# SEISMIC STABILITY ANALYSIS OF FOOTING ADJACENT TO SLOPES BY SLIP LINE METHOD

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**ABSTRACT :**

Many structures have to be placed on sloping grounds by shallow footings. Evaluation of seismic stability and bearing capacity of such footings is a prominent matter that rarely is considered by researches. In general there are two slopes next to a footing. Apart from symmetric cases, one of the two sides of footing is more likely to fail (first side) and most researches ignore the effect of the other side of footing (second side) in their analysis. In this paper stress characteristic methods (slip line method) has been employed to show the effect of second side on the bearing capacity of footing under both static and dynamic loads. Both horizontal and vertical earthquake coefficients have been imposed to the system. It is shown that slope of the other side aggravates the stability of footing in comparison with footing placed between two symmetric slopes. Besides the present study has taken in to account the inclination of load on footing. Results show that as the inclination of load on footing increases, the bearing capacity of footing decreases.

**KEYWORDS:** Seismic coefficients, slope, first side, second side, partial mobilization, bearing capacity

## 1. INTRODUCTION

In practice very often foundations have to be placed on top or on the surface of slopes. Structures such as retaining walls, bridge abutments and transmission towers usually are put on slopes by shallow footings. Besides in hilly regions there is no way to escape from putting foundations on top of slops. The problem of evaluating static bearing capacity of these kinds of footings has been discussed in various studies by limit state methods (Meyerhof, 1957, 1963; Hansen, 1970; Vesic, 1973; Reddy, 1976; Saran et al., 1989; Narita, 1990). Just small number of researches can be found in evaluating seismic stability of footing on slopes. Zhu (2000) applied upper bound approach to estimate $N\gamma$ for footing on sloping ground during earthquake. Using method of characteristics, Kumar (2003) evaluated bearing capacity factors of footing on slopes under earthquake body forces.
Most studies incorporate single-side mechanism of failure to asses bearing capacity of footing and ignore the effect of second side of the footing. Budhu & Al-karani (1993) employed limit equilibrium method to estimate seismic bearing capacity of footings on horizontal ground, concerning a both-side failure mechanism.
Saran. et al. (1989) utilized both-side failure mechanism to find Nc, Nq and Ng for footing on the surface, on top or next to top of slopes. They also considered partial mobilization of strength parameters in the side of footing which was less susceptible to failure than the other side.
The matter of partial mobilization of strength parameters which is shown by a coefficient between zero and unity (m) was also used to find seismic bearing capacity of footing on horizontal ground (Budhu & Al-karani, 1993).
Using stress characteristic method (SCM), Kumar (2003) studied the static and seismic bearing capacity of footing on top of slopes and determined Ng from both-side mechanism and Nc and Nq from single-side mechanism.
In this study, assuming both-side mechanism, static and seismic bearing capacity of footings on slopes are evaluated using SCM. Both horizontal and vertical seismic coefficient is taken in to account to do a pseudo static analysis on a $c-\phi$ soil.
It is assumed that at a side of footing which is less likely to collapse (second side); just a portion of strength is mobilized. In addition, angles of slopes next to footing are varied to evaluate the effect of second side of footing on the bearing capacity.

### *1.1. Definition of the problem*

This study aims at finding the ultimate bearing capacity of a horizontal footing with width B in the presence of vertical and horizontal earthquake acceleration $K_v\,g$ *and* $K_h\,g$. Generally two sloping ground with different inclination angle (i1 and i2) are supposed on each side of footing (Fig 1). Arbitrary surcharges, q1 and q2, with different inclination angle to normal on surface, j1 and j2 can be applied on slopes. In determination of bearing capacity of footing, inclination of load on footing is also incorporated, That is bearing capacity has been determined by this method while tan (H/V) #0.

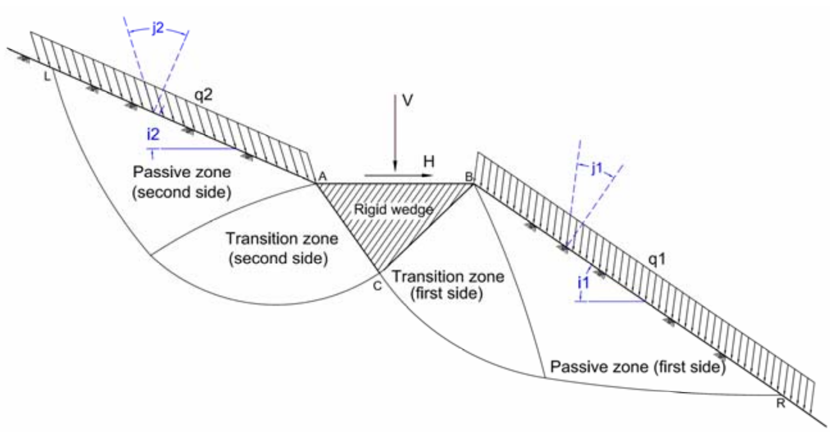


Fig1. Geometry and loading condition of a footing on top of slope

## 2. STRESS CHARACTERISTICS METHOD (SCM)

Stress characteristics method is a well-known approach to solve stress boundary value problem on their limit state. This method assumes equilibrium of stresses everywhere in the system while components of stress at every point of the body satisfy the yield criterion. Using SCM, Sokolovski (1960), provided the solutions of different problems under plane strain condition and Cox *et. al*. studied the axi-symmetric case.
A typical soil element has been shown in Fig 2(a) and corresponding Mohr circle of stress is drawn on Fig 2(b). On assumption that soil obeys Mohr-Coulomb Yield criterion, two failure surfaces that are depicted by lines SCL (+) and SCL (-) on Fig 2(b), can be recognized on stress space where P is the pole. SCL (+) and SCL (-) are called positive and negative stress characteristics lines respectively and they make angle $\mu = (\pi/4) - (\phi/2)$ with major principal stress $\sigma_1$.

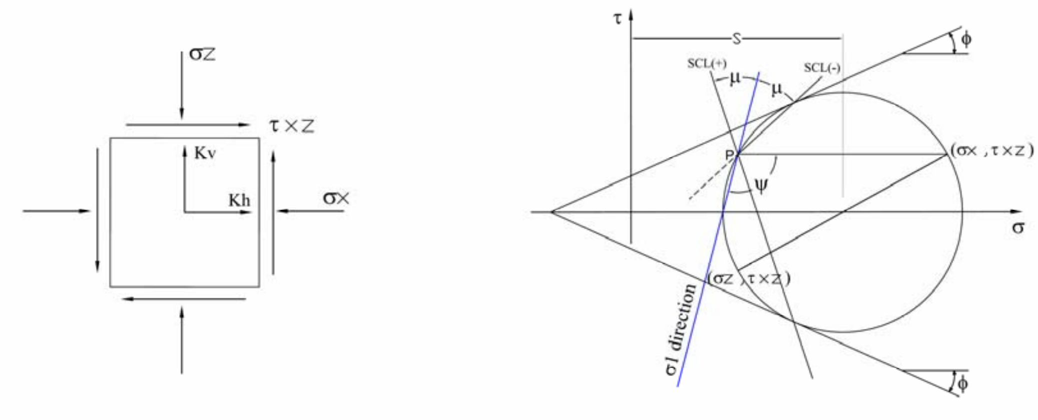


Figure 2 (a) A typical soil element under seismic loads. (b) Mohr circle of stress of element shown on part a.

It is possible to relate normal and shear stresses to S and $\psi$. S is the mean stress and $\psi$ is the angle that major principal stress makes with positive direction of axis x (Eqn. 2.1).

$$\begin{cases} \sigma_x = S + (S.\sin\phi + c.\cos\phi).\cos 2\psi \\ \sigma_z = S - (S.\sin\phi + c.\cos\phi).\cos 2\psi \\ \tau_{xz} = (S.\sin\phi + c.\cos\phi).\sin 2\psi \end{cases} \quad (2.1)$$

Equilibrium equations for element in Fig 2(a) can be stated as:

$$\begin{cases} \dfrac{\partial\sigma_x}{\partial x} + \dfrac{\partial\tau_{xz}}{\partial z} = f_x \\ \dfrac{\partial\tau_{xz}}{\partial_x} + \dfrac{\partial\sigma_x}{\partial z} = f_z \end{cases} \quad (2.2)$$

In Eqn 4.2 , $f_x = K_h\,\gamma$ and $f_z = (1 - K_v)\gamma$.
Substituting Eqn. 2.1 in to Eqn. 2.2 and solving the resulting set of equations, a couple of failure directions will be found (Eqn. 2.3 and Eqn. 2.4).

$$dz / dx = \tan(\psi + \mu) \tag{2.3}$$

$$dz / dx = \tan(\psi - \mu) \tag{2.4}$$

Eqn. 2.3 and Eqn. 2.4 indicate positive and negative stress characteristics direction respectively (Fig 3). As mentioned earlier $\mu = (\pi / 4) - (\phi / 2)$ .

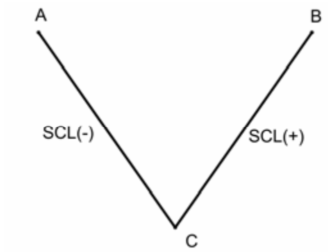


Figure 3 Positive and negative stress characteristic lines (SCL)

Further result of Eqn. 2.1 and Eqn. 2.2 is that the following equations are governing equations along positive and negative characteristics lines respectively.

$$\begin{aligned} &dS + 2(S\tan\phi + c)d\psi = -f_x(\tan\phi\, dz - dx) + f_z(\tan\phi\, dx + dz) + \\ &(S - c\tan\phi)\left(\frac{\partial\phi}{\partial z}dx - \frac{\partial\phi}{\partial x}dz\right) + \left(\frac{\partial c}{\partial z}dx - \frac{\partial c}{\partial x}dz\right) \end{aligned} \tag{2.5}$$

$$\begin{aligned} &dS - 2(S\tan\phi + c)d\psi = f_x(\tan\phi\, dz + dx) - f_z(\tan\phi\, dx - dz) - \\ &(S - c\tan\phi)\left(\frac{\partial\phi}{\partial z}dx - \frac{\partial\phi}{\partial x}dz\right) - \left(\frac{\partial c}{\partial z}dx - \frac{\partial c}{\partial x}dz\right) \end{aligned} \tag{2.6}$$

Equations 2.3 to 2.6 are a set of partial differential equations and can be solved by an appropriate numerical approach such as finite difference method.
suppose A and B are two arbitrary points on the boundary of slope (Fig 3); having quantity of variables x, z , S and $\psi$ on A and B and solving for equations 2.3 to 2.6 the same variables will be found on point C inside the slope body. Then point C becomes a known point and can be used to find stress state of other points on the body at limit state. In other words, by application of equations of SCM along characteristic lines and having stress boundary conditions, stress field at limit state can be calculated.

## 3. SOLUTION PROCEDURE

Every foundation on slope and in 2D cases is under the influence of two slopes, one on the right side and another on the left side. A side which is more likely to be run by footing is called first side and another side is named second side. Mathematically for an isotropic and homogeneous soil mass, if an arbitrary point M is chosen on each slope, the one which has the greatest algebraic value of the following equation (Eqn. 3.1) is more susceptible to failure and so is the first side.

$$\tan(SC) = \frac{z_M}{|x_M| - \frac{B}{2}} \tag{3.1}$$

In Eqn. 3.1, B is the width of footing. Based on the assumed inclination angle of applied load on footing, it is possible to find angles $\alpha_1$ and $\alpha_2$ of rigid wedge ABC under the footing (Fig 4). The calculated $\alpha_1$ and $\alpha_2$ must satisfy the relation $\alpha_1 + \alpha_2 = \pi/2 + \phi$. Internal angle $\beta$ of wedge ABC is assumed to be formed from intersection of a pair of positive and negative slip lines, therefore its value is equal to $\pi/2 - \phi$. After determination of internal angles of rigid wedge ABC, the coordinate of vertex C, can be calculated readily.

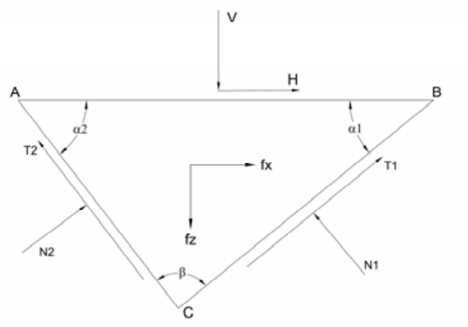


Figure 4 Forces diagram of rigid wedge under the footing.

Boundary conditions on slopes are known. So using equations 2.3 to 2.6 and solving for active and transition zones, stresses and then tangential and normal forces (T1, T2, N1, N2) on sides AC and BC of rigid wedge under the footing will be emerged. Satisfying equilibrium equations of rigid wedge along x and z directions, give the values of H and V respectively.
H and V must be checked to see if tan (H/V) is identical to inclination of assumed load on footing. If it was the case, the solution ends and if not, coefficient of partial mobilization of strength (m) on the second side of footing must be changed. By try and error, the exact value of m is found so that tan (H/V) becomes equal to load inclination on footing. Assumed inclination angle of load on footing must be smaller than friction angle between soil and footing, otherwise H and V have to be determined based on the friction angle soil-footing interface.

## 4. RESULTS

To evaluate the effect of angle i2 (angle of second side slope) on vertical bearing capacity of footing, certain slope and soil type $(i_1 = 30^\circ, B = 1m, \gamma = 18\,KN/m^3, c = 40\,KPa, \phi = 30^\circ)$ have been assumed and calculated under various horizontal seismic coefficients.

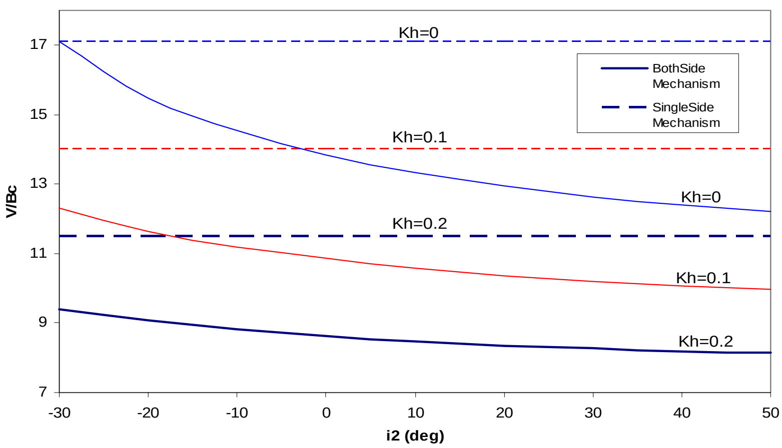


Figure 5 Effect of second side slope on bearing capacity of footing.

As depicted results on Fig 5 show, for all cases, increase in i2, leads to decrease of vertical bearing capacity V/Bc (c is cohesion of soil). Here i2 decreases when the algebraic value of Eqn. 3.1 at second side goes down. The vertical bearing capacity tends to increase, as the algebraic value of Eqn. 3.1 at second side approaches the one of first side.

Effect of horizontal seismic coefficient on bearing capacity of footings has also been assessed. To achieve this, different Kh are applied on a footing with certain value of i1 and i2. Results (Fig 6) indicate that by increasing Kh (toward first side), vertical bearing capacity decreases, but the rate of descending is lower for larger first side angle. To evaluate and compare the effects of both vertical and horizontal seismic coefficients on bearing capacity of footings, for some Kh's (Kh=0.05, 0.1, 0.15, 0.2, 0.25), various Kv's are applied and problem is solved (i1 and i2 values are constant). Results show that increasing Kv contrary to gravity direction leads to decrease in bearing capacity for each value of Kh (Fig 7). In addition, if certain increase in Kv and Kh are applied to system, rate of decrease of bearing capacity due to Kh is larger than the one of Kv (Fig 7).

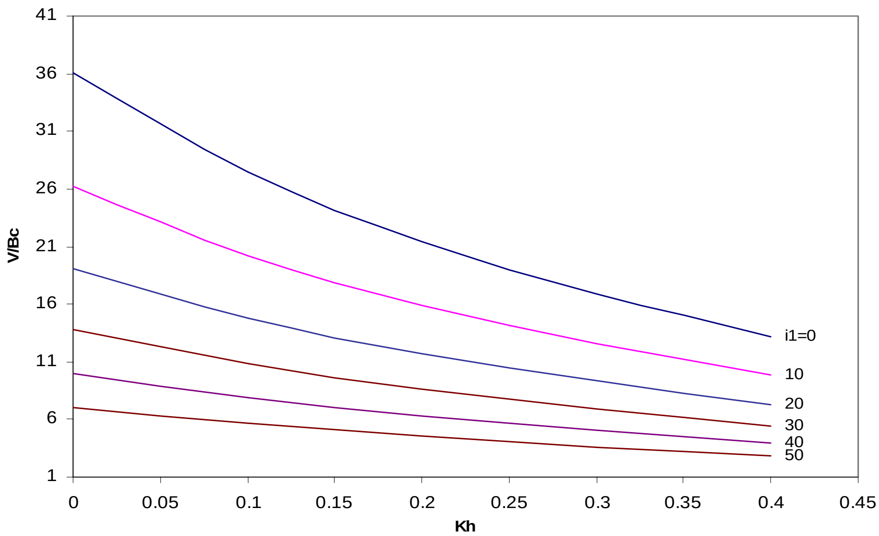


Figure 6 Effect of horizontal earthquake coefficients on bearing capacity of footing $(i_2 = 0^\circ, B = 1\,m, \gamma = 20\,KN/m^3, c = 50\,KPa, \phi = 30^\circ)$.

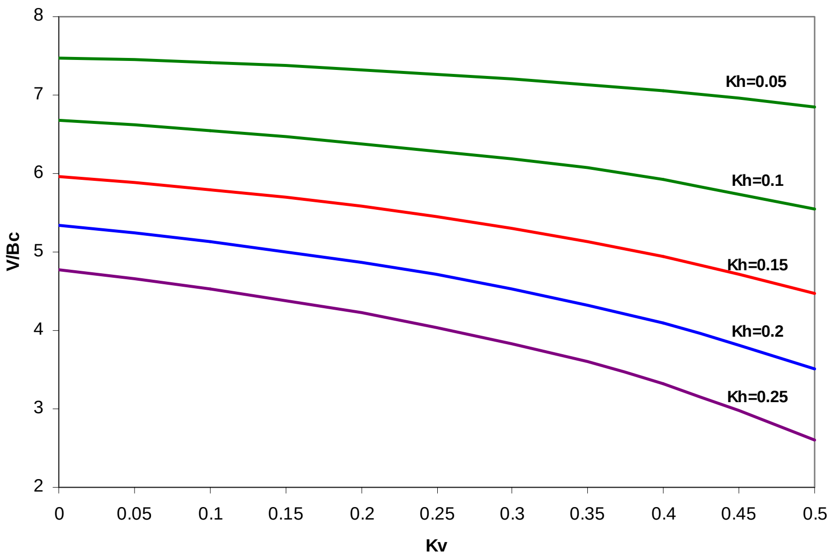


Figure 7 Effect of horizontal and vertical earthquake coefficients on bearing capacity of footing.
$(i_1 = 45^\circ,\ i_2 = 0^\circ, B = 1\,m, \gamma = 20\,KN/m^3, c = 50\,KPa, \phi = 30^\circ)$

Results of present study are compared to those of other researches. Method of Saran et al. (1989) is more similar to present study since both studies, consider partial mobilization of strength parameters at second side. Results show that in all cases, calculated $N\gamma$ of present study is smaller than that assessed by the other researches.

Results of Meyerhof (1957) are closer to values estimated by the present study (Table 1) and the worst difference is with Saran et al. (1989). In Table 4.1, De is the distance between footing corner and top of slope.
Nc values for various internal friction angles and in different first side angle (i1) are found and compared to Hansen solutions (Fig 8). At i1=0 both study give the same result while for other values of i1, Hansen solutions are always greater than present study. To make precise comparison, the value of i2 is supposed to be zero for all cases.

Table 4.1 Comparison of present study to those of other researches (Saran, 1989).

| $\phi$ | i1 | De/B | Meyerhof (1957) | Mizuno (1960) | Siva Reddy and Mogaliah (1975) | Chen (1975) | Saran (1989) | Present Study |
|---|---|---|---|---|---|---|---|---|
| 30 | 15 | 0 | 10 | 11 | 13.76 | 12 | 15.25 | 10.55 |
| 30 | 20 | 0 | 7.5 | 8 | - | 10 | 11.61 | 7.475 |
| 30 | 30 | 0 | 3.1 | - | 5.01 | - | 6.14 | 2.273 |
| 40 | 20 | 0 | 34 | 44 | - | 55 | 53.47 | 33.6 |
| 40 | 20 | 1 | 55 | - | - | - | 85.98 | 50.065 |
| 40 | 20 | 2 | 70 | - | - | - | 121.22 | 63.933 |
| 40 | 30 | 0 | 20 | - | - | 19.5 | 25.37 | 14.721 |
| 40 | 30 | 1 | 40 | 17 | - | - | 62.2 | 26.785 |

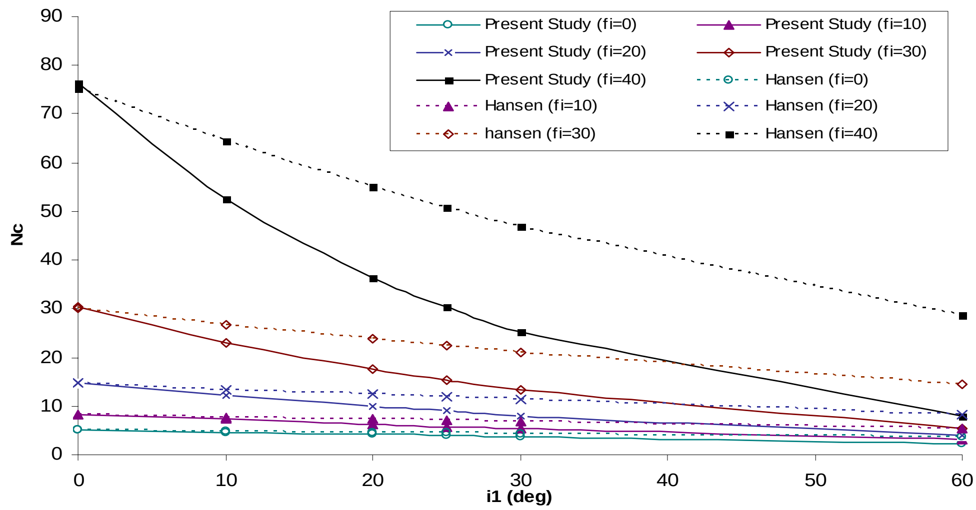


Figure 7 Comparison of Nc value of present study with Hansen solution.
$(i_2 = 0^\circ, B = 1m, K_v = 0, K_h = 0)$

## 5. CONCLUSIONS

The results of the present study can be summarized as follows:

1- Increase in second side slope angle, leads to decrease of static and seismic vertical bearing capacity of footing on slopes.
2- As the horizontal seismic coefficient in direction of first side slope increases, regardless of first and second side slope angles, vertical bearing capacity decreases.
3- Bigger vertical seismic coefficient, contrary to gravity direction, results in smaller vertical bearing capacity of footing.
4- Partial mobilization of strength parameters or m closes to unity as first and second side slopes tend to be symmetric with respect to z axis.
5- Bearing capacity of footing resulted from this study is always smaller than those from other studies. This fact is due to partial mobilization of strength parameter in the second side of footing.

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