SCATTERING OF PLANE WAVES BY A 3-D CANYON IN LAYERED HALF-SPACE

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ABSTRACT :

This paper presents a solution for plane waves scattering by a 3-D canyon in layered half-space in frequency domain by indirect boundary element method (IBEM), based on the exact dynamic stiffness matrices and the dynamic Green's functions for uniformly distributed loads acting an inclined plane in 3-D layered half-space by the authors. The free-field response is carried out to give the displacements and stresses on the curved plane which forms the boundary of the canyon. The fictitious uniformly distributed loads are applied on the same curved plane to calculate the Green's functions for the displacements and stresses. The amplitudes of the loads are determined by the boundary conditions. The displacements due to the free field and due to the fictitious uniformly distributed loads are added to obtain the whole motion. The accuracy of the solution is verified by comparison with the known solutions. The numerical calculations and analyses are performed for a hemispherical canyon in one single layer over half-space for incident waves. The results show that there exist distinct differences between the surface motion of a canyon in layered half-space and that in homogeneous half-space. It is found that the dynamic characteristics of the layered half-space significantly affect both the amplitudes and frequency spectrum of the surface motion.

KEYWORDS: Scattering, plane waves, 3-D canyon, layered half-space, IBEM

1. INTRODUCTION

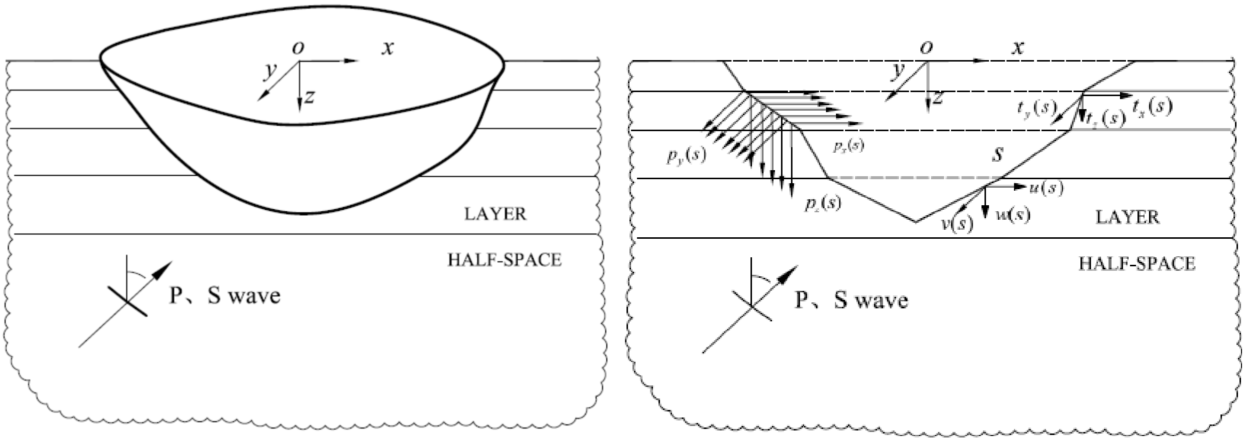
The effect of local site conditions, such as canyons, valleys and hills, on seismic wave propagation is one of the most fundamental subjects in seismology and earthquake engineering. Trifunac (1973) gave an exact analytical solution for the scattering of plane SH waves by a semi-cylindrical canyon, and delineated the amplification pattern for various frequencies and incident angles. Then many studies were carried out on this topic using either analytical or numerical methods, e.g., finite difference methods, finite element methods, boundary element methods, etc., and the details may be found in a review by Sanchez-Sesma, et al (2002). In recent years elastic wave scattering by 3-D canyons has attracted more and more attention (e.g., Lee, 1982; Sanchez-Sesma, 1983; Mossessian and Dravinski, 1989; Gil-Zepeda and Luzon, 2002; Niu and Dravinski, 2003).

It should be noted that most of the contributions are still limited to homogeneous half-space. However, in reality, soil is not homogeneous, but often layered; and the soil layers determine dynamic characteristics of the site, which may affects wave scattering around canyons. This paper presents a solution for plane waves scattering by a 3-D canyon in layered half-space in frequency domain by indirect boundary element method (IBEM), based on the exact dynamic stiffness matrices and the dynamic Green's functions for uniformly distributed loads acting an inclined plane in 3-D layered half-space by the authors.

2. METHOD

Figure 1(a) shows the model for scattering of plane waves by a 3-D canyon in layered half-space. The indirect boundary element method (IBEM) in frequency domain is applied.

At first, the free-field response is calculated to determine the displacements and stresses on the surface S which will form the surface of the canyon. Then fictitious source loads are applied on the same surface S in the free-field system. Figure 1(b) shows the positions and orientations of the applied fictitious source loads. The corresponding Green's functions for the displacements and stresses are then calculated. The amplitudes of the source loads are determined by the boundary conditions that the stresses arising from the waves in the free field and from the fictitious loads on the canyon's surface vanish. These conditions can be satisfied only in an average sense by using the method of weighted residuals. The displacements arising from the waves in the free field and from the fictitious distributed loads are summed up to obtain the solutions.



(a) (b)

Figure 1 The model

2.1. Free-Field Response

By assembling the individual layers and half-space dynamic-stiffness matrices (Liang and Ba, 2007a) and load vectors, the equation of motion in the frequency-wave-number domain of the layered half-space results in

$$[S_{P-SV-SH}]\{U\} = \{Q\} \tag{1}$$

Where $[S_{P-SV-SH}]$ is the global stiffness matrix of a multi-layered site. $\{U\}$ is the vector of the displacements. $\{Q\}$ is the vector of the external loads. Solving this system leads to the free-field response.

2.2. Green's Functions in Layered Half-Space

Figure 1(b) shows distributed loads acting on an inclined plane in a soil layer. Distributed loads $\{p_x(s)\}$, $\{p_y(s)\}$ and $\{p_z(s)\}$, which are selected as piecewise constant over each element, acting on S in the free-field system are introduced. In contrast to the point loads used by other researchers, the distributed loads can be chosen to act directly on the canyon surface without leading to any singularities. To calculate the Green's functions, it is appropriate to introduce fictitious interfaces so that each resulting layer is loaded uniformly. At first the two interfaces of each layer are assumed to be fixed and the corresponding reaction forces are calculated. These reaction forces are then applied with the opposite sign as loads of the discretized equation of the layered half-space to determine the global response. The total response can then be found by adding the result of the analysis of fixed layers. Details can be found in Liang and Ba (2007b). It should be mentioned that these calculations are performed in the wave-number domain, so the Green's functions in the space domain can be obtained by inverse Fourier transformation for the displacements and stresses.

2.3. Boundary Conditions

The conditions can be expressed as

$$\int_s [W(s)]^T (\{t_p(s)\} + \{t_f(s)\})\mathrm{d}s = 0 \tag{2}$$

where $[W(s)]$ is the weighting function, and $\{t_p(s)\}$ and $\{t_f(s)\}$ are the stress due to the fictitious loads and free-field response, respectively; and

$$\{t_p(s)\} = [g_t(s)]\{p\} \tag{3}$$

where $[g_t(s)]$ is the Green's function for stress, and $\{p\}$ is the fictitious loads. If we choose the weighting function as 1 in an element and zero in all others, the integral can be evaluated over each element separately. Substituting Equation (3) into Equation (2) results in

$$[T_p]\{p\} + \{T_f\} = 0 \tag{4}$$

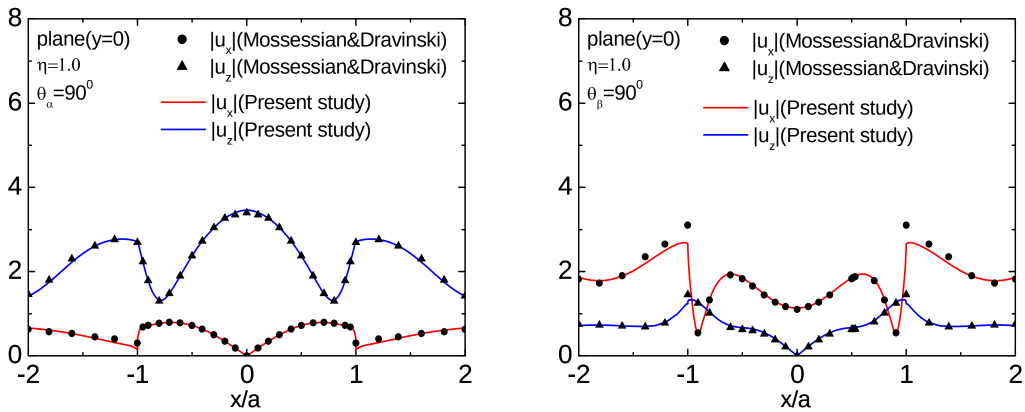
where $\{T_p\} = \int_S [W(s)]^T [g_t(s)]\mathrm{d}s$ and $\{T_f\} = \int_S [W(s)]^T \{t_f(s)\}\mathrm{d}s$. Then the surface displacements of the canyon can be obtained by

$$\{u(s)\} = \{u_f(s)\} + [g_u(s)]\{p\} \tag{5}$$

where $[g_u(s)]$ is the Green's function for displacement.

3. VERIFICATIONS

To verify the precision of the method, Figure 2 shows the surface displacement amplitudes of a hemisphere canyon with radius a in homogeneous half-space compared with the results in Mossessian and Dravinski (1989). The parameters for the half-space are as follows: Poisson's ratio 1/3, damping ratio 0.005, dimensionless frequency $\eta = \omega a / \pi c_s$ =1.0, vertical incidence of plane P and SV waves. The results of the present study agree well with those in Mossessian and Dravinski (1989).



(a) P wave incidence (b) SV wave incidence

Figure 2 Verification of present study

4. NUMERICAL RESULTS

A hemispherical canyon in one layer over half-space is studied for simplicity. The parameters are defined as follows: the canyon radius is a, the layer thickness is H; the velocity, mass density and damping ratio for bedrock and soil layer are C_s^R, ρ^R, ζ^R and C_s^L, ρ^L, ζ^L, respectively; The Poisson's ratio both for bedrock and soil layer are v, and v=1/3 is taken for all following calculations. The dimensionless incident frequency is defined as $\eta = 2a/\lambda^L$, where λ^L is the wavelength of the shear waves.

Figure 3 shows the surface displacement amplitude for a hemispherical canyon in homogeneous half-space for the purpose of comparison (ζ=0.02). Figures 4 to 8 show the surface displacement amplitude respectively for

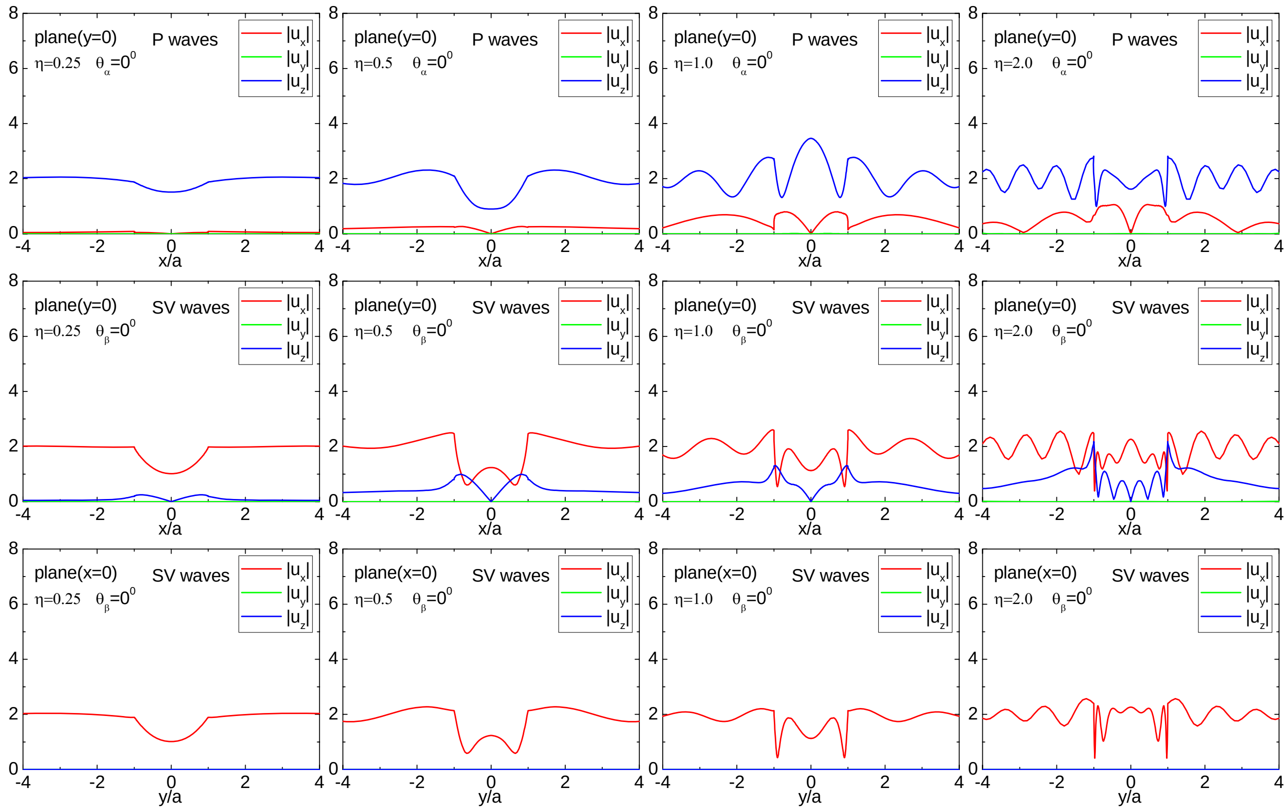


Figure 3 Surface displacement for a hemispherical canyon in homogeneous half-space

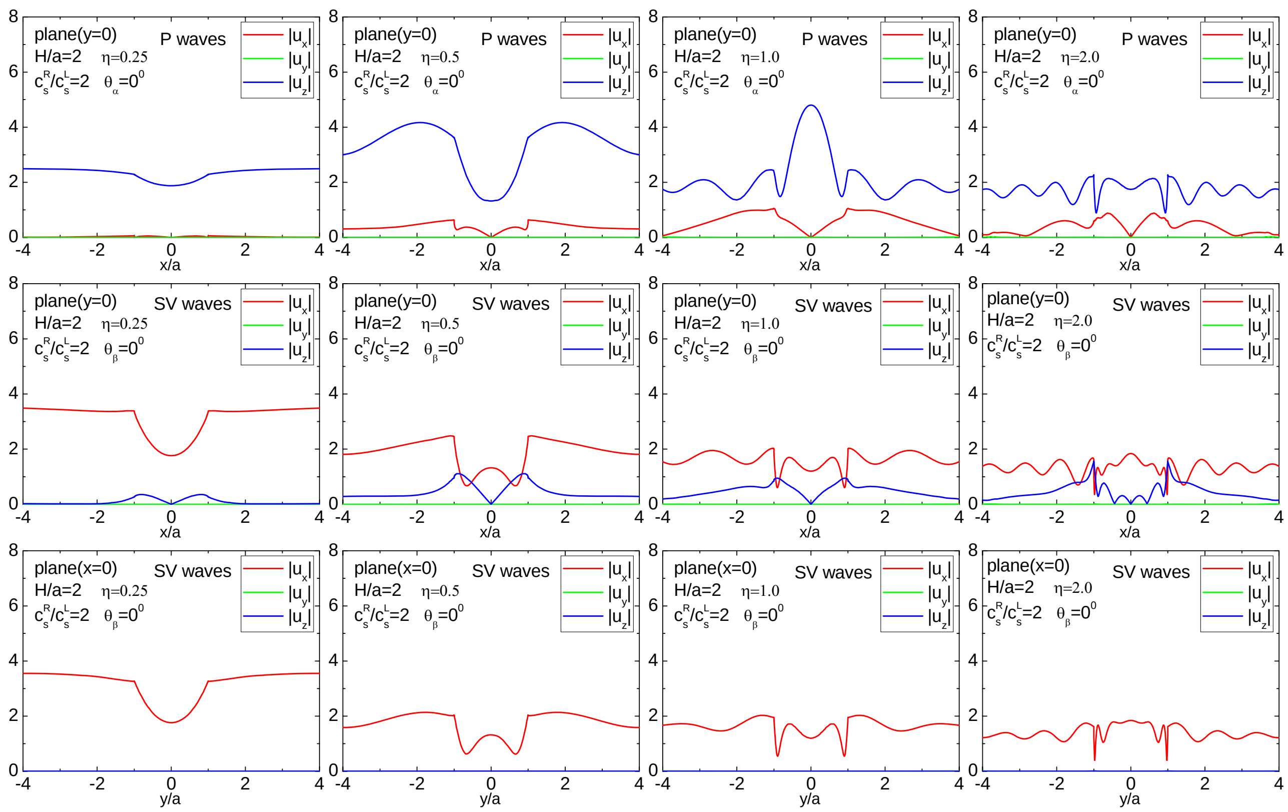


Figure 4 Surface displacement for a hemispherical canyon in layered half-space (H/a=2, C_S^R / C_S^L=2.0)

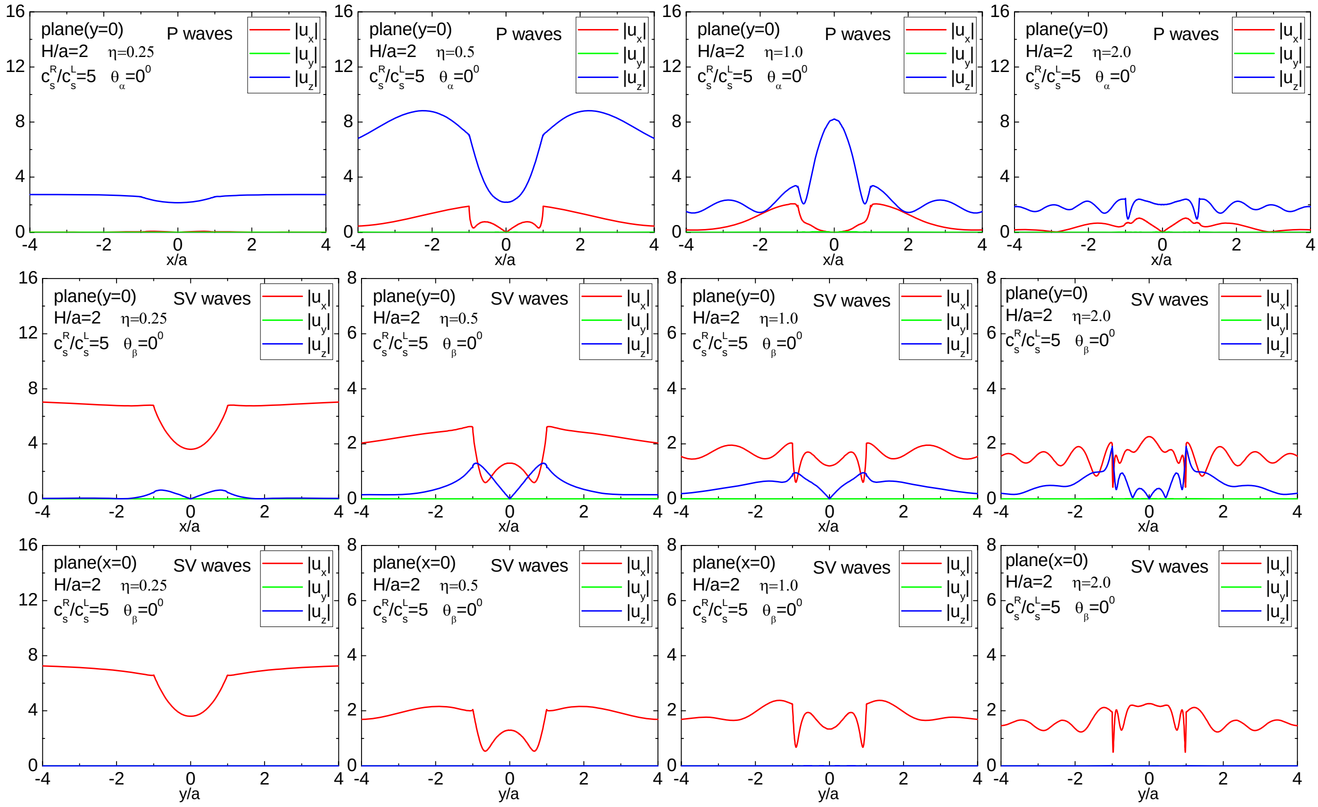


Figure 5 Surface displacement for a hemispherical canyon in layered half-space (H/a=2, C_S^R / C_S^L =5.0)

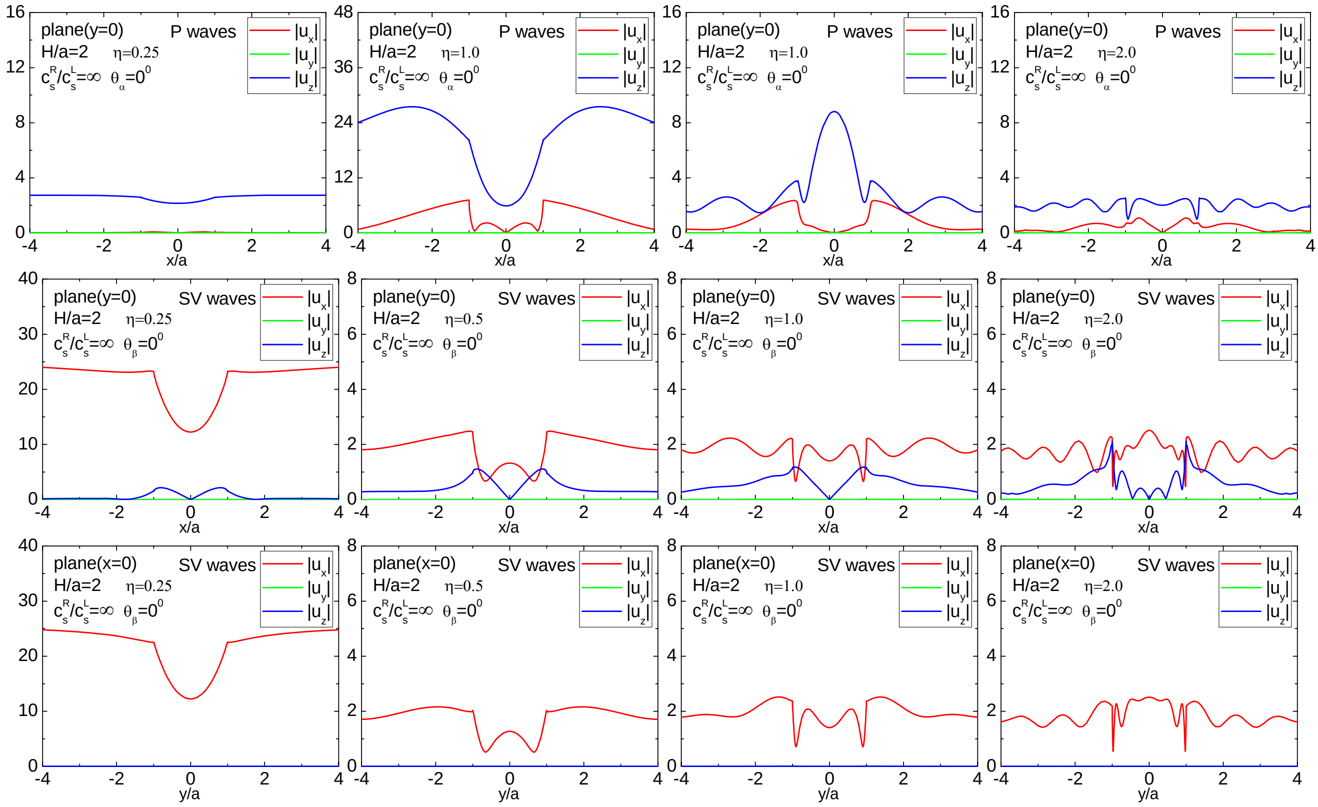


Figure 6 Surface displacement for a hemispherical canyon in layered half-space (H/a=2, $C_S^R / C_S^L = \infty$)

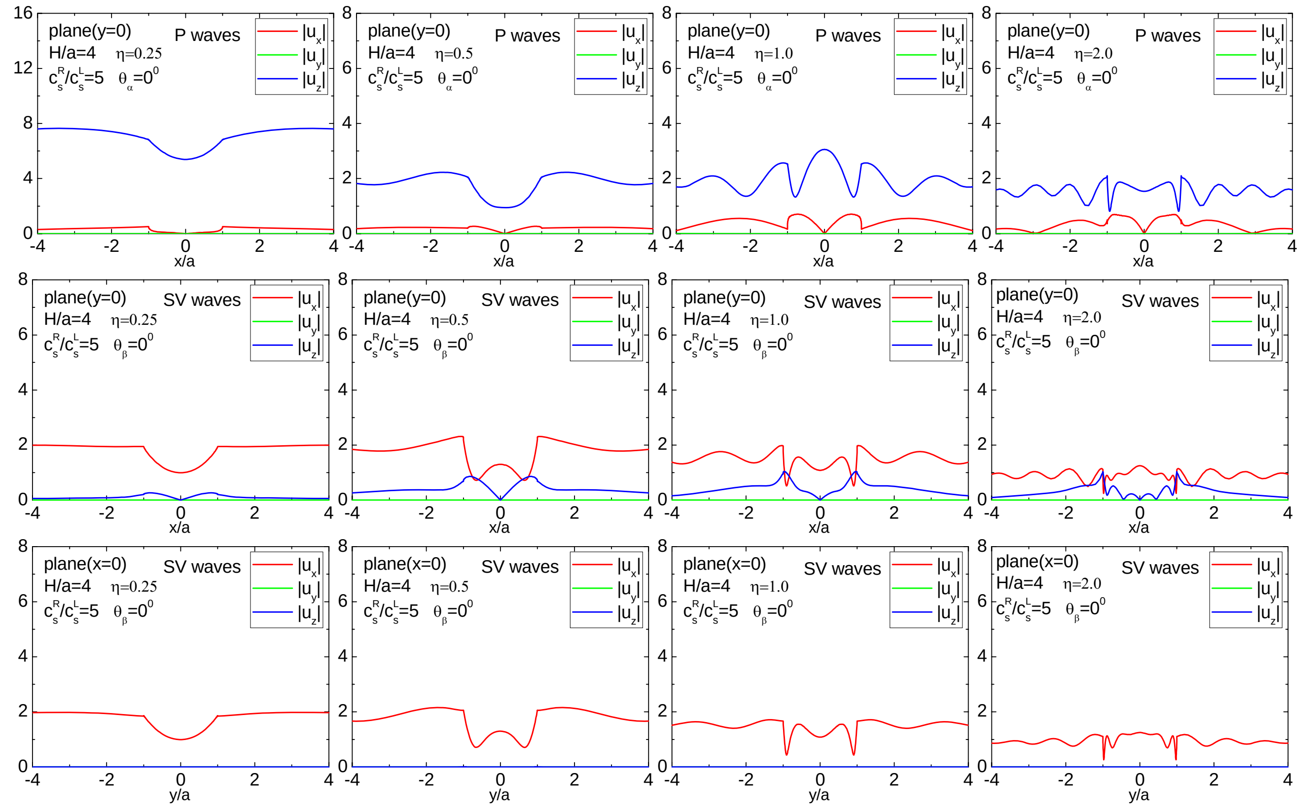


Figure 7 Surface displacement for a hemispherical canyon in layered half-space (H/a=4, C_S^R / C_S^L=5.0)

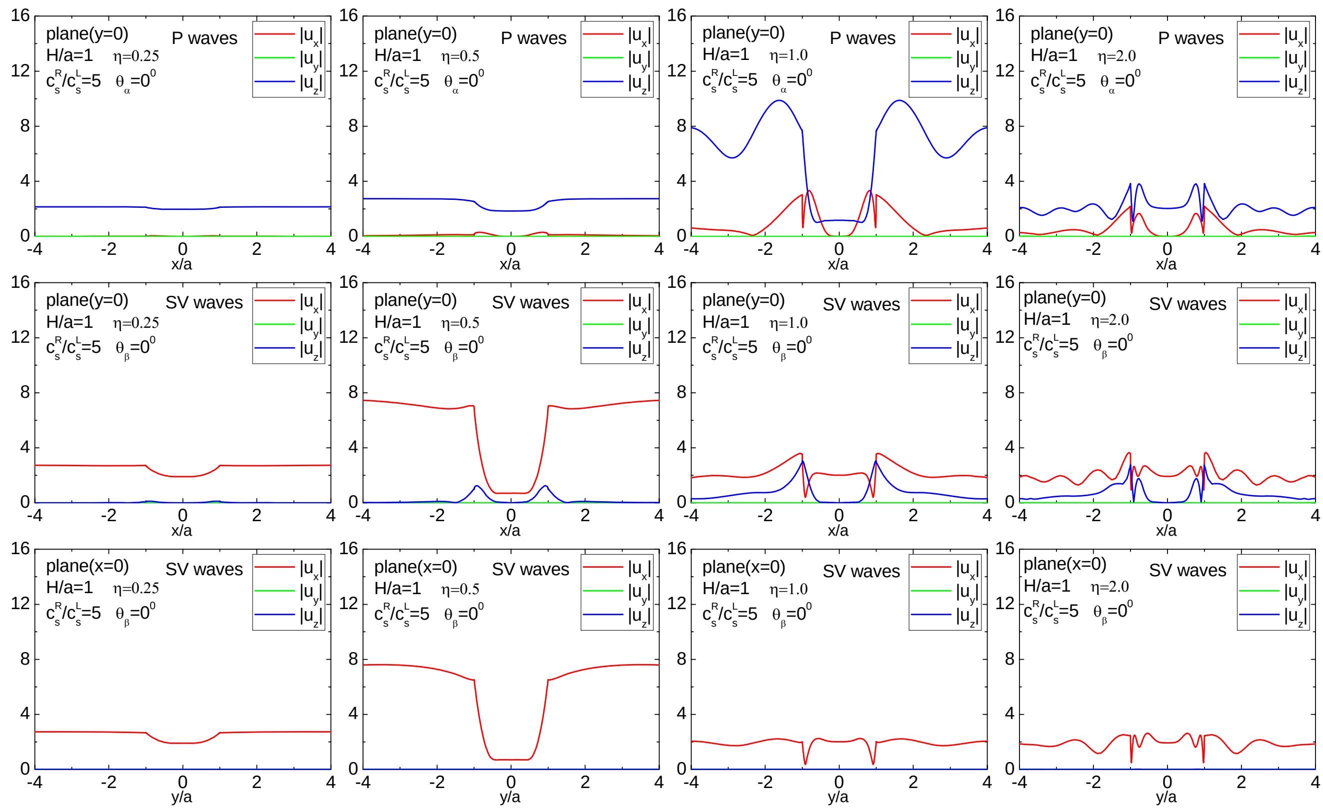


Figure 8 Surface displacement for a hemispherical canyon in layered half-space (H/a=1, C_S^R / C_S^L=5.0)

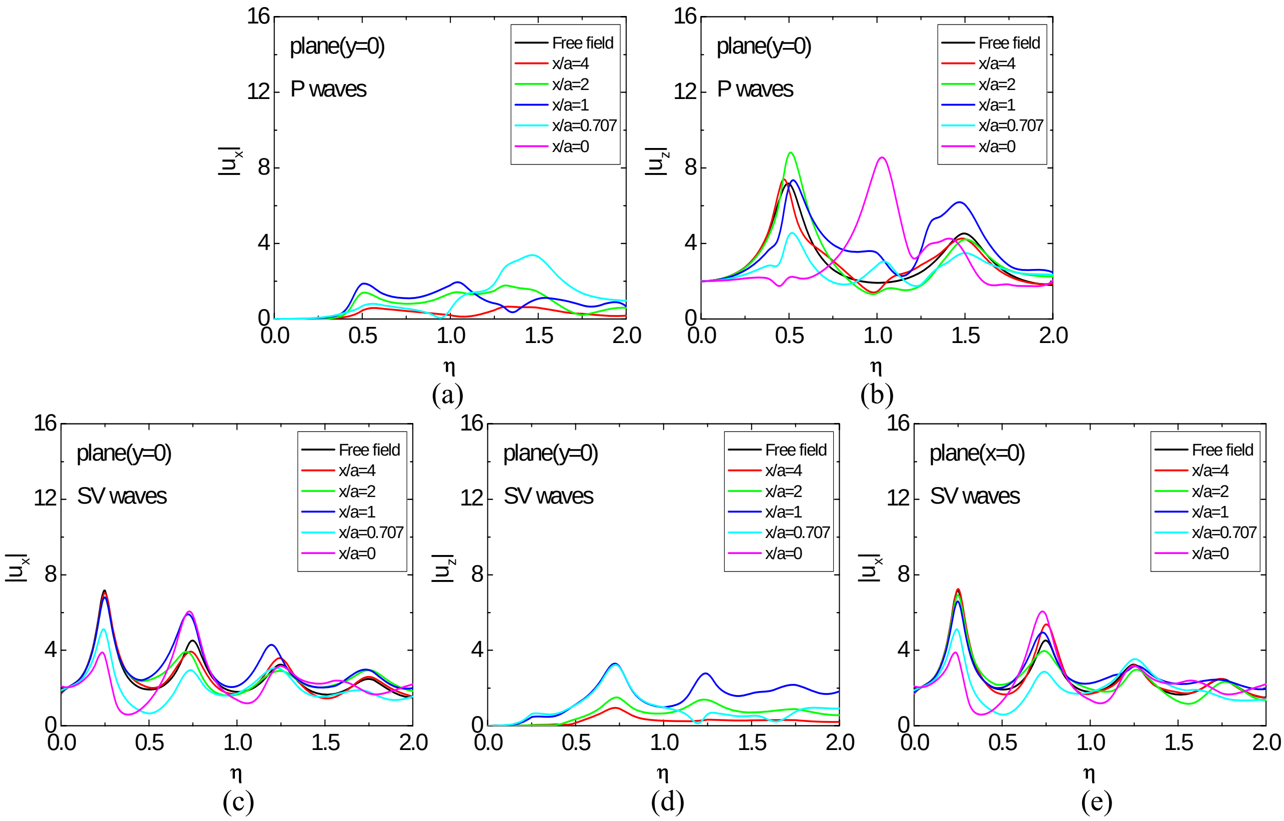


Figure 9 Spectral amplification of surface displacement amplitudes (H/a=2, C_S^R/C_S^L=5.0)

H/a=1, 2 and 4, C_S^R/C_S^L =2.0, 5.0 and ∞, ρ^R/ρ^L=1, ζ^R=0.02, ζ^L=0.05. All these results are for vertical incidence of P and SV waves and the incident frequency η=0.25, 0.5, 1.0 and 2.0, respectively. In these figures, Figure 4 to 6 illustrate the effect of variation of the soil layer velocity on the surface displacement amplitudes, and Figure 5, 7 and 8 illustrate the effect of variation of the soil layer thickness. It is shown that there are significant differences between the surface displacement amplitude for homogeneous (Figure 3) and layered half-space (Figures 4-8); the surface displacement amplitude for layered half-space also highly depends on the thickness and velocity of the layer, or, the resonance characteristics of the layered half-space (the fundamental frequency of the layered half-space for P and SV waves are η=0.5 and 0.25, respectively, for the case H/a=2), besides the incident frequency. In other words, the surface displacement amplitudes depend on both the scattering of incident waves by the canyon and the dynamic characteristics of layered half-space.

Figure 9 shows the spectral amplification of surface displacement amplitudes at different positions in or around the canyon. It is shown that most of the displacement amplitude peaks are located near the resonance frequencies (η=0.5, 1.5, … for P waves; η=0.25, 0.75, … for SV waves) of the layered half-space, however, there are frequency shifts for different positions, and some are located at other frequencies, e.g., the peak for vertical displacement amplitude at x/a=0 for incident P waves is located at η=1.0. Therefore, it can be concluded that there are interaction between the scattering of waves by the canyon and the dynamic characteristics of layered half-space.

5. CONCLUSIONS

This paper presents a solution for plane waves scattering by a 3-D canyon in layered half-space in frequency domain by indirect boundary element method (IBEM), based on the exact dynamic stiffness matrices and the dynamic Green's functions for uniformly distributed loads acting an inclined plane in 3-D layered half-space by the authors. The accuracy of the solution is verified by comparison with the known solutions. The numerical calculations and analyses are performed for a hemispherical canyon in one single layer over half-space for incident waves. It is shown that there exist significant differences between the surface motion of a canyon in layered half-space and that in homogeneous half-space, the surface displacement amplitudes depend on both the scattering of incident waves by the canyon and the dynamic characteristics of layered half-space, and there are interaction between the scattering of waves by the canyon and the dynamic characteristics of layered half-space, the dynamic characteristics of the layered half-space significantly affect both the amplitudes and frequency spectrum of the surface motion.

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