A formula for Calculating Liquefaction Probability Based on Chinese Seismic Code

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ABSTRACT :

The method of liquefaction prediction is very important to seismic design of foundations and engineering structures. The liquefaction prediction method in Chinese seismic code has formed more than 30 years and has been applied in engineering comprehensively. As the rapid development of engineering construction in China, more demand for liquefaction evaluation is raised. There are two questions should be answered. One is the reliability of liquefaction prediction results and only the simple answer of “yes” or “no” liquefaction is not enough. The other is that when the liquefaction prediction result is close to the critical state of discriminant the engineers often feel confusion. They want to know the conservation extent of the discriminant. However, the existing method is deterministic and can not answer the questions.

The purpose of this paper is promoting the deterministic method of Chinese code discriminant to the probability method of liquefaction evaluation. Firstly, the liquefaction limit state is established according to the Chinese code discriminant, and the variable parameters are determined considering the influence factor of liquefaction, and then a probability formula is formed using first order and second moment principle. In addition, the safety factor of liquefied cases and non-liquefied cases is calculated using Chinese code discriminant and its distribution is statistically analyzed, respectively, the cure of liquefaction probability versus SPT ratio (namely safety factor) is proposed according to Bayesian theory. The extent of the conservation of Chinese code discriminant is attained from this cure. Finally, the two results deriving from these two theories are discussed and the discrepancy, merit and demerit of the new formula are pointed out.

KEYWORDS: liquefaction prediction, probability, Chinese code

1. INTRODUCTION

The deterministic liquefaction evaluation method can only answer whether the soil liquefy or not by "yes" or "no". As the rapid development of engineering construction in China, more demand for liquefaction evaluation is raised. Liquefaction Reliability analysis urgently needs to be given comparing to the upper structure reliability design.

The variable parameter and its distribution need to be given firstly, and then the probability can be got by directly integrating. The liquefaction influence factors contain earthquake indeterminacy and the soil characteristic. It is very difficult to calculate liquefaction probability by directly integrating, because the statistical parameter of random variable can not be determined in engineering, and even the type of variable parameter distribution was judged by the expert experience. While the mean (first order) and the standard deviation (second moment) are relative easy to obtain. The first order and second moment method is that the failure probability is calculated based on the statistical parameter of random variable and the liquefaction critical state function. Bayes method is that given the deterministic liquefaction evaluation to calculate the safety factor and the liquefaction probability calculate by statistic the safety factor.

The liquefaction prediction method in Chinese seismic code has formed more than 30 years and has been applied in engineering comprehensively. It is significance to improve the code method to the probability calculation forma.

2. PROBABILITY CALCULATION MODEL

The influence factors and distribution characteristics of the liquefaction should be considered in calculating the total probability. The influencing factors on liquefaction contain two aspects. One is seismic factor such as earthquake magnitude, focal depth, epicenter distance and duration time, those reflects the force and time. The other is soil property such as buried depth, groundwater level, dense degree and clay content, which reflects the liquefaction resistance of the soil layers. The parameter $X_1, X_2, \cdots, X_n$ was used to express the liquefaction influence factors, and the function $Z = g(X_1, X_2, \cdots, X_n)$ could be used to express the state of the soil liquefaction.

$$Z = g(X_1, X_2, \cdots, X_n)\begin{cases} < 0 \cdots \text{liquefaction state} \\ = 0 \cdots \text{limit state} \\ > 0 \cdots \text{nonliquefaction state} \end{cases} \tag{1}$$

The liquefaction probability could be obtained by integrating:

$$P_L = P(Z < 0) = \iint_{Z<0} \cdots \int f_X(x_1, x_2, \cdots, x_n) dx_1 dx_2 \cdots dx_n \tag{2}$$

The function $f_X(x_1, x_2, \cdots, x_n)$ is the joint probability density of random variables (continuous assumption).

The influencing factors on liquefaction of soil have strong randomness and discreteness. The statistical parameters of random variables are difficult to be got, so it difficult to deduce the distribution types of the random variables. Some statistical parameters and distribution types of the random variables are even got by experts' experience. So, the simplified model is needed.

2.1 FIRST ORDER AND SECOND MOMENT MODEL

Using the first order and second moment model to calculate the failure probability is depending on the statistical characteristics and limit state equation of the random variables. The distribution types of liquefaction influence factors are difficult to be determined. The mean value and standard deviation of state function are difficult to calculate. It needs to make approximate conversion about the state function to get mean value and standard deviation simply. The state function $Z = g(X_1, X_2, \cdots, X_n)$ can be expanded as Taylor series at mean value of the random variables.

$$Z = g(\mu_{X_1}, \mu_{X_2}, \cdots, \mu_{X_n}) + \sum_{i=1}^{n}(X_i - \mu_{X_i})\frac{\partial g}{\partial X_i} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(X_i - \mu_{X_i})(X_j - \mu_{X_j})\frac{\partial^2 g}{\partial X_i \partial X_j} + \cdots \tag{3}$$

Where, μ_{X_i} is the mean value of X_i. If neglecting all the high order term and preserving the linear term, and assuming $X_1, X_2 \cdots, X_n$ as statistical independence, and then mean value and variance can be approximately expressed as,

$$\mu_Z = E(Z) \approx g(\mu_{X_1}, \mu_{X_2}, \cdots, \mu_{X_n}) \tag{4}$$

$$\sigma_Z{}^2 \approx \sum_{i=1}^{n}(\frac{\partial g}{\partial X_i})^2_{\mu_{X_i}}\sigma^2_{X_i} + \sum_{\substack{i,j=1 \\ i \neq j}}^{n}(\frac{\partial g}{\partial X_i})_{\mu_{X_i}}(\frac{\partial g}{\partial X_j})_{\mu_{X_i}}\mathrm{cov}(X_i, X_j) = \sum_{i=1}^{n}(\frac{\partial g}{\partial X_i})^2_{\mu_{X_i}}\sigma^2_{X_i} \tag{5}$$

Where, $(\frac{\partial g}{\partial X_i})^2_{\mu_{X_i}}$ is the square value of state function's partial derivation to X_i at mean value of μ_{X_i}.

The liquefaction probability P_L is shown as,

$$P_L = \Phi(-\beta) = 1 - \Phi(\beta) = 1 - \Phi(\frac{g(\mu_{X_1}, \mu_{X_2}, \cdots, \mu_{X_n})}{\sqrt{\sum_{i=1}^{n}(\frac{\partial g}{\partial X_i})^2_{\mu_{X_i}}\sigma^2_{X_i}}}) \tag{6}$$

2.2 BAYES MODEL

To mutual exclusion events, the Bayes gave the expression to calculate the condition probability,

$$P(B_i / A) = \frac{P(AB_i)}{P(A)} = \frac{P(B_i)P(A/B_i)}{\sum_{j=1}^{n}P(B_j)P(A/B_j)} \quad (i = 1, 2, \cdots n) \tag{7}$$

Where, $B_1, B_2, \cdots B_n$ are the mutual exclusion events, A is arbitrary event, $P(B_i)$ is prior probability, $P(B_i / A)$ is posterior probability. The safety factor could be defined as,

$$F_S = \frac{N}{N_{cr}} \tag{8}$$

Where, N is the actual measurement SPT-N value, N_{cr} is critical SPT-N value obtained from deterministic liquefaction evaluation method. Liquefied event and non-liquefied event are mutual exclusion events. According the Bayes law, given the safety factor, the liquefied probability could be expressed as follow,

$$P(L / F_s) = \frac{P(F_s L)}{P(F_s)} = \frac{P(L)P(F_s / L)}{P(L)P(F_s / L) + P(NL)P(F_s / NL)} \tag{9}$$

Where, $P(L / F_S)$ is liquefaction probability given the safety factor, $P(F_S / L)$ is the distribution function of safety factor for liquefaction site, $P(F_S / NL)$ is the distribution function of safety factor for non-liquefaction site, $P(L)$ is prior probability for liquefaction site, $P(NL)$ is prior probability for non-liquefaction site $P(F_S / L)$ and $P(F_S / NL)$ could be determined as,

$$P(F_s / L) = \int_{F_s}^{F_s + \Delta F_s} f_L(x)dx = F_L(F_s + \Delta F_s) - F_L(F_s) = \frac{F_L(F_s + \Delta F_s) - F_L(F_s)}{\Delta F_s} \cdot \Delta F_s \tag{10a}$$

$$P(F_s / NL) = \int_{F_s}^{F_s+\Delta F_s} f_{NL}(x)dx = F_{NL}(F_s + \Delta F_s) - F_{NL}(F_s) = \frac{F_{NL}(F_s + \Delta F_s) - F_{NL}(F_s)}{\Delta F_s} \cdot \Delta F_s \quad (10b)$$

Where, $f_L(x)$ is probability density function for liquefaction site; $f_{NL}(x)$ is probability density function for non-liquefaction site; $F_L(x)$ and $F_{NL}(x)$ are the integral functions of $f_L(x)$ and $f_{NL}(x)$. As $\Delta F_S \to 0$, liquefaction probability could be expressed as,

$$P(L / F_S) = \frac{f_L(F_S)P(L)}{f_L(F_S)P(L) + f_{NL}(F_S)P(NL)} \quad (11)$$

If the prior probabilities $P(L)$ and $P(NL)$ are known, the liquefaction probability could be calculated at the given safety factor. The assumption $P(L) = P(NL)$ could be made according the maximum entropy law, so

$$P(L / F_S) = \frac{f_L(F_S)}{f_L(F_S) + f_{NL}(F_S)} \quad (12)$$

The liquefaction probability can be got by statistical analysis if the safety factor is known. But the method can not consider the distribution and variation of the parameter.

3. THE CHINESE CODE-BASED LIQUEFACTION PROBABILITY METHOD

The First order second moment and Bayes probability method need the statistical characteristics of the random variables, and more over theoretical limit state of liquefaction could not be got. In many conditions, the pseudo limit state was given according the deterministic method. The Chinese code method discriminate liquefaction by calculating the critical SPT-N value. So, the state function can be established according the code method.

3.1 THE FORMULA ACCORDING TO THE FIRST ORDER AND SECOND MOMENT METHOD

According the Chinese code method, the critical SPT-N value could be calculated as,

$$N_{cr} = N_0\left[0.9 + 0.1(d_s - d_w)\right] \quad (13)$$

Where, N_{cr} is the critical SPT-N value, N_0 is reference value of standard penetration, d_s is the sand layer depth, d_w is groundwater level. If N value is larger than N_{cr} value, then it is discriminated as non-liquefaction site, otherwise it is discriminated as liquefaction site.
According to the Chinese code method, the state function can be established as,

$$Z = g(N, d_s, d_w) = N - N_0\left[0.9 + 0.1(d_s - d_w)\right] \begin{cases} < 0 \cdots \text{liquefaction state} \\ = 0 \cdots \text{limit state} \\ > 0 \cdots \text{nonliquefaction state} \end{cases} \quad (14)$$

The mean value and standard deviation of the state function can be got according the formula (4) and (5),

$$\mu_Z = g(\mu_N, \mu_{d_s}, \mu_{d_w}) = \mu_N - N_0[0.9 + 0.1(\mu_{d_s} - \mu_{d_w})] \quad (15)$$

$$\sigma_Z = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i}\right)_\mu^2 \sigma_{X_i}^2} = \sqrt{\sigma_N^2 + (0.1N_0\sigma_{d_s})^2 + (0.1N_0\sigma_{d_w})^2} \quad (16)$$

So the reliability index can be expressed as,

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_N - N_0[0.9 + 0.1(\mu_{d_s} - \mu_{d_w})]}{\sqrt{\sigma_N^2 + (0.1N_0\sigma_{d_s})^2 + (0.1N_0\sigma_{d_w})^2}} \quad (17)$$

The formula is feasible under the condition that the state function is normal distribution. So, it's necessary to normal distribution test the state function. 203 standard penetration cases history were used to calculate the state value according to the state function. Histogram and accumulative curve are figured by statistic analyzing the safety factor, shown in figure 1.

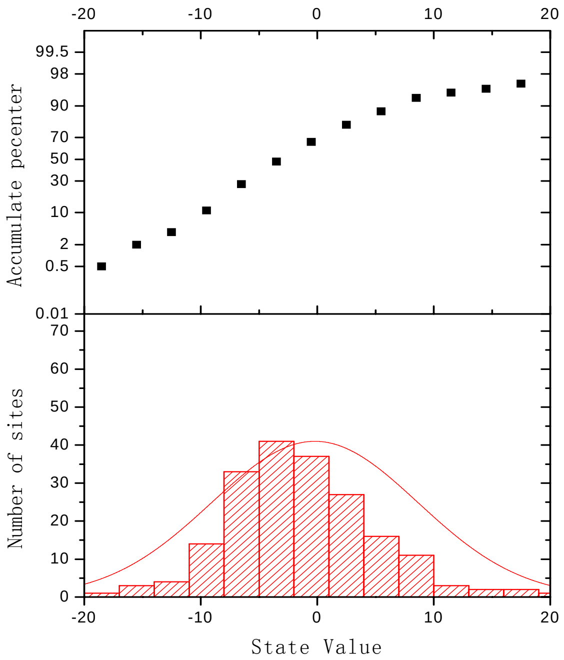

Fig.1 Normal distribution test of the state function Z

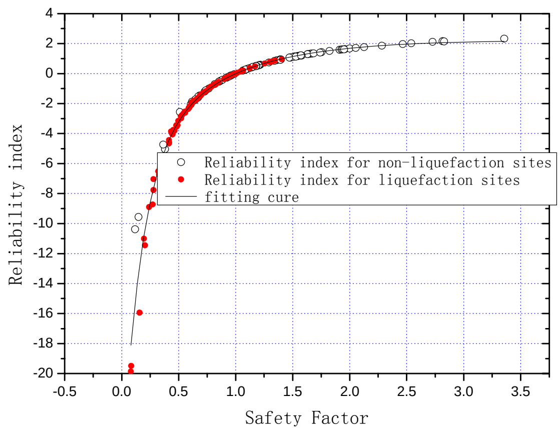

Fig.2 The co-relationship between reliability index and the safety factor

From fig.1, cumulative curve is close to straight line, so the state function Z could be regarded as normal distribution according the characteristics of normal distribution function. Some deviation coefficients ($\operatorname{cov} N = 0.30$, $\operatorname{cov} d_s = 0.10$, $\operatorname{cov} d_w = 0.15$) are assume according the reference of C.Hsein Juang(1999). So $\sigma_N = \operatorname{cov} N \cdot \mu_N$, $\sigma_{d_s} = \operatorname{cov} d_s \cdot \mu_{d_s}$, $\sigma_{d_w} = \operatorname{cov} d_w \cdot \mu_{d_w}$. So the reliability index β could be calculated according the formula (17), the correlation between reliability index and safety factor was shown in fig.2. the fitting cure is express as function,

$$\beta = 2.24 - 8.71\exp\left(\frac{-Fs}{0.72}\right) - 20.12\exp\left(\frac{-Fs}{0.17}\right) \tag{18}$$

The Fs is safety factor. The correlation between liquefaction probability and safety factor was shown in fig.5 based on the formula (6).

3.2 THE FORMULA ACCORDING TO THE BAYES METHOD

The 203 cases history can be divided into two groups: liquefied cases and non-liquefied cases. The frequency histogram can be got by calculating the safety in each site and comparing the results between the liquefied site and non- liquefied site, shown in figure 3 and 4.

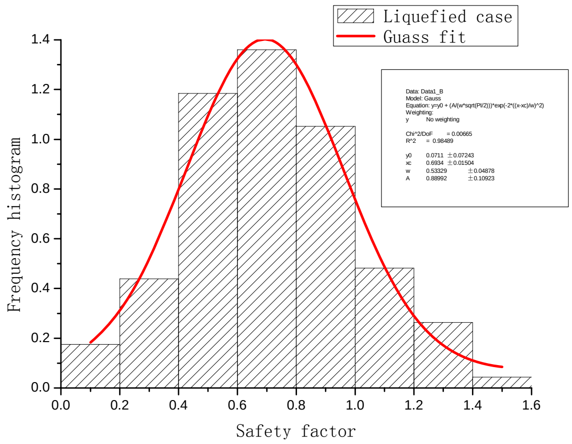

Fig.3 Safety factor frequency histogram for liquefied site

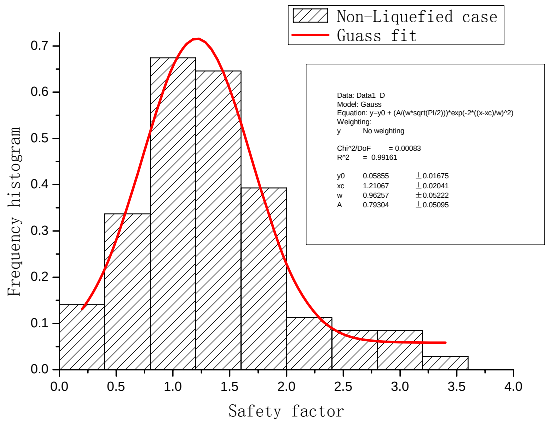

Fig.4 Safety factor frequency histogram for non-liquefied site

The liquefaction probability could be got according to fig.3, fig.4 and the formula (12), which shown in fig.5.

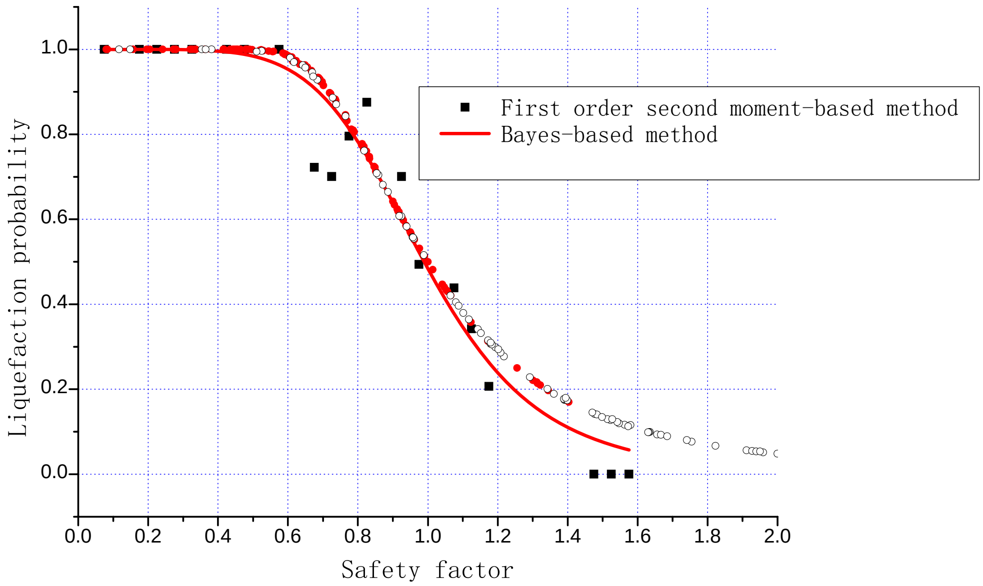

Fig.5 The result comparison of first-order second-moment and Bayes method

In fig.5, the large discreteness is exhibited. One reason may be generated by deficiency data and discontinuity of frequency histogram. The other is lack of the site acceleration value data. The disperse points were fitted as,

$$P_L = \frac{1}{1+(F_s/A)^B} \tag{19}$$

Fitting result are $A = 0.9897 \pm 0.022$ and $B = 6.020 \pm 0.791$.

The result of the first-order second-moment is figured in fig. 5 for comparing. From the fitting curve and result getting from first-order second-moment, intersection at 70%, the liquefied probability getting from first-order second-moment is light higher when safety factor is the same.

4. CONCLUSION

(1) It is very difficult to calculate liquefaction probability by directly integrating, because the statistical parameter of random variable and their distribution can not be determined in engineering. While the first order and second moment method and the Bayes method avoid the tedious mathematical calculation by ingenious assumption.
(2) If mean and the standard deviation were given, the liquefaction probability could be calculated using the first order and second moment method by establishing the liquefaction limit state.
(3) Bayes method also need to obtain the safety factor by determining the liquefaction limit state, and the liquefaction could be calculated by statistic analyzing the distribution of the safety factor.
(4)It is difficult to directly establish liquefaction limit state, and the first order and second moment method and the Bayes method were adopted in this paper by assuming the code forma as liquefaction limit state, so some deviation can't be avoided.
(5) The liquefaction probability could be directly calculated according to the proposed forma after calculating the safety factor through the code method.

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REFERENCES

Yongxiang Chen (1987). Disaster analysis on liquefaction. Institute of Engineering Mechanics, China administration. Master's Degree Dissertation. In Chinese.
Guoxing Chen (1988). Reliability and disaster analysis on liquefaction. Institute of Engineering Mechanics, China administration. Master's Degree Dissertation. In Chinese.
Junfei Xie, Jinren Jiang (2000). Fundamental research for compiling the seismic code. Report of Institute of Engineering Mechanics, China administration. In Chinese.
Chinese standard (2001). Seismic code for structure design. Chinese structure industry express. In Chinese.
Guopang Zhao, Weiliang Jing, Jingxin Gong (2000). Structure reliability theory. Chinese structure industry express. In Chinese.
C. Hsein Juang, David V. Rosowsky, Wilson H. Tang (1999). Reliability-Based Method for assessing liquefaction potential of soils. Journal of geotechnical and geoenvironmental engineering, ASCE, August 1999.
C.Hsein Juang, Caroline J.Chen, Tao Jiang (2001). Probability framework for liquefaction potential by shear wave velocity. Journal of geotechnical and geoenvironmental engineering, August 2001.
C.Hsein Juang, Tao Jiang, Ronald D. Andrus (2002). Assessing Probability-based Methods for Liquefaction Potential Evaluation. Journal of Geotechnical and Geoenvironmental Engineering. July 2002. 580-589.
Jin-Hung Hwang, Chin-Wen Yang (2001). Verification of Critical Cyclic Strength Cure By Taiwan Chi-Chi Earthquake Data. Soil Dynamics and Earthquake Engineering 21 (2001) 237-257.
J. Michael Duncan (2000). Factors of safety and reliability in geotechnical engineering, Journal of geotechnical and geoenvironment engineering, April 2000,
K. Onder Cetin, Raymond B. Seed, Armen Der Kiureghian et (2004). Standard Penetration Test-Based Probabilistic and Deterministic Assessment of Seismic Soil Liquefaction Potential. Journal of geotechnical and geoenvironmental engineering, December 2004.
R.D.Andrus , K.H.Stokoe (1996) . Evaluation of liquefaction Resistance of soils. National Center for Earthquake Engineering Research (NCEER) Workshop. January 5-6,1996