s$) in the second hybrid step is called a 'replication test'. It gives wave field $\vec{u}_c$ as being identical to the first step $\vec{u}_b$ in the whole interior of the EB and zero $\vec{u}_s$ in its exterior (see Eqns. 2.12,2.13)

For practical use it means that the structure of interest is to be located always within the EB interior. Then the replication test of the hybrid method is the way to check the correctness of the wave field injection. If the second step structure is added to perform the computation, then the complete wave field inside the EB is the same as if computed by all-in-one method and the scattered wave field outside the EB is equal to the all-in-one fine structure wave field minus the first-step (background) wave field. For more details see chapters 3.1, 3.2, and 3.3 of Opršal & Zahradník (2002).

2.3. *Specific Implementations of the Hybrid AK Approach*

In principle, the two-step AK coupling allows for a combination of arbitrary methods suitable to compute the wave field generation and propagation, for example FD, discrete wave number (DW) method, ray theory (RT) method, finite elements (FE) or analytical solution. The term 'wave injection' was introduced by Robertsson & Chapman (2000).

Alterman & Karal (1968), who used 2D source wave field (the first step) injection into the 2D FD computation (the second step), were followed by a variety of approaches. Some of them were equivalent to AK, while others were just 'very close' to their principle. For equivalent formulation (Eqn. (2.4)) combining various 1D and/or 2D methods see also Kelly et al. (1976), Levander (1989), Zahradník & Moczo (1996; 2D DW-FD planar free surface), Moczo et al. (1997; 2D P-SV DW-FD hybrid connection; free surface topography treated by non-AK-like connected FE), Opršal et al. (1998; DW-FD), Opršal et al. (1999; 2D RT-FD), Fäh & Suhadolc (1994), Fäh et al. (1994), Vidale & Helmberger (1987; 2D 4th-order FD), Moczo (1989, 2D FD SH), Caserta et al. (1999; 2D DW-FD topography models, stochastically perturbed excitation), Riepl (1997), Riepl et al. (2000), Fäh (1992) and Fäh et al. (1990, 1993; 2D modal summation - FD). While Fäh et al. (1994; propagation from 3D point source by modal summation - 2D FD) used the coupling at the vertical line subdividing the 'source' and 'site' quarter spaces, Zahradník (1995) suggested the extension to envelope the site of interest inside the excitation box, hence reversing the approach of AK using their principle. Zahradník & Moczo (1996) and many others have followed (Eqns. (2.4), (2.5) and (2.7)): Robertsson et al. (1996; acoustic/viscoelastic 2D Gaussian beam-FD, 4th order, staggered FD); Robertsson & Chapman (2000), Robertsson et al. (2000; 2D FD-FD staggered grid, 'wave injection'), Takeuchi & Geller (2003) and Hatayama et al. (2007, DW(3D)-FD(2D staggered-grid P-SV)). Using the source box method with line-to-point source technique, mapping the wave field from 2D to 2.5D, takes into account the spreading over 3D space instead of 2D space: Vidale & Helmberger (1987, 1988), Helmberger & Vidale (1988) with 4th-order 2D FD P-SV, Vidale et al. (1985 2D SH), Pitarka et al. (1994, 2D SH), Pitarka et al. (1996 2D P-SV), Pitarka & Irikura (1996, 2D FD 2nd order SH, 2D 4th order staggered P-SV). Takenaka et al. (2006) used 1D FD plane-wave incidence excitation in 2D or 2.5D FD hybrid computations to eliminate the spurious reflection from the boundaries.

A combination of full 3D methods was recently presented by Opršal & Zahradník (2002; FD-FD), Opršal et al. (2002; 3D RT-FD, DW-FD), Opršal et al. (2004; 3D finite-extent source PEXT - FD), Opršal et al, (2005), Opršal et al. (2007), Fäh et al. (2006) and Sorensen et al. (2006). Hatayama

(2006) combines modal summation (Love and Rayleigh waves) and 3D staggered grid FD (velocity stress formulation) but unlike e.g., Robertsson et al. (2000) the discontinuity on the border is applied only in velocities, not in stresses (L. Eisner, personal communication).

The method principle is also used in a two-step approach of Bielak & Christiano (1984), applied in the 2D FE modelling, was developed theoretically for a volume adjacent to the free surface, preceded by Herrera & Bielak (1977), and further followed by Loukakis (1988), Cremonini et al. (1988) and Gazetas et al. (2002). Recently their method was extended to the 3D FE-FE hybrid modelling as DRM (domain reduction method) by Bielak et al. (2003), Yoshimura (2003), Yoshimura et al. (2003), Yang et al. (2003), Bielak (2005), Faccioli et al. (2005) and Stupazzini et al. (2006).

Lecomte (1996), Gjøystdal et al. (1998, 2002), Hokstad et al. (1998), and Lecomte et. al. (2004) combined ray theory and FD (where RT is an asymptotic solution of the wave equation and 2D FD can be elastic or acoustic). The FD is computed for a buried structure model and the scattered wave field is then propagated to the free surface. The coupling technique, basically described in Lecomte (1996), makes the coupling line non-permeable, however, it can be turned into the Alterman & Karal (1968) treatment by a simple additional operation to fulfill Eqn. (2.4). After that, not only the coupling line becomes permeable (2.1), but the wave field above such a horizontal line would be only scattered (u_s, Eqn. (2.1)) by definition and therefore its propagation to the free surface via RT would also be possible for an arbitrarily long excitation signal. Hence the RT-FD-RT technique of Lecomte et al. (2004) can be added to the group of methods describable by the Alterman & Karal (1968) principle.

Close to the AK method is the paraxial approach of De Martin et al. (2007) and Modaressi (1987) who use FD-FE, where the excitation field is extracted from FD by the paraxial approximation technique applied at the excitation boundary. The scattered wave field is then absorbed at the boundary, again using the paraxial approximation. In principle, the absorbed wave field may be also further propagated outside the complete wave field area in a case of need. The disadvantage of paraxial approximation is lower accuracy in comparison with the AK approach, especially in heterogeneous medium where the FD-FD AK method is practically exact in terms of injection, producing only truncation or possibly interpolation errors. However, the approach can be applied also inside nonlinear medium (Bamberger et al., 1986) where directly employed AK method fails by definition. The failure of the direct AK use is due to the necessity to maintain time dependent discontinuity of the wave field on the EB which is being added or subtracted from so called scattered or complete wave fields, see (2.4).

3 CONCLUSION

The main result of this paper is a derivation of the condition common to a large group of methods given in section 2.3. Derived Eqns. (2.4) to (2.11) generally describe a boundary condition for the Alterman & Karal (1968) principle of wave-field injection. The special or only technical approaches are unified by a common general condition on the wave-field injection boundary. The condition is applicable to arbitrary combination of the two hybrid step methods and naturally also describes the approaches used so far. The main principle consists in keeping the difference between the inner and outer domain to be equal to the background wave field. For more general case e.g, spectral approach or viscoelastic medium the basic idea remains unchanged (Opršal et. al, 2008).

The AK-principle based methods have a fully permeable excitation box (see 2.1) for scattered wave field. Such a wave field can naturally propagate through the EB in both directions since the computed scattered wave field is not 'hard prescribed' and is linearly separable, thanks to the known background wave field, from the complete wave field on the excitation box.

The AK method is directly applicable to linear problems – literally the problem has to be linear outside and on the excitation box, where simple addition and subtraction of the wave field is performed when implemented (see conditions 2.7 and 2.4). The demand for linearity outside the EB arises from the fact that only the scattered wave field is computed outside the EB in the second step and thus its non-linear interaction with the background wave field is not possible. The model inside the excitation box may be nonlinear because the computed wave field is complete (u_c) there.

Advantages of the AK method can be summarized as follows. Since the general description given in (2.4), (2.5) and (2.7) does not limit the use of actual implementations, we believe that it also can combine other methods because the boundary condition, as it is, can be expanded to a zone-description (e.g., Bielak, 2003). As mentioned in Opršal & Zahradník (2002, sect. 3.1.2.2.), the condition virtually means a 'maintenance' of time-dependent discontinuity between the complete and scattered wave fields (being time-dependent as well), called the background wave field, on the excitation boundary. The same philosophy dates back to work by Herrera & Bielak (1977) who established conditions for traction and displacement discontinuity at the soil-structure interface adjacent to free surface, and is continued in Bielak & Christiano (1984), Cremonini et al. (1988) and Bielak et al. (2003).

In a general case, the medium at the excitation boundary (and its vicinity; for example one grid step in second-order FD use) always has to be linear to be able to generally apply the AK method. Possible nonlinearity at the excitation box may be avoided, for example, in the case when the excitation signal duration is very short and thus the background wave field is already zero at the time when a scattered wave field of significantly enhanced amplitudes of non-linear field travels through the excitation box. Lecomte (1996) and Lecomte et. al. (2004) describe a similar situation while avoiding the reflection of the scattered wave field by their excitation box formulation.

Some advantages of the AK method in indirect computation:

- Complex (finite-extend, dynamic) seismic source computations are performed separately
- Repeated computations of the second step in parametric study or waveform inversion. The excitation (background wave field) is the same while model alterations are done for each of multiple second-step computations.
- Source-site distance may be very large, second step model can be (depending on situation) a very small fraction of the whole model and thus computation may be done up to engineering frequencies (Opršal & Fäh, 2007, 0-12Hz).
- First step is typically computed for a relatively simple medium.
- Easy parallelization of (e.g., FD or FE) second step computations.
- Saving computer memory and time
- Relatively small values of the scattered wave field (computed outside the EB; practically equal to zero in successful replication test) compared to direct all-in-one incoming wave field. Hence the spurious reflections from the absorbing boundaries are also smaller.
- Relatively simple implementation of canonical excitations like planar or spherical wave of arbitrary incidence angle and polarization.

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