PRACTICAL EFFECTIVE STRESS ANALYSIS AND DETERMINATION OF ITS DYNAMIC PROPERTY

T. Shiomi[1], Y. Nukui[2] and F. Yagishita[3]

[1] *Assistant to General Manager, R&D Institute, Takenaka Corp, Chiba. Japan*
[2] *General Manger, R & D Center, Tokyo Electric Power Company, Kanagawa, Japan*
[3]*General Manager, Arch. & Struct. Eng. Operation Center, Tokyo Electrc Power Serv. Co., Ltd., Tokyo, Japan*
Email: : shiomi.tadahiko@takenaka.co.jp, nukui.y@tepco.co.jp, yagisita@tepsco.co.jp

ABSTRACT :

A practical earthquake response analysis method based on effective stress analysis is proposed. The method, to consider material nonlinearity of soil, required only soil parameters such as the dynamic deformation curve (G-γ and h-γ curve) and liquefaction strength curve in addition to elastic parameters. Both data curve are directly obtained by ordinary site investigations.

KEYWORDS: Effective stress analysis, Parameter identification, Free-field dynamic analysis

1. INTRODUCTION

Earthquake response analysis of soft soil layer has important role for seismic engineering to evaluate force and motion act to buildings and civil structures. In this context, evaluation of wave propagation of a free field is quit important engineering practice. In case no liquefaction occurs, the equivalent linear analysis is the most popular method and the program SHAKE [Schnabel 1972] is so frequently used. However, in case liquefaction occurs, there is no standard method for earthquake response analysis. Only methods, obtaining the safety factor of liquefaction, which determines that liquefaction occurs or not, are well known.

Several amendments of the equivalent linear analysis have proposed for liquefaction. A most simple method is that degrading the soil stiffness according to the liquefaction [Miwa et al 1998]. In this method, it is difficult for engineers to determine the material properties for liquefaction analysis, in which two soil conditions have to be analyzed, non liquefied condition or liquefied condition. Then average value of response is used for the design.

Another and maybe a little better method of this kinds is that two analyses are done for the soil materials, which are no liquefiable case and liquefiable case [Shiomi et al 1997]. Then the larger response is chosen for the design as a safer case. In these cases, it is necessary to always perform two analyses and choose the proper case depend on the problem such as estimation of the ground motion, lateral spring of soil to pile analysis or improvement of ground so on. That leaves ambiguous points on engineering judgment and obviously both analyses do not represent the real situation. The liquefaction failure is a progressive phenomenon and this should be analyzed.

To overcome this defect, we propose a method, which uses only "liquefaction strength curve" in addition to the popular equivalent linear analysis for liquefaction phenomena. The method is based on dynamic effective stress analysis and nonlinearity is considered by Cycle-Wise Equivalent Linear Liquefaction (CWELL) method. The method actually performs as well as equivalent linear analysis during half-cycle of shear stress history. Liquefaction process is modeled by accumulated damage parameter similar to the method proposed by Seed [Seed et al 1976]. The details of the method are summarized in the following section.

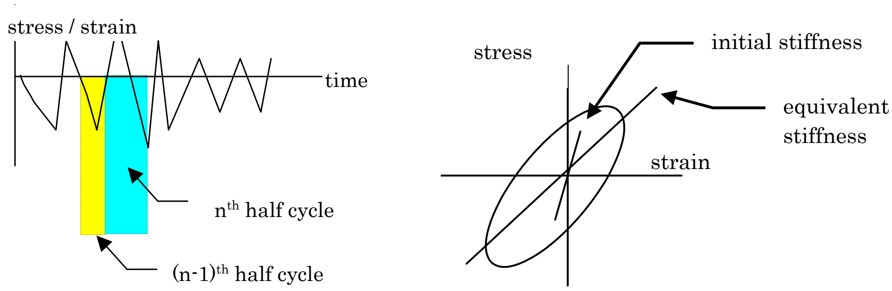
Our proposed method bridges the conventional equivalent linear analysis and requirement of liquefaction analysis. But there are some more requirements remained. That is determination or guess of the material properties before the complex site investigations are done. The equivalent linear method uses the shear stiffness - strain curve (G-γ and h-γ curve). And our proposed method requires only the liquefaction curve (stress ration vs. number of cycle, R-N curve) in addition to them. And more, these parameters are not always obtained at the early stage of the civil structure design, so empirical formula, which based on the standard penetration test, is proposed. These two subjects, CWELL method and the empirical formula, are the topics on this paper.

2. DYAMIC EFFECTIVE STRESS ANALSYIS BY CWELL METHOD

Proposed dynamic effective stress analysis is based on evaluation of equivalent shear modulus and accumulation of pore pressure during a half cycle of shear stress history.

2.1 Integration of dynamic Equation for CWELL

Basic idea of equivalent linear analysis is to solve the non-linear problem using linear analysis with a reduced scant stiffness, so it can simulate a response at the largest amplitude of a cycle. To obtain the proper stiffness, the linear analysis is repeated until the stiffness and the maximum shear strain response at each layer satisfy the material relationship of G-γ and h-γ curve. Therefore the equivalent linear analysis well simulates the response for the cycle in which maximum magnitude is obtains. Nevertheless, the response of the other cycle has error since the stiffness used is not correct for the amplitude of the other cycle. This problem is solved by the proposed method here, since the method uses the shear modulus equivalent to each half cycle. The concept of half cycle is shown in Fig. 2.1.



1. set the possible stiffness K_i.
2. do dynamic linear analysis and obtain the maximum shear strain
3. evaluate the stiffness K_{i+1} for the maximum shear strain from G-γ curve
4. if the $K_{i+1} = K_i$, exit. otherwise repeat from step 2 with new stiffness K_{i+1}

Figure 2.1 Concept of Cycle-wise Equivalent Linear Method

At first, theoretical background of the half-cycle equivalent linear method is explained. In general, the digitalized dynamic Eqn. can be written as Eqn. 2.1 at any time t_n.

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}_n\dot{\mathbf{u}} + \mathbf{K}_n\mathbf{u} = -\mathbf{M}\mathbf{c}\ddot{u}_0(t) \tag{2.1}$$

where $\mathbf{M}$ is mass matrix, $\mathbf{C}_n$ is damping matrix, $\mathbf{K}_n$ is internal force matrix. $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$, $\mathbf{u}$, t are acceleration, velocity, displacement vector and $\ddot{u}_0$ is input motion, $\mathbf{c}$ is a component vector for excitation direction. Eqn. 2.1 should also satisfied at a time t_{n+1} as Eqn. 2.2.

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}_{n+1}\dot{\mathbf{u}} + \mathbf{K}_{n+1}\mathbf{u} = -\mathbf{M}\mathbf{c}\ddot{u}_0(t) \tag{2.2}$$

where Eqns. 2.1 and 2.2 should be continuous so, the following conditions must be satisfied.

$$\mathbf{K}_{n+1}\mathbf{u}_n = \mathbf{K}_n\mathbf{u}_n \tag{2.3}$$

$$\mathbf{C}_{n+1}\dot{\mathbf{u}}_n = \mathbf{C}_n\dot{\mathbf{u}}_n \tag{2.4}$$

To satisfy both Eqns. 2.3 and 2.4, $\mathbf{K}_{n+1} = \mathbf{K}_n$ or $\mathbf{u}_n = 0$. If $\mathbf{u}_n \neq 0$, the following condition must be satisfied.

$$\mathbf{K}^e_{n+1}\mathbf{u}_n = \mathbf{K}^e_n\mathbf{u}_n = \mathbf{0} \tag{2.5}$$

where e is an index for an element. Eqn. 2.5 is satisfied if the internal force of the element is zero, in which shear strain γ of a soil layer is also zero. The term $\mathbf{K}_n\mathbf{u}_n$ for the one dimensional soil layer are represented as

$$\begin{bmatrix} K_1 & -K_1 & & & \\ -K_1 & K_1+K_2 & K_2 & & \\ & K_2 & K_2+K_3 & K_3 & \\ & & K_3 & K_3+K_4 & \cdots \\ & & & \cdots & \ddots \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_{\dots} \end{Bmatrix} \tag{2.6}$$

If this equation is satisfied at j^{th}-element, Eqn. 2.5 is satisfied. At the timing that internal force becomes zero and Eqn. 2.3 is satisfied, $\mathbf{K}_n$ can be changed to $\mathbf{K}_{n+1}$. In other word, stiffness can be changed if shear strain $(u_{j+1} - u_j)/L_j$ is zero and dynamic equation is satisfied. Here, L_j is the length of j^{th}-element. The half cycle is defined as the interval of the shear strain equal zero.

About damping term, same logic can be applied. If Eqn. 2.7 is satisfied, Eqn. 2.4 is satisfied.

$$\mathbf{C}_{n+1}^{e}\,\dot{\mathbf{u}}_n = \mathbf{C}_n^{e}\dot{\mathbf{u}}_n = \mathbf{0} \tag{2.7}$$

That is means if $\mathbf{C}_e$ is Rayleigh damping ($\mathbf{C}^e=\alpha_s^e\ \mathrm{M}^e+\beta_s^e\,\mathrm{K}^e$) and strain velocity $\dot{\gamma}$ become zero, $\mathbf{K}_e$ and β_s^e can be changed. And if the time $\dot{\mathbf{u}}_n^{node}$ become zero, change of α_s^e and $\mathbf{M}^e$ does not violate the consistency of the dynamic equation. In other words, this algorithm allows the change an element stiffness at the zero crossing of shear strain (= zero crossing of shear stress). For example if stiffness of a element at the zero crossing of its strain, internal force does not change and no effect occurs at other element since shear stress and strain is zero at the time.

2.2 Material nonlinearity modeled by CWELL method

It was shown that the stiffness of a finite element can be changed at any each half cycle above. Therefore it is possible for the stiffness to change to an equivalent stiffness related to the maximum strain and the pore pressure ration. This makes for the CWELL method to simulate progressive non-linearity such as liquefaction phenomena and take advantage to the original equivalent linear method, which use the same stiffness through all duration of the earthquake. The algorithm of the calculation taking into account non-linearity of strain and pore pressure non-linearity is shown in Fig. 2.2 [Shiomi et al. 1999 & 2000, Nukui et al. 1999].

Solving a free field soil layer problem as shown at the top of left hand side corner, shear strain and stress are obtained. Scant stiffness and damping factor due to strain hardening is obtained using G/G_0-γ and h-γ curve from the shear stress (stress ratio) as shown at the top right hand side.

$$\frac{G}{G_0} = F_G\left(\frac{\gamma}{\gamma_{50_{ref}}\left(\sigma'_{m_{ref}}\right)^n}\right),\ h = F_h\left(\frac{\gamma}{\gamma_{50_{ref}}\left(\sigma'_{m_{ref}}\right)^n}\right) \tag{2.8}$$

where these curves are obtained by a ordinary dynamic property test such as "Method for Cyclic Tri-axial Test to Determine Deformation Properties of Geo-materials" (JGS 0542-2000) of Japanese Geotechnical Society. And excess pore pressure due to liquefaction is calculated from the accumulated damage parameter, which is calculated from the shear strain. Here the dependency of initial shear stiffness to pore pressure is defined by empirical formula shown in Eqn. 2.9.

$$G_0 = G_{0\_ref}\left(\frac{\sigma'_{m0}}{\sigma'_{m\_ref}}\right)^n \tag{2.9}$$

where σ'_m is effective mean stress, G_{0_ref} is G_0 at reference effective mean stress σ'_{m_ref}, G_0 is elastic shear stiffness, γ is shear strain, γ_{50_ref} is γ_{50} at the reference effective mean stress, γ_{50} is the strain where the shear stiffness is half of the initial value. The mean pore pressure ratio is obtained from the pore pressure ratio r_u

$$\sigma'_m = \sigma'_{mo}\left(1 - r_u\right) \tag{2.10}$$

The r_u is obtained from the accumulated damage parameter D. The liquefaction ratio r_u can be obtained by an empirical formula of the accumulated damage ratio D. This formula was originally proposed by Seed [Seed et al,1976] as Eqn. 2.11 and modified to Eqns. 2.12a and Eqn. 2.12b for cases of "before cyclic mobility" and "after cyclic mobility" respectively by the authors.

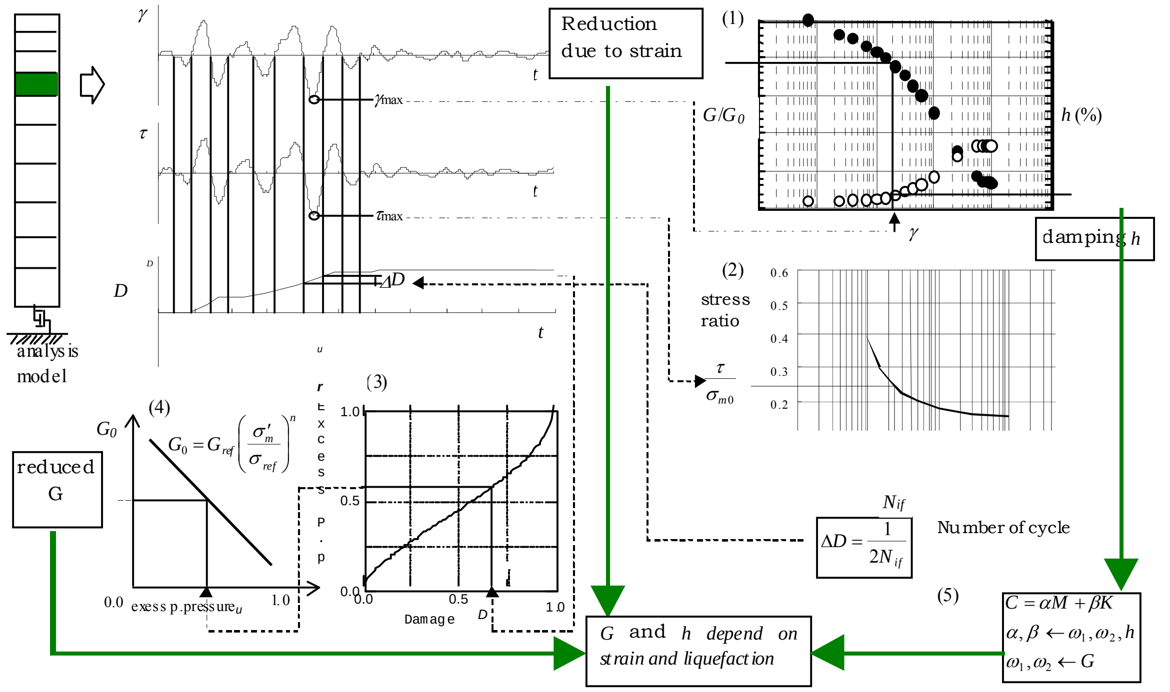


Figure 2.2 Schematic flow to calculate stiffness under liquefying process

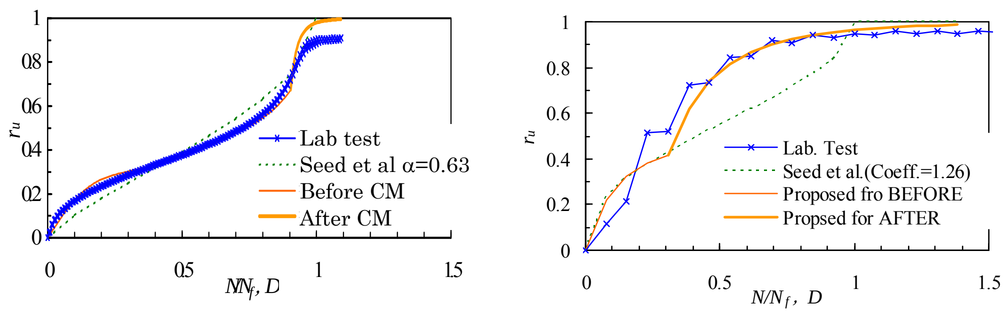
$$r_u = \frac{1}{2} + \frac{1}{\pi}\sin^{-1}\left(2D^{\frac{1}{\alpha}} - 1\right) \tag{2.11}$$

$$r_u = f_{ru}(D,\alpha) = \frac{1}{2} + \frac{1}{\pi}\sin^{-1}\left(\left(2D^{\frac{1}{\alpha}} - 1\right)\left(\frac{\left((\cos(2\pi D)+1)/2\right)^n + \beta}{1+\beta}\right)\right) \tag{2.12a}$$

$$r_u = 1 - \left(1 - r_{u(D=D_{CM})}\right)\cdot\left(\frac{D - D_{CM}}{1 - D_{CM}} + 1\right)^4 \tag{2.12b}$$

where $\alpha=(0.0177D_r)^3$, β, n are constants 2.0 and 5.0 respectively. D_r is relative density (%) of sand. $r_{u(D=DCM)}$ is the pore pressure ratio at $D=D_{CM}$ and D_{CM} is the accumulated damage parameter when cyclic mobility starts. Those equations are well adapted to a sample sand as shown in Fig. 2.3.

In the above formulation, the accumulated damage parameter D is calculated from the stress ratio using the liquefaction strength curve that is also one of a standard soil test. The increment of D is obtained as $\Delta D=1/(2N)$ where N is a number of cycle, with which liquefaction occurs under the stress ratio $R=\tau/\sigma_m$'. It is possible to calculate the equivalent stiffness for each half cycle taking the strain and stress of the previous half cycle to calculate the current shear stiffness. It is also possible to iterate the whole process of the response analysis, so that the strain and stress of the previous calculation can be used.



(a) In case of excitation force is small (b) In case of excitation force is large

Figure 2.3. Excess pore pressure ratio r_u against damage parameter D

3. LIQUEFACTION STRENGTH CURVE

Soil parameters required for the liquefaction analysis by CWELL method are density ρ, elastic coefficient G_0, strain dependency curve of stiffness and damping (G/G_0-γ curve, h-γ curve) and liquefaction strength curve. They can be directly obtained through ordinary site investigation tests. However, it is not easy to directly obtain measured data in the most of structural design practices. So some of them can be obtained indirectly such as G_0 from PS wave investigation. There are some empirical formula for the dependency curve of stiffness and damping for several types of sand and clay. But very few formulas are proposed for the liquefaction strength curve. So we introduce a set of empirical equations for a liquefaction strength curve. There are some empirical equations for liquefaction strength at specific number of cycle but not for liquefaction curve. For example, R_{20} (stress ratio which produces liquefaction with 20 cycles) is defined by Japanese Highway Code as follows.

$$R_{20} = \begin{cases} 0.0882\sqrt{N_a/1.7} & (N_a < 14) \\ 0.0882\sqrt{N_a/1.7} + 1.6\times10^{-6}(N_a - 14)^{4.5} & (N_a \geq 14) \end{cases} \tag{3.1}$$

where N_a is,

$$\begin{aligned} &N_a = C_1 N_1 + C_2 \\ &N_1 = \frac{1.7N}{\sigma_v'/98 + 0.7} \\ &C_1 = \begin{cases} 1 & (0 \leq FC < 10) \\ (FC+40)/50 & (10 \leq FC < 60) \\ FC/20 - 1 & (60 \leq FC) \end{cases} \\ &C_2 = \begin{cases} 0 & (0 \leq FC < 10) \\ (FC-10)/18 & (10 \leq FC) \end{cases} \end{aligned} \tag{3.2}$$

For gravel,

$$N_a = \left\{1 - 0.36\log\left(\frac{D_{50}}{2}\right)\right\} N_1 \tag{3.3}$$

N is the number of the standard penetration test, N_1 is value of N at 1/9.8 kPa, σ_v' is overburden, N_a is N value modified with content ratio of fine particle, C_1 and C_2 are coefficients, FC is content ratio of the fine particle, D_{50} is the average value of sand particle.

This formula is determine only a point of liquefaction strength but does not define the liquefaction strength curve. Therefore we assume the liquefaction curve can be written as

$$R = \frac{a}{N^c} + b \tag{3.4}$$

This equation has three unknowns a, b and c. These can be determined if three points are fixed. Obviously point (R_{20}, N_{20}) can be used. Adding more two points of (R_4, N_4) and (R_{100}, N_{100}), three unknowns a, b and c are determined as Eqn. 3.5.

$$a = 20^c(R_{20} - b), \qquad b = \frac{R_4 R_{100} - R_{20}{}^2}{R_4 + R_{100} - 2R_{20}}, \qquad c = \frac{1}{\log_{10} 5}\log_{10}\frac{R_4 - R_{20}}{R_{20} - R_{100}} \tag{3.5}$$

To determine the R_4 and R_{100}, the following equations are adopted similar to R_{20}.

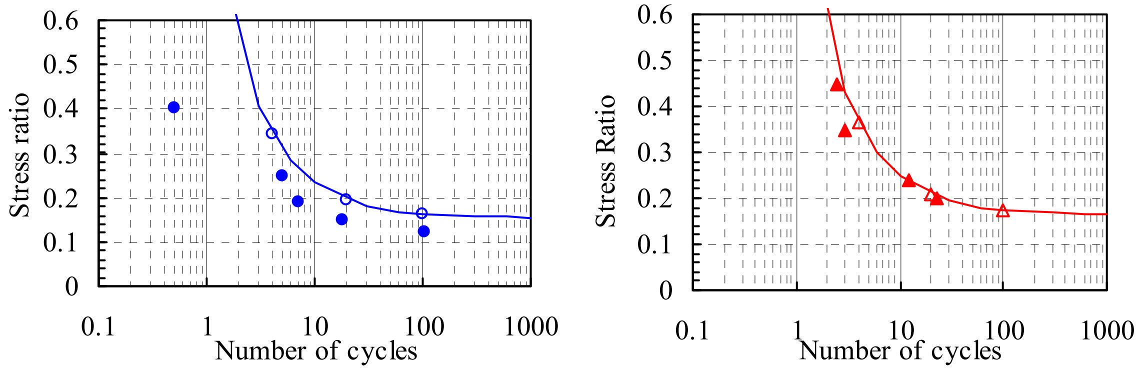
$$R_L = \begin{cases} \alpha\sqrt{N_a/1.7} & (N_a < 14) \\ \alpha\sqrt{N_a/1.7} + \beta(N_a - 14)^{\gamma} & (N_a \geq 14) \end{cases} \tag{3.6}$$

where R_L is cyclic strength of triaxial test, L is number of cycles. The coefficients α, β and γ are 0.0882, $1.6\text{x}10^{-6}$ and 4.5 respectively for R_{20}. For R_4 and $R_{100,}$ these coefficients α, β and γ are obtained by fitting many cases of laboratory test data. Coefficients obtained are shown in Table 3.1.

Table 3.1 Coefficients of the Empirical Equation for R_4 and R_{100}

Coeff	R_4			R_{100}		
	α	β	γ	α	β	γ
Allvium sand	0.156	$1.3\text{x}10^{-6}$	4.67	0.074	$1.6\text{x}10^{-6}$	4.08
Dillvium sand	0.175	$1.6\text{x}10^{-6}$	3.90	0.079	$1.3\text{x}10^{-6}$	4.47

The formula is examined for the data of the soil layer at Kobe Port Island, which is obtained after the Hyogoken-nanbu earthquake 1995. The comparison of the predicted curve and the liquefaction strength of soil data are shown in Fig. 3.1. The predicted curve is shown by a solid line and the soil data are shown by dots. The predicted curves by the proposed equation agree well to the soil data.



Upper layer Middle layer

Figure 3.1 Liquefaction strength curve; Eqn.(solid line) vs Data(dot)

4. VERIFICATION OF CWELL METHOD AND LQUEFACTION STRENGTH CUURVE

The CWELL method is implemented into a web based program as a trial with a graphical user interface (http://taklab.takenaka.co.jp/liqsmart-e/) as shown Fig. 4.1. In case of the liquefaction analysis for earthquake, there are many data, which are commonly used such as earthquake data, G-γ and h-γ curve, and liquefaction curve. These data may provided by researchers or institutes. This program can can manage these as a data base.

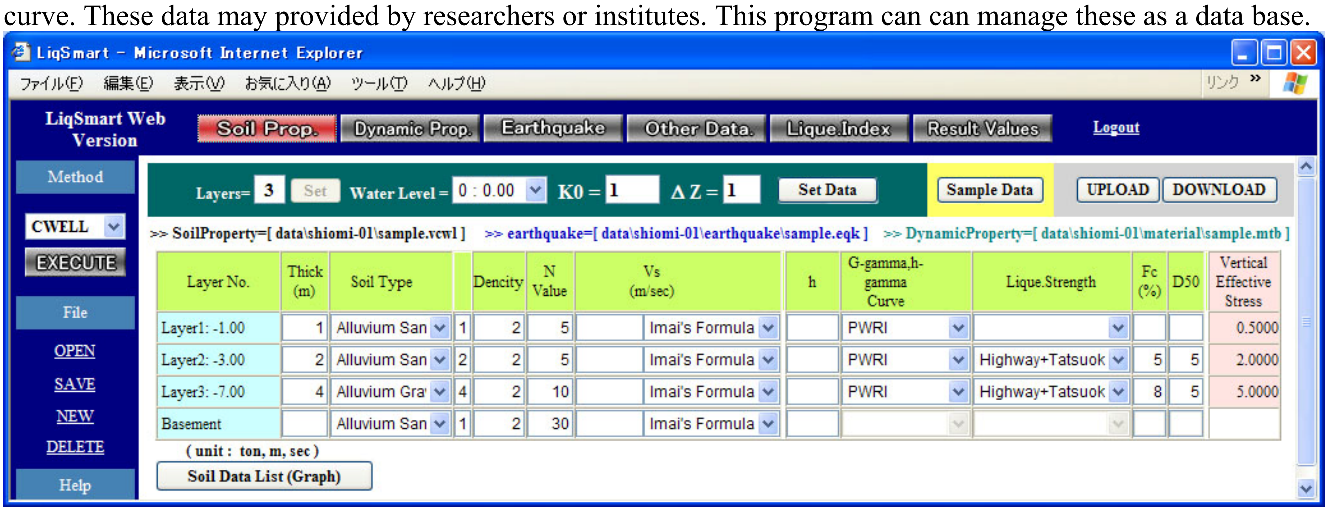


Figure 4.1 Web based effective stress analysis with CWELL method

5. VERIFICATION OF CWELL METHOD AND LQUEFACTION STRENGTH CUURVE

To verify the above mentioned method of effective stress analysis, the recorded earthquake at Kobe Port Island on 1995 Hyogoken Nanbu earthquake [Iwasaki & Tai 1996] is simulated. The soil layer model is taken from the data obtained through the frozen sampling by Hatanaka et al. [Hatanaka et al. 1997]. The material properties of the soil layers are shown in Table 5.1. In this case, both G-γ and h-γ curves and liquefaction curve have been obtained by measurements, but the predicted liquefaction curves via N value are used to verify the proposed method instead of these site data. So the proposed method includes the cyclic mobility effect, modification of r_u-D curve and the prediction of liquefaction strength curve.

The material properties of the soil layer at Kobe Port Island is shown in Table 5.1. The water table is -2.3m and the soil layers consist of the upper liquefiable layer (-10m to -18.6m) and the middle layer consists of clay and dense sand and gravel. The layer from -67m to -83.8m consists of dilluvial clay. The liquefiable layer is investigated by the frozen sampling method, so the liquefaction strength curve KPU, KPM and KPL are obtained by the most accurate acquisition method at the current state of arts as shown Fig. 5.1. Their shear velocities are 210m/sec and not too soft layers.

Table 5.1 Material properties of soil layer

Layer	Depth (m)	V_s (m/sec)	γ (kN/m^3)	Shear modulus G_0 (kPa)	G~γ curve	Liquefaction strength curve
sand and gravel	-2.3	170	19.60	57800	S-1	
	-10	210	19.60	88200	S-1	KPU
	-14.7	210	19.60	88200	S-2	KPM
	-18.6	210	19.60	88200	S-2	KPL
alluvial clay	-23	180	16.07	53136	C-1	
	-27	180	16.17	53460	C-2	
sand	-32.8	245	17.15	105044	S-2	
sand and gravel	-50	305	17.93	170236	S-2	
sand	-61	350	18.23	227850	S-2	
dilluvial clay	-67	303	16.66	156075	C-5	
	-79	303	16.66	156075	C-5	
sand and gravel	-83.8	320	19.60	204800		

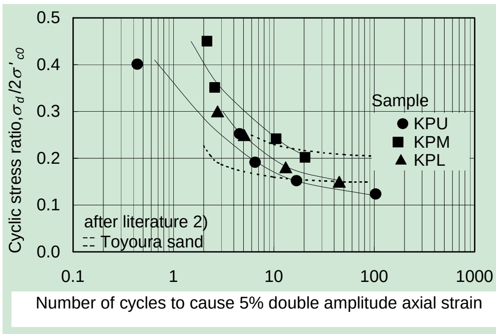
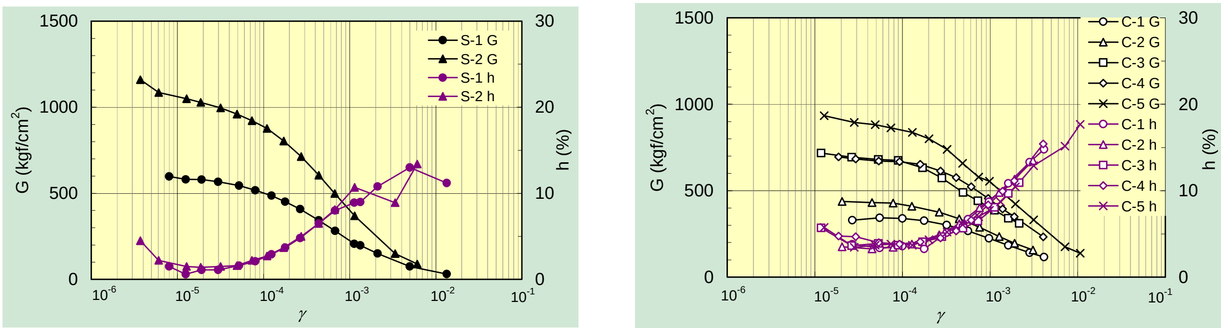


Figure 5.1 Liquefaction strength of sand

The G-γ curve is also obtained with the frozen sampling. Figure 5.2 shows the shear modulus of the layers of sand and clay. Although shear modulus is directory obtained by the laboratory test but only reduction ratio G/G_0 is used for the analysis. Strain when shear modulus become half of the initial (γ_{50}) is about 0.05% for sand and about 0.2% for clay. The strain at damping coefficient 10% is about 0.1% for sand and about 0.15% for clay.



(a) shear modulus of strain dependency of sand (b) shear modulus of strain dependency of clay

Figure 5.2 Scant shear modulus for soil layers

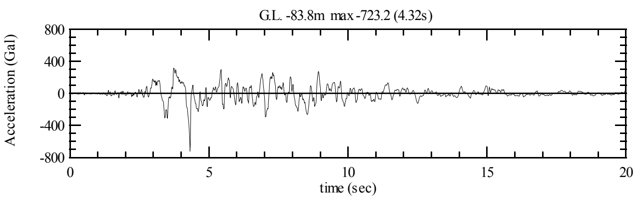


Figure 5.3. Recorded earthquake at Port Island -83.8m

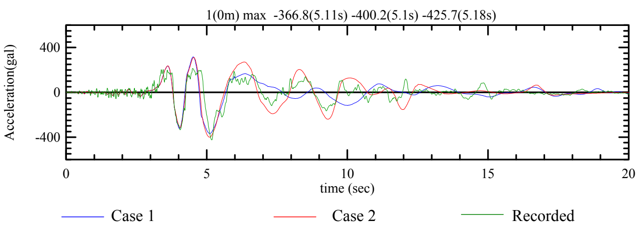


Figure 5.4 Acceleration at the ground surface

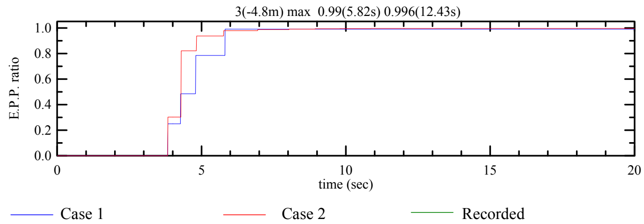


Figure 5.5 Time history of excess pore pressure

Input motion is applied at the depth -83.3m as the motion of incoming + outgoing wave. The wave is shown in Fig. 5.3. Impulsive wave takes place at about 4 sec. that may trigger liquefaction phenomenon. Response accelerations on surface are calculated by the result by the original CWELL method (Case 1) and the proposed method (Case 2) as shown in Fig. 5.4. Both accelerations agree well with the recorded acceleration till the largest peak acceleration. But after that, the acceleration calculated by the proposed CWELL method shows large wave with long period, which is a typical liquefaction behavior and closer to the recorded data than one obtained by the original CWELL method. The time that difference starts at the time history of acceleration is the time that the full liquefaction starts as shown in Fig. 5.5. This may be caused that the proposed method considers the cyclic mobility phenomenon but the original is not, so the less stiffness remains after the full liquefaction starts for the original CWELL method. In other word, consideration of the cyclic mobility is important and well modeled by the proposed method.

6. CONCLUDING REMARKS

Cyclic-Wise Equivalent Linear Liquefaction analysis method (CWELL method), which is a simpler dynamic effective stress analysis than ordinary time marching integration method, is proposed. It is a renewal of the CWELL method proposed in 2000 by the authors, considering the cyclic mobility and modifying the relationship between excess pore pressure and accumulated damage parameter. The new method is verified by the simulation of the recorded earthquake at Kobe Port Island 1995. In addition to these basic modifications, the determination method of the liquefaction strength curve is also proposed. This is also verified with the simulation of Kobe earthquake.

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