STOCHASTIC GREEN'S FUNCTION: ESTIMATION OF DURATION AND MASTER ENVELOPE DUE TO THE PROPAGATION PATH EFFECT

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ABSTRACT :

Average envelopes of small earthquakes represent a convenient basis for the construction of semi-empirical stochastic Green's functions, used for prediction of future strong ground motions. Petukhin and Gusev (2003) proposed method for estimation of the average envelopes at rock sites based on assumption of self-similarity of envelopes and applied it for Kamchatka. Self-similarity means that average envelopes for small earthquakes have similar shapes and controlled by two scaling parameters: amplitude and duration. It is also assumed that the high-frequency envelope at a site could be represented as the convolution of source, path and site envelopes. For records of small earthquakes, source envelope is the δ-function. Path envelope can be estimated by the generalization of observed envelopes at bedrock sites. In this work, we continue study of the average envelope shapes and estimate average envelopes of the wavelet coefficients for a region adjoining to the Tonankai earthquake for bedrock sites (path effect). First, we slightly modified methodology. (1) To estimate the duration scaling parameter, instead of using the RMS-duration in earlier work, here we use the envelope-delay time T_{env} (also known as group-delay time). The new duration parameter is less affected by the coda waves or by the asymmetry of record. (2) In order to select "bedrock" sites, i.e. sites having negligible duration site corrections, T_{env} site corrections were estimated by the regression analysis of the observed T_{env} values. Second, we divided studied region adjoining to the source region of the Tonankai earthquake, Japan, into three large zones according to tectonical and geological structure: accretion prism (AP), middle zone (MZ), and fault zone along the Hanaore-Nojima fault system (FZ). Third, to study depth dependence of the average envelopes we also analyzed records from deep events, having paths cross to the Lower Crust (LC). In this work we estimate the dependence of scaling parameter T_{env} vs. hypocentral distance R, and the master envelope shapes for the bedrock sites. The results show strong regional dependence of the relationship $T_{env}(R)$. The smallest T_{env} values are observed in the LC zone. For AP zone T_{env} values are larger than in other zones; they increase with distance at $R < 60$km and than stabilize at $R > 60$km. Master envelopes are more robust: they are practically the same for all studied zones.

KEYWORDS: Envelope Green's function, master envelope, envelope delay time, path effect, wavelet

1. INTRODUCTION

Average envelopes (mean square amplitude time histories) of small earthquakes represent a convenient basis for the construction of semi-empirical stochastic 'Green's functions', used for prediction of future strong ground motions. In (Petukhin and Gusev, 2003) it was proposed a method for the estimation of the average envelopes at rock sites based on the assumption of self-similarity of envelopes (adaptive stacking technique). Method was applied for Kamchatkan data. Self-similarity means that average envelopes of small earthquakes have similar shapes and controlled by two scaling parameters: amplitude and duration. Petukhin and Kagawa (2003), see also (Petukhin et al., 2006), applied this method for Kinki area in Japan; they also introduced and for the first time estimated average envelope site corrections for the sedimentary sites in the Osaka basin.

In this study, for the stochastic Green's function simulations, similar to Petukhin et al. (2006), we use simple but effective approach that is based on the convolution rule of the source, path and site envelopes. Implicitly, envelope convolution rule was considered in many studies of the source-path two-element model, starting from the work of (Kopnichev, 1977) in Russia and (Midorikawa and Kobayashi, 1978) in Japan (see also Gusev and Pavlov, 1991, Zeng et al., 1993, Kakehi and Irikura, 1997, Nakahara et al., 1999). In all these works, the envelope convolution rule was used to study high-frequency source radiation process by elimination of the path envelope. Similar to the amplitude spectrum, envelope must account for the effect of site, as well as source and path.

Due to scattering of high-frequency waves in heterogeneous crust, the pulse of seismic waves broadens with distance during the path propagation (Sato and Fehler, 1998). This effect could be negligible in case of relatively small, close/near earthquakes, and site envelope could be estimated by simple stacking of envelopes of records of such earthquakes. In low seismicity regions, to have enough data for stacking, we need to use abandon distant earthquakes. In this case deconvolution of the path envelope becomes necessary. To estimate path envelope we assume the self-similarity of envelopes of small earthquakes at bedrock sites: path effect envelope can be calculated as the master envelope E^0_{path} at a reference distance R_0, scaled in time domain to the duration at target hypocenter distance R. As duration parameter, it is convenient to use the envelope delay time T_{env} (Boore, 2003). Using small earthquake data, effect of source envelope can be neglected.

In this study, we will consider envelopes of wavelet series. It will be more easy to understand wavelet approach if at first we introduce more traditional envelope approach. In next part, we introduce the convolution rule and the self-similarity principle for path envelopes. Then, we will show some results of using this approach.

2. ENVELOPE APPROACH

In this part, we introduce the envelope convolution rule and then show how this rule could be used to estimate path envelope. In case of the full source – path – site model, observed envelope is

$$E_{obs}(t\mid f) = E_{source}(t\mid f) * E_{path}(t\mid f) * E_{site}(t\mid f)\,, \tag{2.1}$$

here $E_{obs}(t)$ – observed envelope, $E_{source}(t)$ – source envelope, $E_{path}(t)$ – path envelope, $E_{site}(t)$ – envelope site effect, f - frequency. In case of small earthquake, we can assume that source envelope is δ-function $E_{source} = K_{source}\delta(t)$. In case of the bedrock site $E_{site} = K_{site}\delta(t)$. In this case:

$$E_{obs}(t\mid f) = K_{source}(f)K_{site}(f)E_{path}(t\mid f) \tag{2.2}$$

Using Eqn. 2.1 and the self-similarity assumption, envelope E_{path} can be estimated in the next way (Petukhin and Gusev, 2003). Using preliminarily estimated $T_{env}(R)$ relationship, E_{obs} of records of small earthquakes in the target region can be reduced to the same reference distance R_0. Then, after reducing them to the same intensity I_0, they can be stacked into the master envelope E^0_{path}. Combination of the master envelope and $T_{env}(R)$ relationship defines the $E_{path}(t|R)$ model.

3. WAVELET APPROACH

It is expected that using wavelets for simulation of strong ground motions is easier and gives better waveforms than by the using of envelope approach. Example of wavelet transform, using the Meyer wavelet (Meyer, 1989), is shown in Figure 1. Figure 1 shows original S-wave record, wavelet coefficients at 4 levels and expected envelope for wavelet coefficients (average amplitude envelopes). The wavelet coefficient series have the same properties as average amplitude envelops: steep leading edge and gradually decreasing coda part.

In order to apply the wavelet approach, we neglect by the source effect for small earthquakes and by the site effect for bedrock sites, similar to Eqn. 2.4. In this case, by averaging the wavelet series from small earthquakes recorded at bedrock sites, we can estimate average path effect for wavelet coefficients. Similar to the method of Petukhin and Gusev (2003), we assume that envelopes of wavelet series at different hypocentral distances are self-similar. Now lets consider method of estimation of path effect for envelopes of wavelet coefficients (see Figure 2).

Step 1. Wavelet transform of observed records

$$A_j(i) = \sum_{k=0}^{2^j-1} \alpha_{j,k} \sqrt{\frac{1}{2^{n-j}\Delta t}}\,\psi\!\left(\frac{i}{2^{n-j}\Delta t} - k\right). \quad (3.1)$$

Indexes here show: i – time, j – frequency band, k –time shift. $\alpha_{j,k}$ is named series of wavelet coefficients.

Step 2. Estimation of the relationships between duration and hypocentral distance. Petukhin and Gusev (2003), used the RMS duration T_{rms} as the time scaling parameter. In this study, instead of parameter T_{rms} we will use the average envelope delay time T_{env} , proposed by Boore (2003), also known as group delay time. In Step 2, we will calculate T_{env} values for all records and estimate by regression analysis the relationship $T_{env}(R)$. We will also try to estimate depth dependence of the relationship $T_{env}(R)$ by considering shallow seismogenic crust events and deep subduction slab events separately.

Step 3. Reduction of individual wavelet series to the same duration value. Using the relationship $T_{env}(R)$, estimated in Step 2, we can estimate duration T_{env0} at a reference distance R_0 (for example R_0 = 100km). In this case, it is become possible to reduce an observed wavelet series to the same duration T_{env0} (actually to the same distance R_0):

$$A_j^{T_{env0}}(i) = \sum_{k=0}^{2^j-1} \alpha_{j,k} \sqrt{\frac{1}{2^{n-j}\Delta t}}\,\psi\!\left(\frac{i}{2^{n-j}\Delta t} - k\frac{T_{gr0}}{T_{gr}}\right) \quad (3.2)$$

$$\left\{\alpha_{j,k}^{T_{env0}}\right\} = spline\left(\left\{k\frac{T_{env0}}{T_{env}}\right\}, \left\{\alpha_{j,k}\right\}, \left\{k\right\}\right) \quad (3.3)$$

Step 4. Reduction of individual wavelet series to the same intensity. Before averaging of many different wavelet series, it is necessary to reduce them to the same intensity level I^0:

$$\alpha_{j,k}^0 = \alpha_{j,k}^{T_{env0}} \sqrt{\frac{I_j^0}{I_j}}, \qquad I_j = \sum_k \left(\alpha_{j,k}^{T_{env0}}\right)^2 \quad (3.4)$$

For simplicity we assume that $I_j^0 = 1$.

Step 5. Averaging and smoothing. In order to get stable envelope estimate, we average wavelet series calculated in Steps 3 and 4, and then smooth the result by moving window. The result is smooth average wavelet series:

$$\left\langle \alpha_{j,k}^0 \right\rangle. \quad (3.5)$$

In order to apply for stochastic simulations, we can use backward the reduction scheme above, and reduce the wavelet series from Eqn. 3.5 to the hypocentral distance R to the target site, and then to the theoretical intensity level I_j.

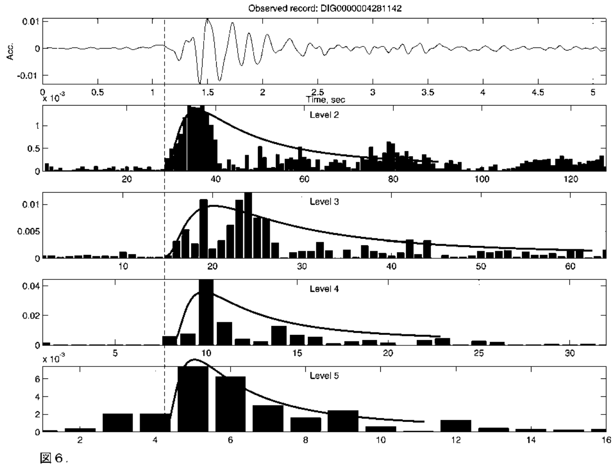

Figure 1 Example of the wavelet transform. Absolute values of the wavelet coefficients are shown.

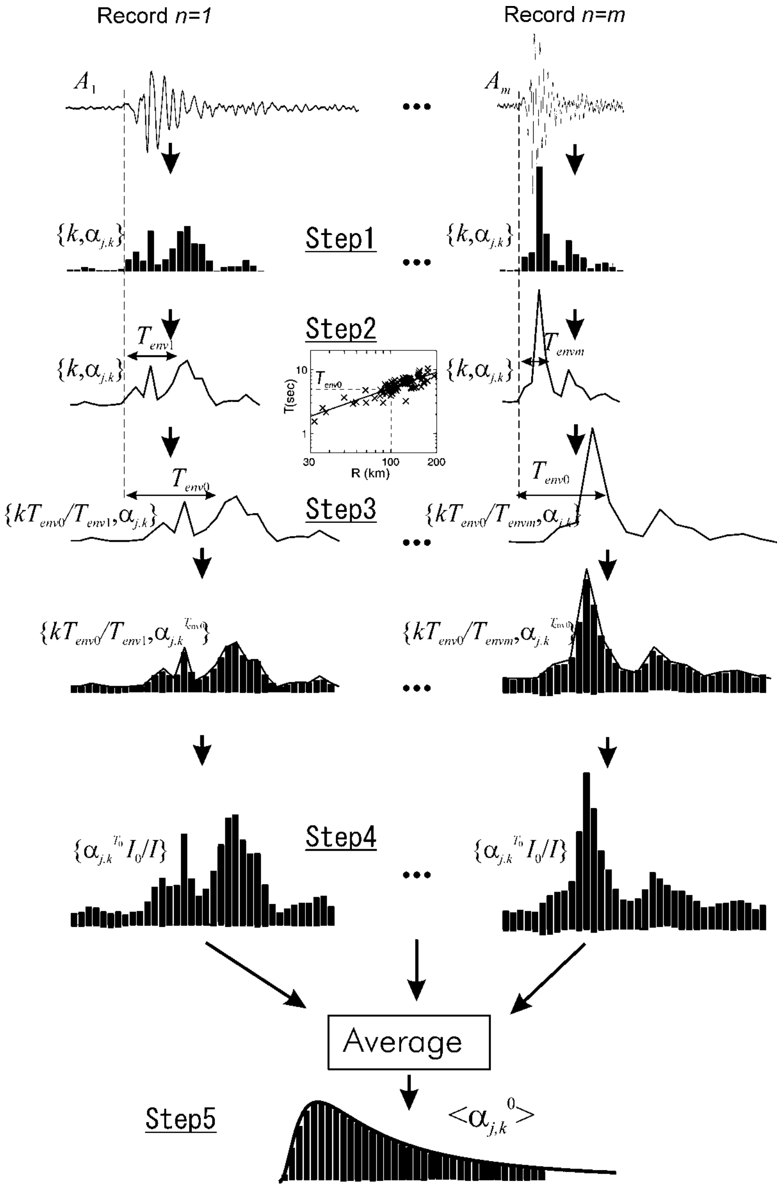

Figure 2 Algorithm of estimation of the average (master) envelope.

4. $T_{env}(R)$ REGRESSION

Following to Sawada (1998), under assumption that observed Fourier spectrum is the product of the spectral source, path and site effects, group delay time of the observed record is simply sum of the group delay time values due to source, path and site effects. From next equation

$$S_{obs}(f) = S_{source}(f) \cdot S_{path}(f) \cdot S_{site}(f) \tag{4.1}$$

it follows that

$$T_{gr,obs}(f_l) = T_{gr,source}(f_l) + T_{gr,path}(f_l) + T_{gr,site}(f_l)\,. \tag{4.2}$$

Considering that the envelope delay time is the group delay time averaged in some frequency range (Boore, 2003) we come to the next equation

$$\langle T_{gr,obs}(f_l)\rangle = \langle T_{gr,source}(f_l)\rangle + \langle T_{gr,path}(f_l)\rangle + \langle T_{gr,site}(f_l)\rangle\,, \tag{4.3}$$

or

$$T_{env,obs} = T_{env,source} + T_{env,path} + T_{env,site}\quad. \tag{4.4}$$

From Eqn. 4.4, assuming that for small earthquakes $T_{env,source}$ = 0 and that from scattering theory it is expected that in an ideal uniform scattering medium $T_{env,path} = R^b$, where b is constant (e.g. Sato and fehler, 1998), and also supposing that some nonlinearity is possible in the dependence of $T_{env,path}$ vs. R in real nonuniform scattering medium, next equation is proposed to estimate envelope delay time including the site effect $T_{env,site}$.

$$T_{env,obs,i}(f_K) = T_{env,site,i}(f_K) + T^{R_c}_{env,path} \cdot \begin{cases} (R/R_c(f_K))^{b_1(f_K)} & \text{if } R < R_c(f_K) \\ (R/R_c(f_K))^{b_2(f_K)} & \text{if } R > R_c(f_K) \end{cases} + \varepsilon(f_K) \tag{4.5}$$

Here T_{env,obs_i} and $T_{env,site\_I}$ are the observed envelope delay time and the envelope delay time due to site effect at the i'th observation site. R_c is a corner hypocenter distance, accounting the non-linearity, exponent b is showing rate of growing of the envelope delay time with distance R, due to scattering.

To solve Eqn. 4.5 it is necessary to apply some nonlinear regression analysis. In case of a small number of the unknown parameters $\{T_{env,site\_i}\}$, b_1, b_2 and R_c the grid search method can be used. In case of large number of parameters Genetic Algorithm (GA) or, if reliable initial model could be available, minimum search methods like the steepest descent method or the simplex method can be used. In this study we use simplex method.

5.DATA AND RESULTS

In case of giant earthquakes, like Nankai – Tonankai earthquake, because of large source size, seismic waves propagate large distance before attenuating to a negligible level. In this case, there is a large probability that they will pass subregions with different geological and tectonic conditions. Scattering parameters in the subregions also can be different. To investigate how large can be difference of scattering in various subregions, we divided studied region into a few large zones according to tectonical and geological structure: accretion prism (AP) in the south of the Median Tectonic Line (MTL), middle zone (MZ, north of MTL) and Hanaore-Nojima fault zone (FZ), see Figure 3. Records of shallow, h = 5-17 km, events in upper crust were studied for these zones. In addition, to study depth dependence of the average envelopes, we also analyzed records from deep lower crust events, h = 17-40 km (LC), having paths cross to the Philippine Sea subduction zone, see Figure 3. We used combined data set of nationwide networks K-NET, Kik-net and Hi-net, enlarged by local networks Denkyoken (managed by 11 electric power companies in Japan) and CEORKA (Committee for the Earthquake Observation and Research in Kansai Area), rock sites only. Total number of records - 1355. In case of Kik-net, Hi-net and Denkyoken, high-quality borehole data were used. At the next step, each record were filtered in 4 frequency bands: f_k = 1-2, 2-4, 4-8, 8-16Hz and T_{env} values were calculated. Figure 4 shows example of calculation of T_{env}.

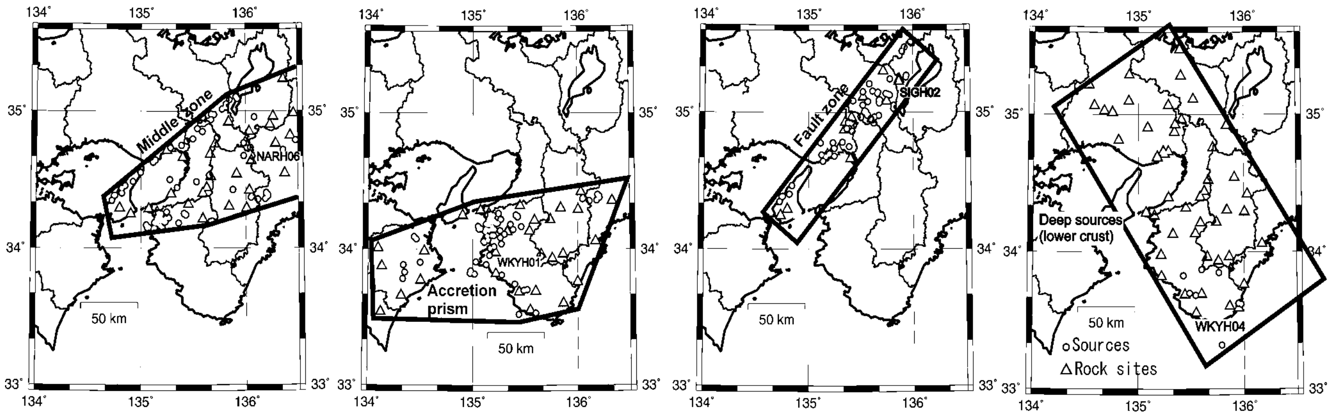

Figure 3 Distribution of the sources and sites in the studied areas: MZ, AP, FZ and LC.

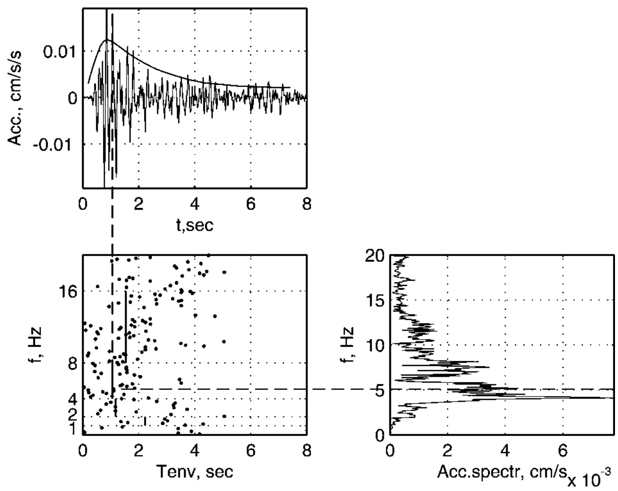

Figure 4 Example of calculation of the envelope delay time T_{env}(lower left plot). Vertical bars in the plot shows average values of the group delay time values (points) in the frequency intervals 1-2, 2-4, 4-8 and 8-16Hz. Observed record (upper plot) and its amplitude spectrum (lower right plot) are also shown. T_{env} value in the frequency band, corresponding to the maximum of the spectrum, correlates well (dashed line) with the delay of maximum amplitudes of the record.

In order to estimate relationship $T_{env}(R)$, nonlinear regression analysis using Eqn. 4.5 was performed in 3 steps. In the 1st step T_{env_site} corrections are estimated for all rock sites and then bedrock sites having T_{env_site} d 0 are selected. In the 2nd step the nonlinear relationship $T_{env\_path}(R)$ is estimated using only the bedrock sites. Analysis of results of the regression analysis in 2nd step shows, that they generally can be divided into 2 groups. In first group corner distance R_c is very close to one of the ends of data distance range. This indicates that $T_{env\_path}(R)$ dependence should be approximated by the one-segment relationship instead of assumed in Eqn. 4.5 two-segment relationship. In the second group, the relationship is the two-segment type with R_c ~ 60km, practically independently on frequency range or studied area. For this reason, to get final result, in the 3rd step regression analysis was performed one more time employing the one-segment model for 1st group and the two segment model with R_c = 60km for 2nd group. Based on preliminary results and assuming that corner distance reflect scattering properties of the crust and should be the same in the same area we assigned FZ, MZ and LC areas to group 1, and AP area to group 2 at all frequencies. Figure 5 compares final results of regression analysis.

Then we applied results of regression analysis to the Step 3 of algorithm in Figure 2 and estimated master envelopes. In spite of high variations of observed envelopes (see realistic examples in Figure 2), proposed algorithm of estimation of the average envelope results in a smooth master envelope, consistent with theoretical shapes. Results for studied areas are compared in Figure 6. Generally, master envelope demonstrates high robustness: their shapes are very similar in different studied areas, at different earthquake depths and in different frequency ranges. The noticeable differences are only in $T_{env\text{-}path}(R)$ relationships.

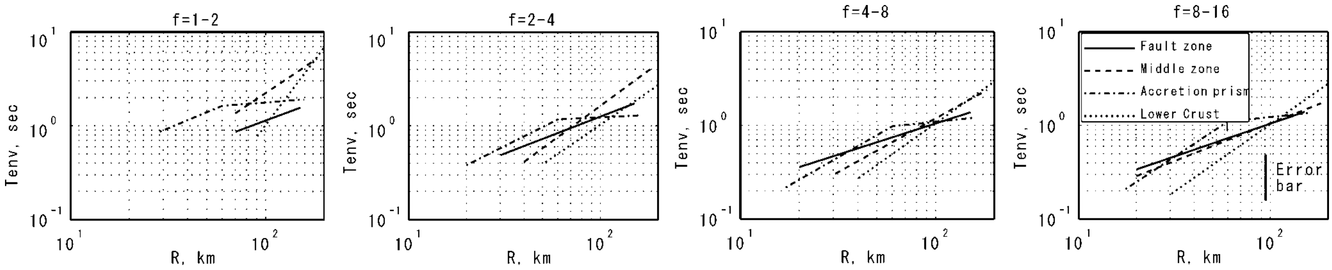

Figure 5 Estimated relationships $T_{env}(R)$ for the all frequency intervals and all regions.

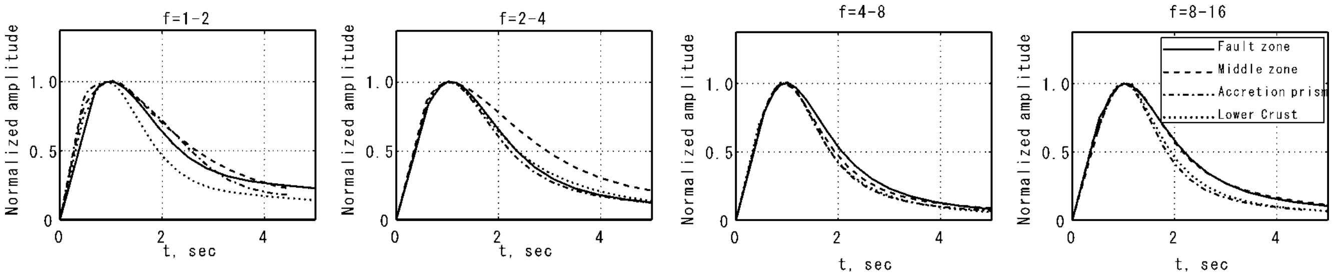

Figure 6 Master envelopes of the wavelet series. For the clarity, master envelopes in this figure are normalized to the peak amplitude equal 1.0 and the peak delay time equal 1.0 sec.

6. CONCLUSIONS

Results of estimation of relationship of the envelope delay time versus hypocentral distance $T_{env}(R)$ show that regional variations of scattering can be strong (relationships $T_{env\text{-}path}(R)$ have different level) and that the lower crust events have shorter duration, indicating that scattering properties of the lower crust and upper mantle are lower. In case of shallow earthquakes, T_{env} values are larger in the Accretion prism (MTL south) and Fault zone (Hanaore-Nojima fault zone), indicating higher scattering in these zones. However, these last differences are smaller than the standard deviation of errors. Master envelope shapes of wavelet coefficients (wavelet path effect) were estimated employing the self-similarity assumption. They demonstrate high robustness: their shapes are very similar in different studied areas, at different earthquake depths and in different frequency ranges. The results indicate that for stochastic simulation of Tonankai earthquake, as a first approximation, we can use the same $T_{env}(R)$ relationship and the same master wavelet series.

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REFERENCES

Meyer, Y. (1989), Orthonormal wavelets, in Wavelets, Springer, 21-37.

Boore, D.M. (2003), Phase derivatives and simulation of strong ground motions, *Bull.Seism.Soc.Am.*, **93**, 1132-1143.

Gusev, A.A. and Pavlov, V.M. (1991), Deconvolution of squared velocity waveforms as applied to the study of a noncoherent short-period radiator in the earthquake source, *Pure Appl. Geophys.*, **136**, 235-244.

Kakehi, Y. and Irikura, K. (1997), High-frequency radiation process during earthquake faulting – envelope inversion of acceleration seismograms from the 1993 Hokkaido-Nansei-oki, Japan, earthquake, *Bull.Seism.Soc.Am.*, **87**, 904-917.

Kopnichev, Yu.F. (1977), A method for determination of structure of radiation of a strong earthquake by envelope shape of p-wave, *Dokl.Acad.Nauk SSSR*, **234**, 794-797 (in Russian).

Midorikawa, S. and Kobayashi, H. (1978), On estimation of strong earthquake motions with regard to fault, Proc.2nd Int.Conf.Microzonation, 2, 825-836.

Nakahara, H., Sato, H., Ohtake, M. and Nishimura, T. (1999), Spatial distribution of high-frequency energy radiation on the fault of the 1995 Hyogo-ken Nanbu, Japan, earthquake (M_w6.9) on the basis of the seismogram inversion, *Bull.Seism.Soc.Am.*, **89**, 22-35.

Petukhin, A.G. and Gusev, A.A. (2003), The duration-distance relationship and average envelope shapes of small Kamchatka earthquakes, *Pure Appl. Geophys*., **160**, 1717-1743.

Petukhin, A. and Kagawa, T. (2003), Study of average envelope shapes of small earthquakes at rock and sedimentary sites, 2003 Joint Meeting of Earth and Planetary Science, Chiba, Japan, CD-ROM, Abstract S046-P007.

Petukhin, A., Kagawa, T. and Tsurugi, M. (2006), Site specific high-frequency envelopes and the stochastic green function simulation of strong ground motions, First European Conference on Earthquake Engineering and Seismology, Geneva, Switzerland, 3-8 September 2006, Paper Number: 751.

Sato, H. and Fehler, M.C. (1998), Seismic Wave Propagation and Scattering in the Heterogeneous Earth, Springer, New York.

Sawada, S. (1998), Phase characteristics on the site amplification of layered ground with irregular interface, The Effects of Surface Geology on Seismic Motion, Irikura, Kudo, Okada and Sasatani (eds.) Balkema, Rotterdam.

Zeng, Yu., Aki, K. and Teng, T.-L. (1993), Mapping of the high-frequency source radiation for the loma prieta earthquake, California, *J.Geophys.Res.*, **98, B7,** 11981-11993.