Scattering of SV waves by a local nonhomogeneous body in a layered half-space

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ABSTRACT :

The scattering of SV waves by a local nonhomogeneous body of arbitrary shape in a layered half-space is modeled using the indirect boundary element method (IBEM) in the frequency domain. The free-field responses are calculated by using the exact dynamic-stiffness matrix of soil layers to determine the displacements and stresses at the interface of the local nonhomogeneous body , and fictitious distributed loads are then applied at the same interface in the free field to calculate the Green's functions for displacements and stresses. The amplitudes of the fictitious distributed loads are determined from the boundary conditions, and the displacements arising from the waves in the free field and from the fictitious distributed loads are summed to obtain the solution. The effects of width and thickness of the local nonhomogeneous body, and the incident angle and frequency on the amplitudes of the surface displacements are studied. These results are compared with that of one-dimensional model. The numerical results indicate: The two-dimensional scattering effect of local nonhomogeneous body is significant and quite different from that of one-dimensional model. The influence of local nonhomogeneous body on determination of design earthquake ground motion parameters should be considered reasonably in evaluation of seismic safety for engineering site.

KEYWORDS: layered site, local nonhomogeneous body, IBEM, SV waves, scattering

1. INTRODUCTION

Local geological conditions may generate significant amplification of ground motion and concentrated damage during earthquakes (Hu Yuxian,2006; Li Xiaojun,2006; Wong Y.L.,1998). Scattering of plane waves by local sit conditions can be solved by numerical methods or analytical methods. The weighted residual method was applied to the scattering of plane SH-waves by an underground inclusion of arbitrary shape in a two-dimensional half-space (Manoogian,1996). The wave functions expansion method was used to the problem of scattering of plane SV waves by different irregular topographies (Liang Jianwen, 2003; 2005b; JI Xiaodong, 2006) and to the problem of Scattering of plane SV (LI WeiHua,2004) and Rayleigh (Wang Lei, 2007) waves by circular-arc alluvial valleys with saturated soil deposits. Based on indirect boundary integral equation approach, diffraction of plane harmonic P, SV, and Rayleigh waves by dipping layers of arbitrary shape was investigated (Dravinski,1987). Seismic responses for alluvial valleys subjected to SH, P and SV waves were analyzed by direct boundary element method (BEM) (Fishman, 1995). Amplification of plane P (Liang Jianwen, 2005a) and SV(You Hongbing,2006) waves by a cavity in a layered half-space was discussed respectively by indirect boundary element method (IBEM).

Most of above studies assumed that the irregular topographies located in homogeneous half-space instead of layered half-space. There is significant difference between the seismic response of the layered site and that of the homogeneous site (Liang Jianwen, 2005a; You Hongbing, 2006). Therefore, it is of importance both in theory and engineering to study the waves scattering in a layered half-space.

Wolf (1985) derived the exact dynamic-stiffness matrices of soil layer and half-space, and formed a complete theory for dynamic soil-structure interaction. By using the IBEM in the frequency domain, Wolf's theory is extended to solve the problem of wave scattering by a local nonhomogeneous body of arbitrary shape in a layered half-space. The effects of width and thickness of the local nonhomogeneous body, the incident angle and frequency on the amplitudes of the surface displacements are studied. Some useful conclusions are stated.

2. CONCEPT

The local nonhomogeneous body of arbitrary shape in a layered half-space is shown in Figure 1 (a), where S denotes the interface. This site can be assumed as the combination of the part outside S in Figure 1 (b) and the part inside S in Figure 1 (c). The scattering of SV waves by a local nonhomogeneous body of arbitrary shape in a layered half-space is modeled using the indirect boundary element method in the frequency domain. At first, the free-field response of site in Figure 1 (b) and (c) due to incident SV waves is calculated respectively to determine the displacements and stresses on the line of the interface S. Then fictitious source loads are applied on the same line in the free-field system, and the corresponding Green's functions for the displacements and stresses are calculated. The amplitudes of the source loads are determined by the boundary conditions that the stresses arising from the waves in the free-field and from the fictitious loads on S vanish. These conditions can be satisfied only in an average sense.

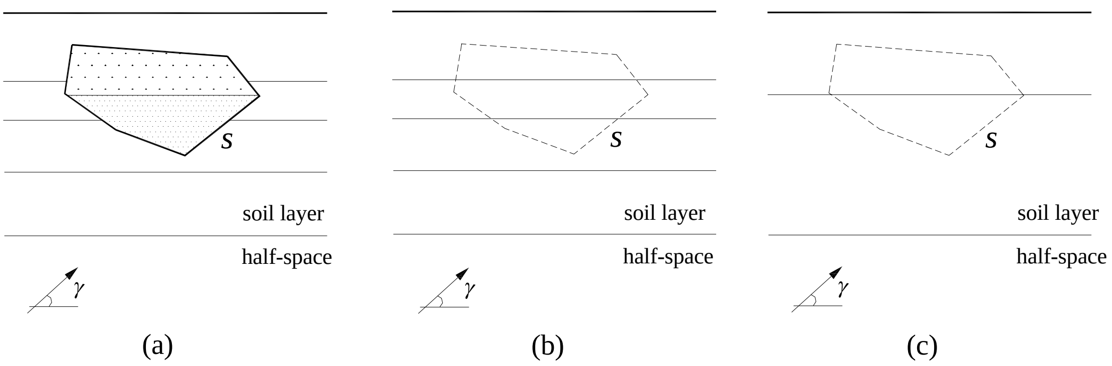


Figure 1 Model of layered system

2.1. Free-field response

By assembling the dynamic-stiffness matrices (Wolf, 1985) and load vectors of the individual layers, the discretized equation of motion in the frequency wave number domain of the layered half-space is constructed. Solving this system leads to the free-field response, consisting of the stresses $t_1^f(s)$, the displacements $u_1^f(s)$ in Figure 1(b) and the stresses $t_2^f(s)$, the displacements $u_2^f(s)$ in Figure 1(c).

2.2. Green's functions

Green's function means the dynamic responses at any point in free-field system when a unit load is applied in somewhere of the system. To calculate the Green's functions, it is appropriate to introduce fictitious interfaces so that each resulting layer is loaded uniformly.

At first the two interfaces of each layer are assumed to be fixed and the corresponding reaction forces are calculated. These are then applied with the opposite sign as loads of the discretized equation of the layered half-space. As only part of the layer is loaded, an additional interface is introduced in the nodes. The layer on which the distributed load acts is first fixed at the two interfaces. The corresponding reaction forces (external loads) are calculated to achieve this condition, whereby the analysis is restricted to the loaded layer. They are then applied with the opposite sign to the total system. To this global response, the result of the analysis for the fixed layers has to be added to calculate the total one. The total system for loads acting at the interfaces can be analyzed using the direct stiffness approach (Wolf, 1985).

These calculations are performed in the wave-number domain k, whereby the load $p(x, z)$ and $r(x, z)$ are transformed as follows:

$$p(k,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}p(x,z)\exp(ikx)dx \tag{2.1}$$

$$r(k,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}r(x,z)\exp(ikx)dx \tag{2.2}$$

The inverse Fourier transformation for the final displacement and stress amplitudes calculated in wave-number domain need to be performed as follows:

$$[F(x)]=\int_{-\infty}^{+\infty}[F(k)]\exp(-ikx)\mathrm{d}k \tag{2.3}$$

Where F denotes displacements or stresses amplitudes.

In Figure 1(b), the fictitious source loads $[p_1]$ and $[r_1]$ are applied on S respectively, and the corresponding Green's functions for the displacements $[g_u^1(s)]$ and stresses $[g_t^1(s)]$ are calculated and can be expressed as

$$\left[u_{g1}(s)\right]=\left[g_u^1(s)\right]\left[p_1 \quad r_1\right]^T \tag{2.4}$$

$$\left[t_{g1}(s)\right]=\left[g_t^1(s)\right]\left[p_1 \quad r_1\right]^T \tag{2.5}$$

In Figure 1(c), the fictitious source loads $[p_2]$ and $[r_2]$ are applied on S respectively, and the corresponding Green's functions for the displacements $[g_u^2(s)]$ and stresses $[g_t^2(s)]$ are calculated and can be expressed as

$$\left[u_{g2}(s)\right]=\left[g_u^2(s)\right]\left[p_2 \quad r_2\right]^T \tag{2.6}$$

$$\left[t_{g2}(s)\right]=\left[g_t^2(s)\right]\left[p_2 \quad r_2\right]^T \tag{2.7}$$

2.3. Boundary condition

The amplitudes of the fictitious distributed loads can be determined from the boundary conditions on S. The displacement continuous condition can be expressed as

$$\left[u_1^f(s)\right]+\left[g_u^1(s)\right]\begin{Bmatrix}p_1\\ r_1\end{Bmatrix}=\left[u_2^f(s)\right]+\left[g_u^2(s)\right]\begin{Bmatrix}p_2\\ r_2\end{Bmatrix} \tag{2.8}$$

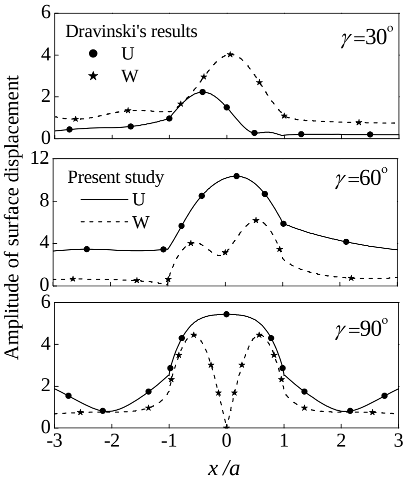
The stress continuous condition can be expressed as

$$\left[t_1^f(s)\right]+\left[g_t^1(s)\right]\begin{Bmatrix}p_1\\ r_1\end{Bmatrix}+\left[t_2^f(s)\right]+\left[g_t^2(s)\right]\begin{Bmatrix}p_2\\ r_2\end{Bmatrix}=0 \tag{2.9}$$

At last, the displacements arising from the waves in the free field and from the fictitious distributed loads $[p_1]$, $[r_1]$ and $[p_2]$, $[r_2]$ are summed to obtain the solution.

3. METHOD VALIDATION

A semi-circular alluvial valley in homogeneous half-space is considered to test the method. A dimensionless frequency is defined: $\eta=2a/\lambda$, where a is the radius of the valley; λ is the wavelength of the incident wave. The ratio of the shear modulus is 1:6. The ratio of the shear wave velocity of the valley soil and half-space is 1:2. Figure 2 shows the present results compared with those of Dravinski (1987), where , $\eta=0.5$, γ is the incident angle. It is shown that the results agree well.



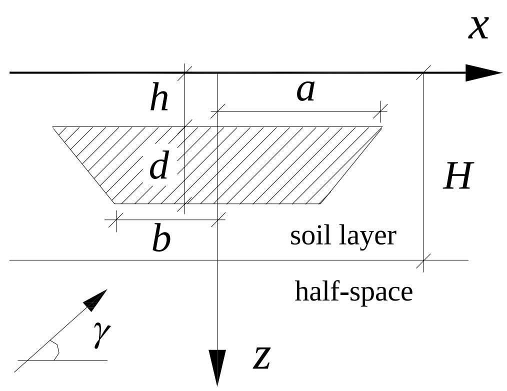


Figure 2 Results compared with those of Dravinski   Figure3 Model of the nonhomogeneous body

4. NUMERICAL RESULTS AND DISCUSSIONS

The model of the nonhomogeneous body is presented in Figure 3, where a and b are the top and bottom half-width of the nonhomogeneous body respectively. d and h are the thickness and depth of the nonhomogeneous body respectively. H is the thickness of the soil layers overlaying on half-space. A dimensionless frequency is defined: $\eta = H/\lambda$, where λ is the wavelength of the incident wave. γ is the incident angle of SV wave. The shear wave velocity of the nonhomogeneous body, soil layer and half-space are 200, 400 and 800 $\mathrm{m/s}$ respectively. The damping ratios are 0.05, 0.05 and 0.02 respectively. The density of all is 2000 $\mathrm{kg/m^3}$.

4.1. The influence of width of the nonhomogeneous body

In order to study the influence of width of the nonhomogeneous body, the following parameters in Figure 3 are selected: $d = 0.2H$, $b = a$, $h = 0$ and $a = 0.5H$, $1.0H$, $2.0H$, $3.0H$ respectively. The other parameters are not varied. Results are shown for SV waves in Figure 4 for two different frequencies ($\eta = 0.5$ and 1.0). $|U|$ and $|W|$ are the horizontal and vertical displacements respectively.

It is evident from Figure 4 that the width of the nonhomogeneous body can magnify SV waves obviously. The width is wider, the range of influence on surface displacement become larger. The two dimensional scattering effect of nonhomogeneous body is significant and quite different from that of one dimensional model. For SV wave vertical incidence, when $\eta = 1.0$ and $a = 0.5H$, the surface horizontal displacements are amplified within $x = \pm 0.5H$, and the maximal amplitudes of horizontal displacement is 3.78. When $a = 3.0H$, the surface displacements are amplified within $x = \pm 3.0H$, and the maximal amplitudes of horizontal displacement is 3.48. These results are greater than that of one-dimensional model: 1.65（$a = 0$, which means no nonhomogeneous body in layered site）and 2.67（$a = \infty$, which means the width of nonhomogeneous body is infinite）.When $\gamma = 60^{\circ}$ and $\eta = 1.0$, the maximal amplitudes of horizontal displacements are 4.87 ($a = 0.5H$), 4.61($a = 1.0H$), 4.41($a = 2.0H$), 4.29($a = 3.0H$) respectively. The corresponding results of free-field responses are 2.09 ($a = 0$) and 2.31 ($a = \infty$) respectively.

4.2. The influence of buried depth of the nonhomogeneous body

To study the influence of buried depth of the nonhomogeneous body The following values in Figure 3 are selected: $b=0.4H$, $a=1.0H$, $d=0.2H$, and $h=0, 0.2H, 0.3H, 0.4H$ respectively. Other parameters are not varied. Results are shown for SV waves in Figure 5 for two different frequencies ($\eta=0.5, 1.0$) as specified in the Figure.

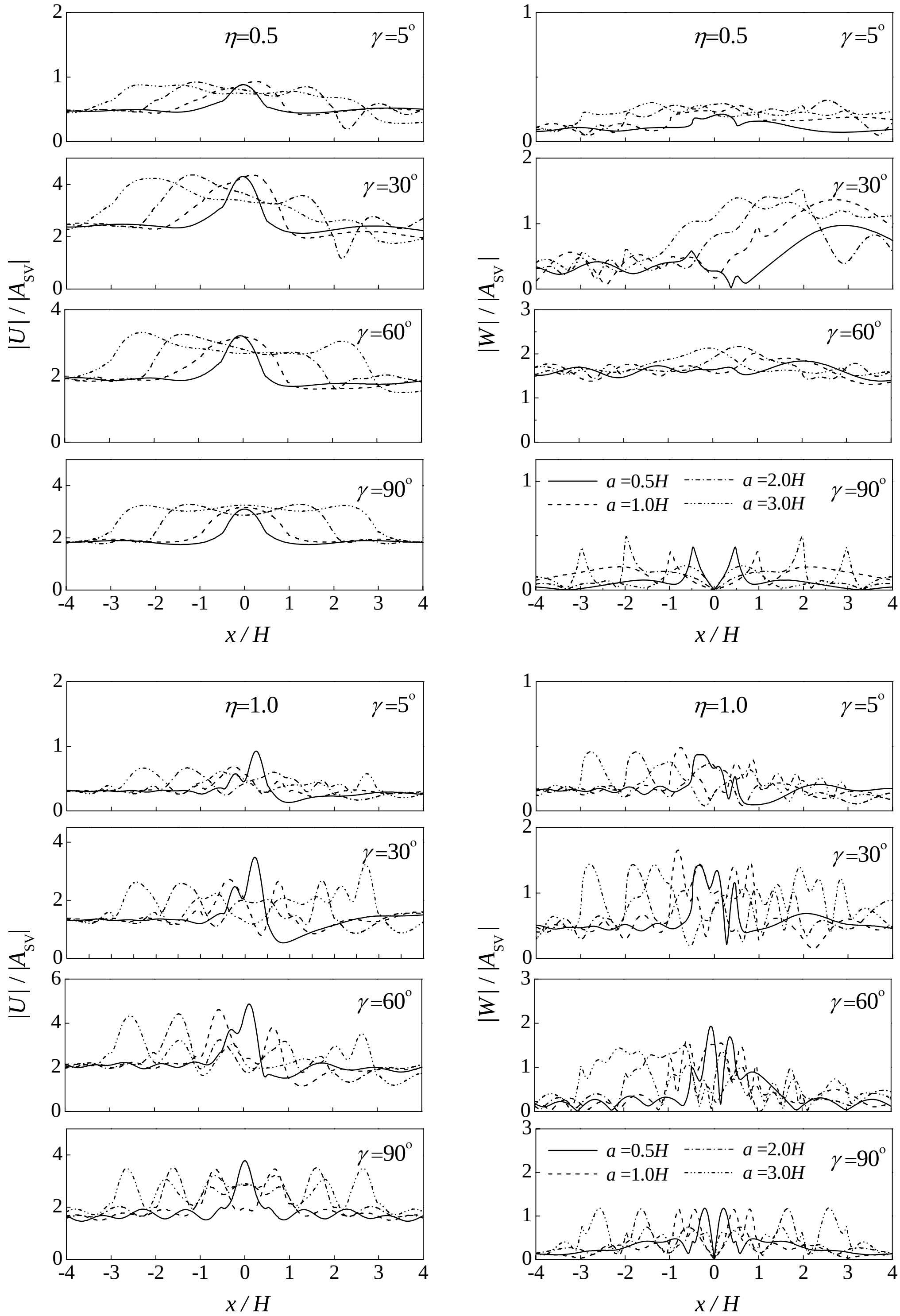


Figure 4 The influence of width of the nonhomogeneous body

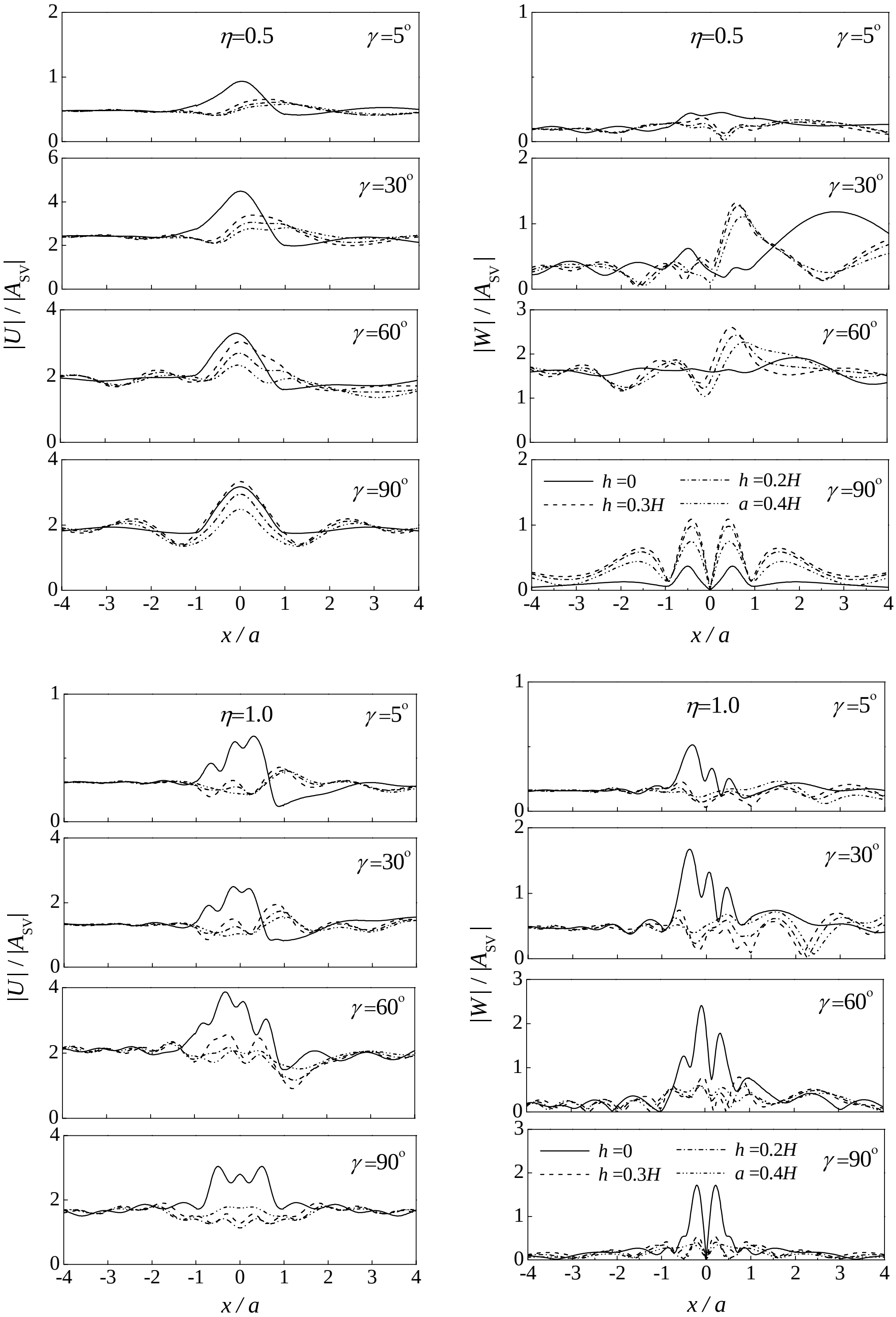


Figure 5 The influence of buried depth of the nonhomogeneous body

It can be seen from Figure 5 that the buried depth of nonhomogeneous body is deeper, the surface displacements are smaller. When $\gamma = 60^{\circ}$ and $\eta = 0.5$, the maximal amplitudes of horizontal displacements are: 3.29 ($h = 0$), 3.03 ($h = 0.2H$), 2.69 ($h = 0.3H$), 2.32 ($h = 0.4H$) respectively. The corresponding results of free-field responses are 1.93 ($a = 0$), 1.69 ($a = \infty$) respectively. The similar instance is seen for $\eta = 1.0$. When the buried depth is shallower ($h < 0.4H$), the surface displacements are larger. When the buried depth of nonhomogeneous body is deeper ($h > 0.5H$), the influence of buried depth of the nonhomogeneous body can be ignored.

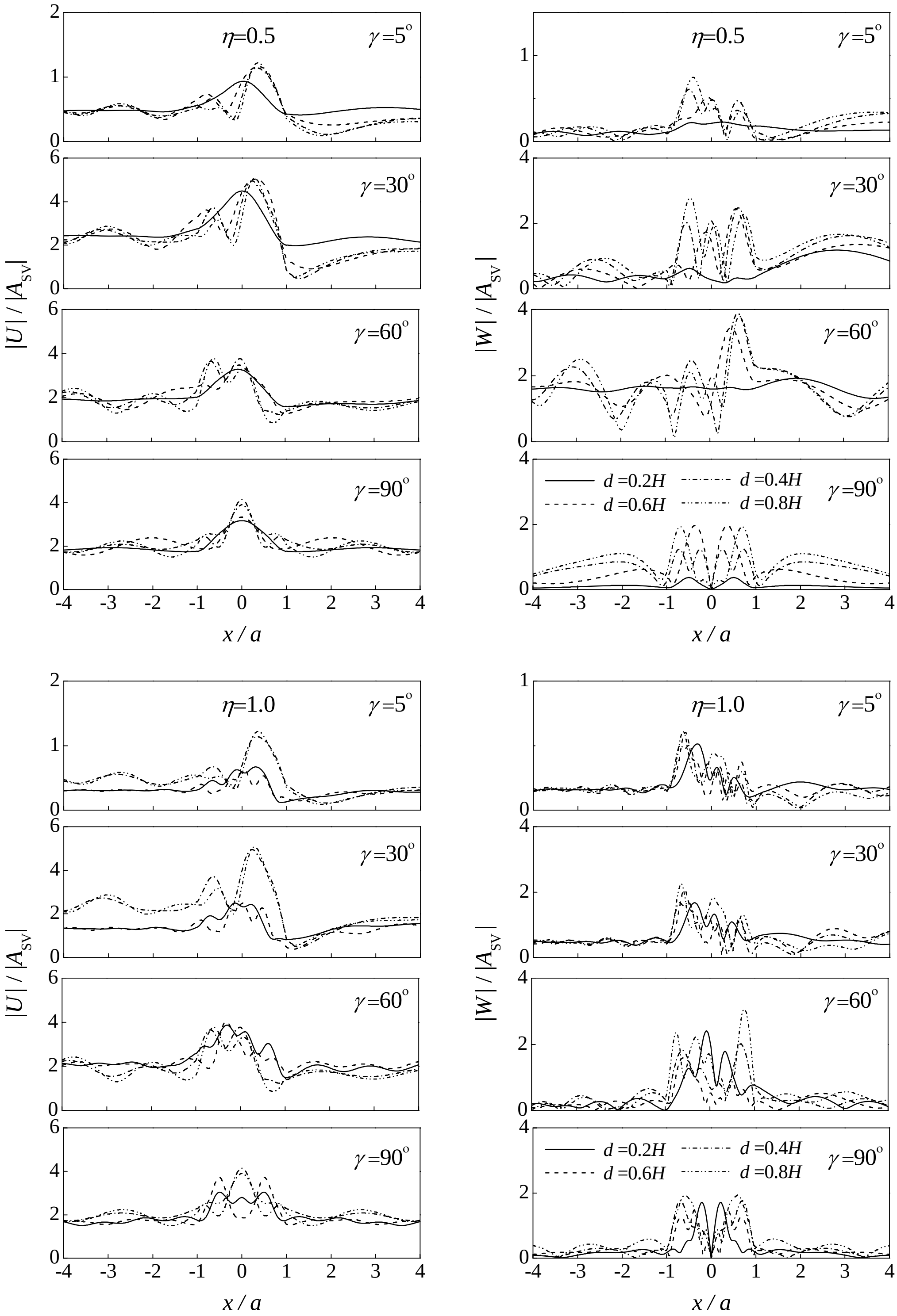


Figure 6 The influence of thickness of the nonhomogeneous body

4.3. The influence of thickness of the nonhomogeneous body

To study the influence of thickness of the nonhomogeneous body, the following values in Figure 3 are selected: $b = 0.4H$, $a = 1.0H$, $h = 0$, and $d = 0.2H$, $0.4H$, $0.6H$, $0.8H$ respectively. Other parameters are not varied. Results are shown for SV waves in Figure 6 for two different frequencies ($\eta = 0.5, 1.0$) as specified in the Figure.

It can be seen from Figure 6 that the thickness of nonhomogeneous body is great, the surface displacements are larger. But this trend is not the linear variation. For example, when $\gamma = 90^0$ and $\eta = 0.5$, the maximal amplitudes of horizontal displacements are: 3.17 ($d = 0.2H$), 3.34($d = 0.4H$), 4.14($d = 0.6H$), 3.9 ($d = 0.8H$) respectively. When $d = 0.6H$, the amplitude of horizontal displacements is the largest.

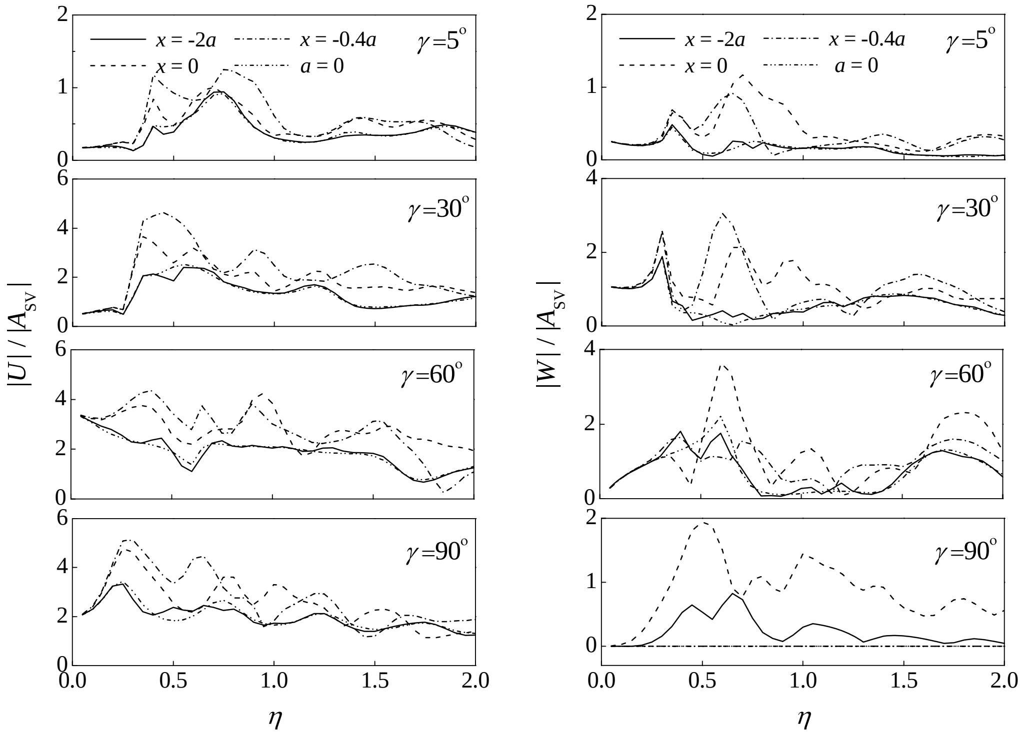


Figure 7 The influence of the frequency of incident wave ($h = 0$)

4.4. The influence of the frequency of incident wave

To study the influence of the frequency of incident wave, the following values in Figure 3 are selected: $b = 0.4H$, $a = 1.0H$ and $d = 0.2H$, other parameters are not varied. Figure7 shows the effect of the frequency of incident wave on amplitudes of surface displacements for $h = 0$. Figure8 shows the effect of the frequency of incident wave on amplitudes of surface displacements for $h = 0.2H$. In these figures, $a = 0$ means no nonhomogeneous body in layered site or the free-field response using one- dimensional model.

It can be seen from Figure 7 and 8 that there is an evident two-dimensional effect for the scattering of SV waves by a local nonhomogeneous body in a layered half-space. The frequency of incident wave has a significant influence on the amplitude of horizontal and vertical displacement. There are several peak values for the amplitudes of displacements. When the incident wavelength is close to the width of the nonhomogeneous body, the amplitude of SV waves are magnified obviously.

For example, when SV wave vertical incidence, the maximal amplitudes of horizontal displacements are: 3.33 ($x = -2a$), 4.77 ($x = -0.4a$), 5.1($x = 0$), respectively (Figure 7). The corresponding result of free-field response is 3.45, and has no change in vary frequency. Therefore the influence of local nonhomogeneous body on the seismic response should be considered properly.

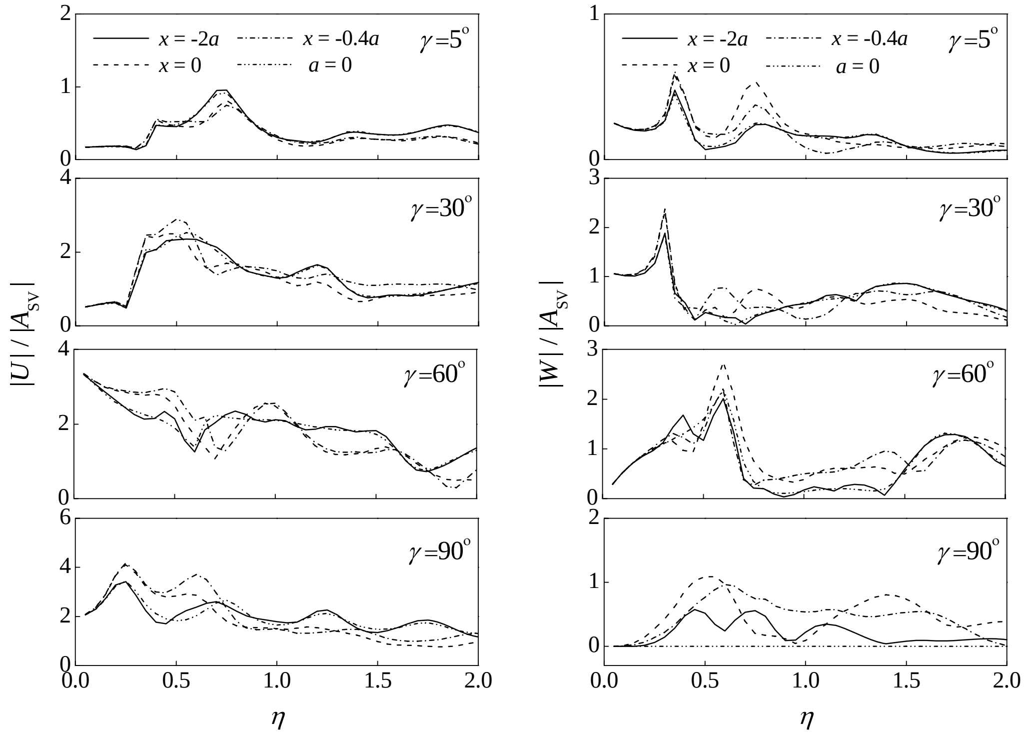


Figure 8 The influence of the frequency of incident wave ($h = 0.2H$)

5. CONCLUSIONS

Based on the exact dynamic-stiffness matrix of soil layers and half-space, the scattering of SV waves by a local nonhomogeneous body of arbitrary shape in a layered half-space is studied using the indirect boundary element method in the frequency domain. The effects of width and thickness of the local nonhomogeneous body, and the incident angle and frequency on the amplitudes of the surface displacements are analyzed. These results are compared with that of one-dimensional model. Some useful conclusions can be stated.

There is an evident two-dimensional effect for the scattering of SV waves by a nonhomogeneous body in a layered half-space. The amplitudes of surface displacements are quite different from the free-field response using one-dimensional model.

The width, thickness and buried depth of the local nonhomogeneous body have a significant influence on the amplitudes of surface displacements. The amplitudes of displacements depend strongly on the frequency and the angle of incident SV waves. Therefore the influence of local nonhomogeneous body on determination of design earthquake ground motion parameters should be considered reasonably in evaluation of seismic safety for engineering site.

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