EFFECT OF CONSOLIDATION RATIOS ON MAXIMUM DYNAMIC SHEAR MODULUS OF SANDS

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ABSTRACT :

The shear modulus is the most basic parameter and can be attained by the experiments in the field or in the laboratory. The maximum dynamic shear modulus obtained in the laboratory is generally for the cases of the isotropic consolidation. The most advanced apparatus for testing the dynamic shear modulus in small strain range in the laboratory now is the resonant column device. However, the most existing resonant column devices are only suitable for specimens under isotropic consolidation. Therefore, the effect of anisotropic consolidation on the maximum dynamic shear modulus is still a question to be discussed further.

A formula for calculating the increment of the maximum dynamic shear modulus of anisotropically-consolidated sands is presented in the paper. The new resonant column testing device with the anisotropic consolidation function is employed to attain the formula for calculating the increment of the maximum dynamic shear modulus. The results here indicate: (1) The effect of the anisotropic consolidation on the maximum dynamic shear modulus is quite remarkable and cannot be neglected; (2) A suitable form to show this effect is to use the power function of the increment of the consolidation ratio, i.e. $(k_c\text{-}1)^B$; (3) The variation of the maximum dynamic shear modulus for the cases of $k_c>1$ can be expressed by the relative increment formula, i.e. $\Delta G_m/G_{0,m}= 1+0.66(k_c-1)^{0.54}$; (4) The formula presented above means that the maximum dynamic shear modulus shows a more rapid rise in the interval of k_c near to 1 and a slower rise in the interval of k_c far away from 1; (5) The increasing degree of the maximum shear modulus due to $k_c>1$ is significantly larger than that described by the Hardin and Black's formula, e.g. the increasing degree for k_c from 1 to 2 in the paper is about 66% while only 15% by the Hardin and Black's formula; (6) The consolidation ratio also should be one of the important reasons on the obvious difference between in the field and laboratory determination of the maximum dynamic shear modulus of the soils..

KEYWORDS: Anisotropic consolidation; Maximum dynamic shear modulus; Increment formula; Sands

1. INTRODUCTION

In the soil property, the shear modulus is the most basic parameter and can be attained by the tests in the field or in the laboratory (Hardin and Black, 1968, 1969; Zen and Higuchi, 1984; Seed and Wong, 1986; Amini *et al*, 1988; Yu *et al*, 1988; He, 1997; Chen *et al*, 2002). The most advanced apparatus for testing the dynamic shear modulus in small strain range in the laboratory now is the resonant column device because of its advantages in the simply mechanical principle, clear stress condition and convenient operation and the small deviation of testing results. However, the most existing resonant column devices are only suitable for specimens under isotropic consolidation. Because the consolidation ratio, k_c, is about from 1.4 to 3 in the actual subsoil, some researchers and engineers try to employ Hardin and Black's formula (1968,1969) to describe the maximum shear modulus of soils under the different Consolidation ratio k_c. In terms of the dynamic triaxial tests, some researchers (He, 1997) have raised doubt whether the formula is correct in describing the soil maximum dynamic shear modulus under the anisotropic consolidation. However, these results are only a few and the dynamic triaxial tests are basically suitable for the moderate and large deformation of soils. Therefore, the effect of consolidation ratios on the maximum dynamic shear modulus is still a question to be discussed further.

2 TEST APPARATUS

To identify the maximum dynamic shear modulus of soils under anisotropic consolidation a new resonant column device as shown in Fig.1 is developed in the Institute of Engineering Mechanics, China Earthquake Administration, in 2002. As usual the resonant column is fixed-free type, but there are two special designs for performing the tests of deviatoric stress consolidation. One is a special transmission mechanism to apply the vertical static deviatoric force to the soil specimens without eccentric force. Another is to set a manipulator inside the pressure vessel to solve the unstable problem during fitting the specimen and loading the vertical deviatoric stress to the soil specimen. Both designs ensure that soil specimen is just right in the axial line of the load. After fitting the specimens and applying the confining pressure and the additional vertical stress, the manipulator can loose by operating outside.

Figure 1 The resonant column device used in tests

3 SOIL SAMPLES AND TEST PROCEDURE

Two kinds of sands, The Fujian standard sand and Harbin sand of China, are employed in the tests of the paper. The physical specifications of the sands are listed in Tables 1 and 2 and the grain compositions of the sands are shown in Figs.2 and 3, respectively. From the grain-size distribution, both two kinds of sands belong to the medium compact sands.

Table 1 Physical specification of the Fujian sand

Specific gravity	Maximum void ratio	Minimum void ratio	Uniformity coefficient	Curvature coefficient
2.66g/cm^3	0.796	0.578	1.64	0.85

Table 2 Physical specification of the Harbin sand

Specific gravity	Maximum void ratio	Minimum void ratio	Uniformity coefficient	Curvature coefficient
2.62g/cm^3	0.756	0.435	2.62	1.06

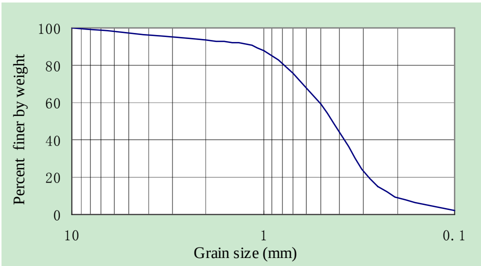

Figure 2 The grading distribution of the Fujian sand

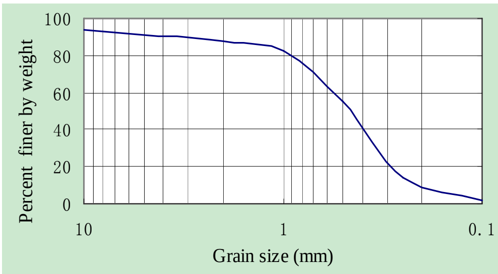

Figure 3 The grading distribution of the Harbin sand

In the tests, three relative densities of the samples, D_r=72.8%, D_r=60% and D_r=30%, are made for the sands. Three confining stresses, σ_3=100kPa, 200kPa and 300kPa, and five consolidation ratios, k_c=1, 1.2, 1.5, 1.7 and

2.0, are employed. In some cases, the tests for k_c=2.4 are performed. For each confining stress, five or six 'identical' sand samples of 3.91cm×8cm are used respectively for the tests of the different consolidation ratio. After finishing the consolidation, the graded loads of the torsional moment are conducted to the soil samples. By the free vibration method, the dynamic shear modulus of the samples is finally obtained.

4 EXPERIMENTAL RESULTS

The tests on the relation of the dynamic shear modulus, G, and the dynamic shear strain, γ, are conducted for the different relative densities of the samples. The hyperbolic equation, $G=1/(a+b\gamma)$, is used to obtain the regression curve of the dynamic shear modulus versus the shear strain. The maximum dynamic shear modulus, G_{max}, can be obtained from the regression equation by $\gamma \rightarrow 0$, which also equals to $1/a$ and the reciprocal of the ordinate interception of $1/G \sim \gamma$ as well.
The maximum dynamic shear modulus G_{max} under the different consolidation ratio can be included by two parts: $G_{0,m}$, representing the maximum dynamic shear modulus for k_c=1 and ΔG_m, representing the increment of the maximum dynamic shear modulus due to k_c>1. In the paper, the focus is placed on the relative increment of the maximum dynamic shear modulus, $\Delta G_m/G_{0,m}$, referring to the effect of consolidation ratios on the maximum dynamic shear modulus.

4.1 Results of the Fujian Sand

The typical result on the test data and the regression curves of $1/G \sim \gamma$ for the Fujian sand of Dr=0.60 under three confining stresses is demonstrated in Fig.4 and it can be seen that the deviation of the test data is quite small. All of the other results show the same good behavior.
According to the above determined maximum dynamic shear modulus, the relations between the maximum dynamic shear modulus and the confining stress are exhibited in Fig.5 by dual logarithm plot and, the relations between the maximum dynamic shear modulus and the void ratio are exhibited in Fig.6.
By regressing the data for the cases of k_c=1 in Figs.5 and 6 a formula for the maximum dynamic shear modulus of the Fujian sand in k_c=1 is formed as

$$G_{0,m} = 117 \cdot \frac{(2.973 - e)^2}{1 + e} \cdot \sigma_3^{0.5} \quad \text{(MPa)} \tag{1}$$

The form of Eq.(1) is consistent with the Hardin and Black's formula except the slight difference in the first coefficient, which is 117 here and 102 in the Hardin and Black's.
Furthermore, it can be seen that all of the lines for the different k_c in Fig.5 are basically parallel and also, the same phenomena occurs in Fig.6. Therefore, the form of $\Delta G_m/G_{0,m}$ can be taken to show the variation of the maximum dynamic shear modulus under the different k_c. The relation of $\Delta G_m/G_{0,m} \sim k_c$ can be taken as the following equation

$$\frac{\Delta G_m}{G_{0,m}} = C \cdot (k_c - 1)^B \tag{2}$$

where C and B are coefficients to be determined. It should be noticed that coefficient C equals to the relative increment of the maximum dynamic shear modulus when k_c=2, and the coefficient B refers to the curvature of $\Delta G_m/G_{0,m} \sim k_c$-1.

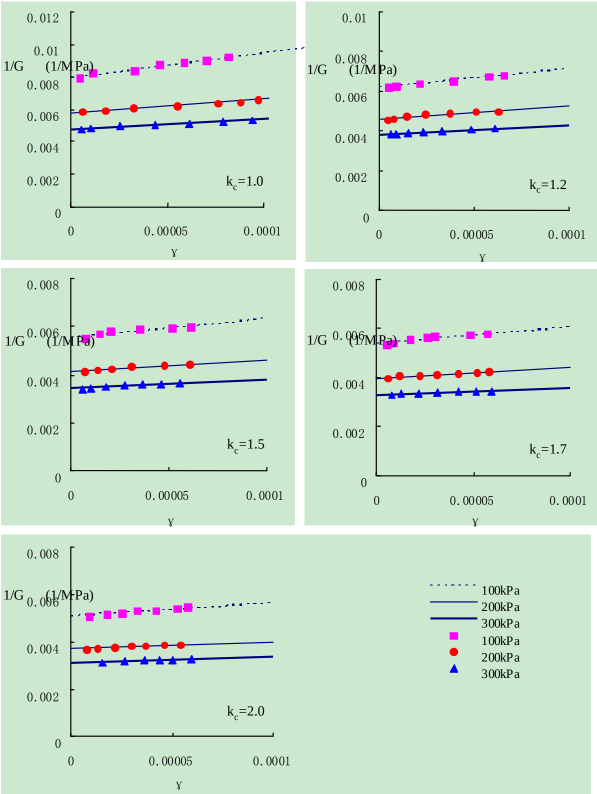

Figure 4 The $1/G \sim \gamma$ for the Fujian sand

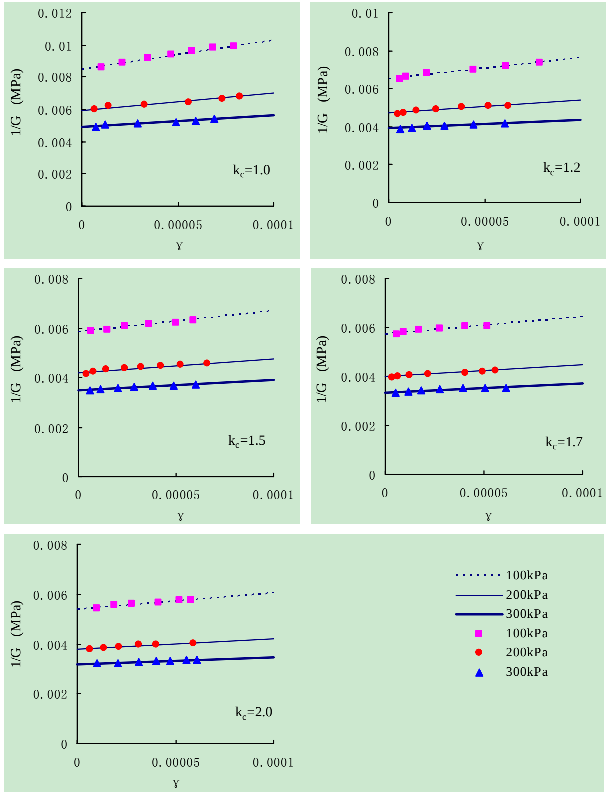

Figure 5 Relations between G_{max} and σ_3

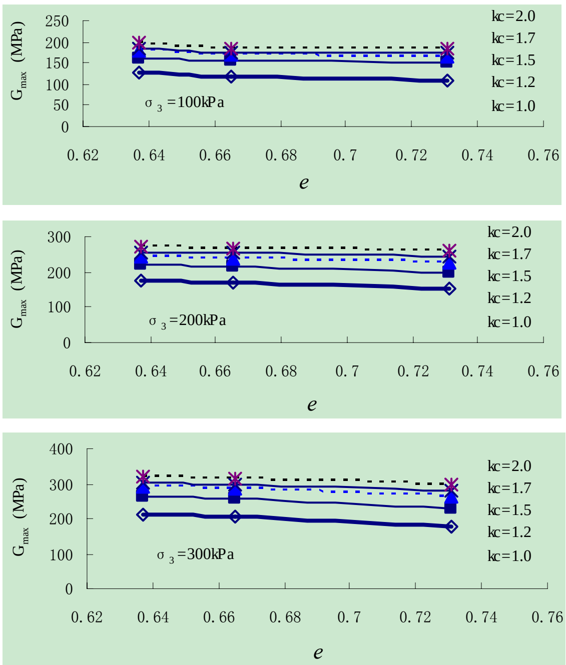

Figure.6 Relations between G_{max} and e

To determine the coefficients B and C, the effects of the confining stress σ_3 and the void ratio e on $\Delta G_m/G_{0,m}$ are illustrated in Figs.7 and 8, respectively. It can be seen that the effect of the confining stress σ_3 can be neglected as shown in Fig.7 and the effect of the void ratio e should be considered as shown in Fig.8. It means that it is only possible that B and C are the function of the void ratio e. Therefore, for each kind of the relative density of the sand, B and C can be taken as the average value of the various confining stresses and based on this, the relations of the relative increment of the maximum dynamic shear modulus, $\Delta G_m/G_{0,m}$, and the increment of consolidation ratio, k_c-1, for three void ratios are illustrated in Fig.9.

It can be seen from Fig.9 that the three lines are basically parallel, which means that coefficient B is independent on the void ratio. Then, the coefficient B can be taken as 0.458, the average value of 0.472, 0.443 and 0.460 as shown in Fig.9. From Fig.9, it can also be seen that the variation of the coefficient C mainly results from the void ratios. By regressing the data between the coefficient C and the void ratio as shown in Fig.10, the coefficient C is obtained as following

$$C = 1.085\, e^{1.548} \tag{3}$$

Finally, the relative increment of the maximum shear modulus due to k_c>1 can be expressed as

$$\frac{\Delta G_m}{G_{0,m}} = 1.085\, e^{1.548} \cdot (k_c - 1)^{0.458} \tag{4}$$

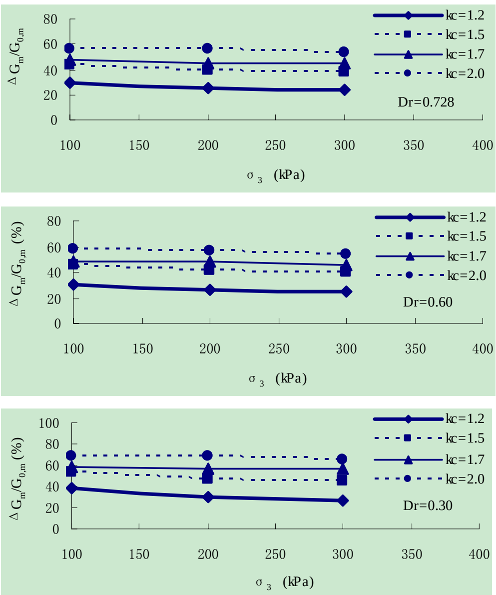

Figure 7 Effects of the confining stress σ_3 on $\Delta G_m/G_{0,m}$

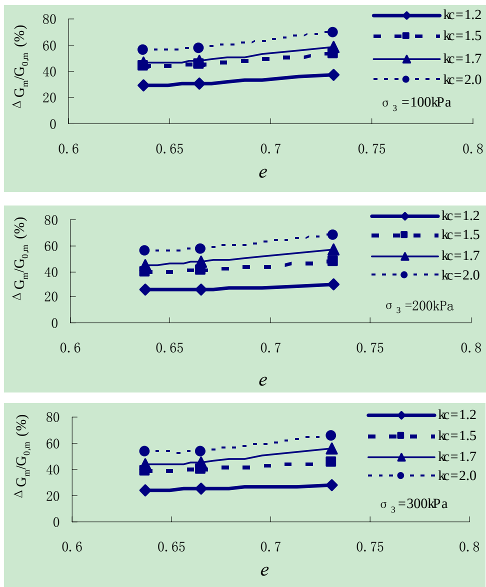

Figure 8 Effects of the void ratio e on $\Delta G_m/G_{0,m}$

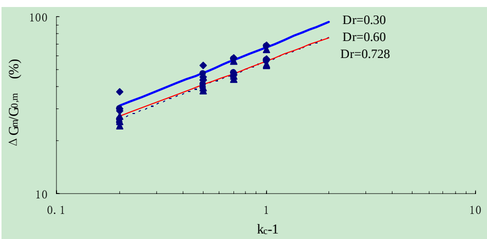

Figure 9 Relations between $\Delta G_m/G_{0,m}$ and k_c-1

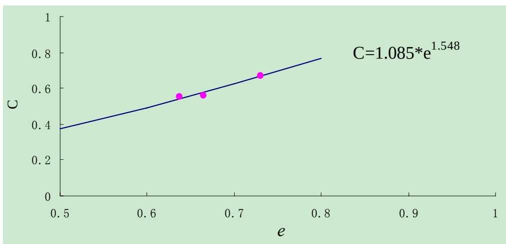

Figure10 The regression of the coefficient C

The maximum shear modulus of the Fujian sand under the different consolidation ratio now can be obtained by combination of Eqs.(1) and (4). The comparison of the maximum shear modulus between the presented formula and the test data for D_r=0.60 is listed in Table 3 and it can be seen that the errors are quite small.

Table 3 Comparison of G_{max} (MPa) between the presented formula and the test data

σ_3 kPa		k_c=1.0	1.2	1.5	1.7	2.0
100	Test	117.8	153.4	171.5	174.5	185.9
	Formula	118.4	151.1	168.2	176.4	186.7
	Error (%)	0.5	1.5	1.9	1.1	0.4
200	Test	168.1	212.3	237.5	251.9	263.9
	Formula	167.4	213.6	237.7	249.5	264.0
	Error (%)	0.4	0.6	0.1	1.0	0.04
300	Test	204.5	256.4	285.7	298.5	314.5
	Formula	205.0	261.6	291.1	305.5	323.3
	Error (%)	0.2	2.0	1.9	2.3	2.8

Furthermore, it can be seen from Fig.10 that the effect of void ratios on the coefficient C is not very notable. To simplicity, the effect of void ratios on the increment of the maximum shear modulus can be neglected and then, the maximum shear modulus of the Fujian sand under different consolidation ratios now can be expressed by averaging the coefficients of C and B in three densities and in this case, C=0.593 and B=0.458. Fig.11 shows the comparison between the calculated and tested maximum shear modulus in this case and it can be seen from it that the power function of k_c-1 is a quite suitable form for describing the variation of the maximum dynamic shear modulus due to k_c>1 because all of the errors between the test data and regression equation is enough small in engineering sense.

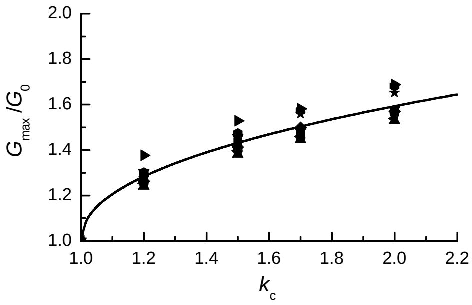

Figure 11 Comparison between the presented formula and the test data for Fujian sands

4.2 Results of the Harbin Sand

Some typical results on the test data and the regression curves of $1/G \sim \gamma$ under two confining stresses for the Harbin sand with Dr=0.60 are illustrated in Fig.12.

According to the above same procedure, the relative increment of the maximum shear modulus for the Harbin sand of Dr=0.60 due to k_c>1 can be obtained as the same form as the Fujian sands, but the coefficients C=0.735 and B=0.626. The comparison between the formula and the test data for Harbin sands is illustrated in Fig.13 and it indicates that the power function of k_c-1 also is a quite suitable form for describing the variation of the maximum dynamic shear modulus for the Harbin sand due to k_c>1.

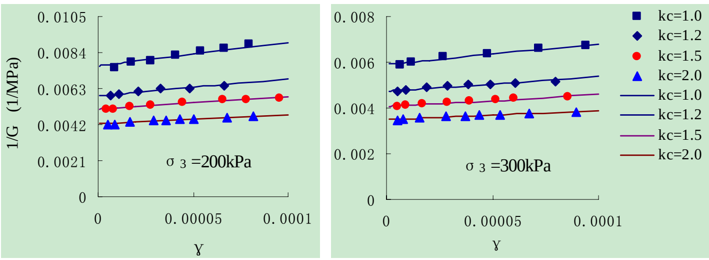

Figure 12 The $1/G \sim \gamma$ for the Harbin sand

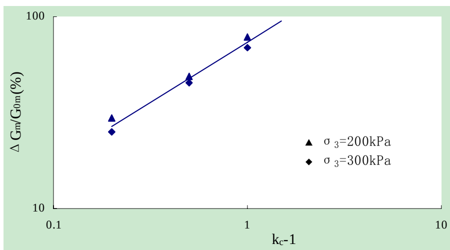

Figure 13 Comparison for Harbin sands

4.3 Recommended Formula of This Paper

By averaging above experimental results of two kinds of sands, a recommended formula for expressing the relative increment of the maximum dynamic shear modulus under the different k_c is presented by

$$\frac{G_{\max}}{G_0} = 1 + 0.66 \cdot (k_c - 1)^{0.54} \tag{5}$$

5 DISCUSSIONS

5.1 Comparison with the Hardin and Black's Formula

Because the effect of consolidation ratios on the maximum dynamic shear modulus is not very clear, some researchers and engineers like to use the Hardin and Black's formula to calculate the maximum dynamic shear modulus of soils under the anisotropic consolidation conditions. If $\sigma_2 = \sigma_3$ and $\sigma_1 = k_c \sigma_3$, according to the Hardin and Black's formula (1968), $\Delta G_m/G_{0,m}$ can be written as

$$\frac{\Delta G_m}{G_{0,m}} = \left(\frac{2 + k_c}{3}\right)^{0.5} - 1 \tag{6}$$

The comparison of Eq.(6) with Eq.(5) in this paper is illustrated in Fig.14.

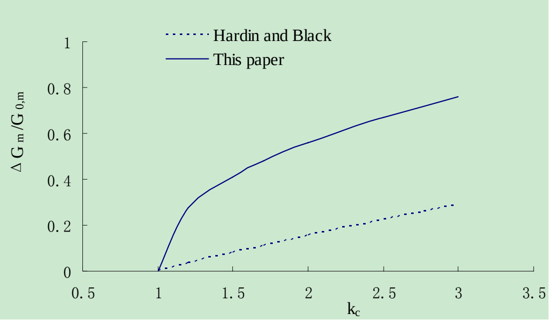

Figure 14 Comparison of $\Delta G_m/G_{0,m} \sim k_c$ between the Hardin and Black's and this paper's

First, it should be noticed that compared with Eq.(5) in the paper, Eq.(6) shows a quite different form as shown in Fig.15. The relation of $\Delta G_m/G_{0,m} \sim k_c$ in Eq.(6) is a nearly linear increase in the maximum dynamic shear modulus in the interval of k_c=1 to 3. While in the paper, $\Delta G_m/G_{0,m}$ is the function of k_c-1 to the power of B. The coefficient B in Eq.(5) is 0.54, much less than 1, which means $\Delta G_m/G_{0,m}$ has a more rapid rise when k_c is near to 1 and then has a slower rise with the increasing of k_c as shown in Fig.15.

Second, it also can be seen from Fig.15 that the effect of consolidation ratios on the maximum dynamic shear modulus of sand is significant. For k_c=1.5 and k_c=2.0 in Fig.15, $\Delta G_m/G_{0,m}$ by the formula presented in the paper is 45% and 66%, respectively, and while $\Delta G_m/G_{0,m}$ by Eq.(6) is only about 8% and 15%, respectively. The increasing degree of the maximum dynamic shear modulus for k_c=2 in the paper is not as small as the value of 15% described by the Hardin and Black's formula.

5.2 Comparison with the Other Results

He (1997) conducts the dynamic triaxial tests for the undisturbed cohesive soils and the disturbed sandy soils under the deviatoric stresses. By employing the test data of Table 3 in his paper, we can attain the relative increment of the compression modulus, i.e. $\Delta E_m/E_{0,m}$. From his results, $\Delta E_m/E_{0,m}$ for k_c=2 is about 40%-100% higher than those for k_c=1. In terms of the formula of E=2(1+v)G, $\Delta G_m/G_{0,m}$ should be quite near to $\Delta E_m/E_{0,m}$ although the Poisson ratio v perhaps is slightly different for the soil specimens under the different consolidation ratio. Then, it can be deduced that $\Delta G_m/G_{0,m}$ for the cases of k_c=2 in his paper may be about 40%-100%. This is coincident in quality with the results in this paper.

Some results (Pitilakis *et al*, 1992; Jiang, 1990) reveal that the maximum dynamic shear modulus by the resonant column tests is always below the values by the velocity tests in the field, and in many cases, the difference between in-situ and in the laboratory is 100%-200% (Jiang, 1990). Some researchers imagine that the reason is the time factor in the consolidation or is that the soil to be tested in the laboratory is disturbed to a certain extent. However, it seems that these explanations are not perfect because the evidence is not enough. From the above results in the paper, the maximum dynamic modulus will rise to a great extent if the actual anisotropic stresses are considered. Therefore, the difference in the consolidation ratio also should be one of the important factors to cause the significant deviation of the maximum dynamic shear modulus between the field and laboratory tests.

If there is an opportunity, more detailed comparison of results between the recommended formula in the paper and in-situ tests should be conducted further to validate Eq.(5).

6 CONCLUSIONS

By the resonant column tests the effect of the consolidation ratios on the maximum dynamic shear modulus for two kinds of sands is investigated and the recommended formula for calculating the increment of the maximum dynamic shear modulus for the cases of k_c>1 is obtained. The conclusions of the paper can be summarized as following:

1. The effect of the anisotropic consolidation on the maximum dynamic shear modulus is quite remarkable and cannot be neglected.
2. A suitable form to show this effect is to use the power function of the increment of the consolidation ratio, i.e. $(k_c\text{-}1)^B$.
3. The variation of the maximum dynamic shear modulus for the cases of k_c>1 can be expressed by the relative increment formula, i.e. $\Delta G_m/G_{0,m}= 1+0.66(k_c\text{-}1)^{0.54}$.
4. The formula presented above means that the maximum dynamic shear modulus shows a more rapid rise in the interval of k_c near to 1 and a slower rise in the interval of k_c far away from 1.
5. The increasing degree of the maximum shear modulus due to k_c>1 is significantly larger than that described by the Hardin and Black's formula, e.g. the increasing degree for k_c from 1 to 2 in the paper is about 66% while only 15% by the Hardin and Black's formula.
6. The consolidation ratio also should be one of the important reasons on the obvious difference between in the field and laboratory determination of the maximum dynamic shear modulus of the soils.

REFERENCES

Amini F, Tawfig KS and Aggour MS (1988), "Cohesionless soil behavior under random excitation conditions," *Journal of Geotechnical Engineering*, *ASCE*, **114**(.8): 896-914.

Chen CL, Hu ZQ, Xie DY and Feng ZY (2002), "Research on dynamic characteristic of sand in foundation of Xiabandi Dam," *Proc. 6th Chinese National Conference on Soil Dynamics,* Nanjing, China, 698-704. (in Chinese)

Hardin BO and Black WL (1968), "Vibration modulus of normally consolidated clay," *Journal of the Soil Mechanics and Foundations Division*, *ASCE*, **94**(2): 353-369.

Hardin BO and Black WL (1969), "Vibration modulus of normally consolidated clay (closure)," *Journal of the Soil Mechanics and Foundations Division*, *ASCE*, **95**(6): 1531-1537.

He CR (1997), "Dynamic triaxial test on dynamic modulus and damping ratio," *Chinese Journal Geotechnical Engineering*, **19**(2): 39-48. (in Chinese)

Jiang ST and Wang XX (1990), "Dynamic modulus and damping ratio of undisturbed soils in Zhengzhou area," *Proc. 3th Chinese National Conference on Soil Dynamics*, Shanghai, China, 151-155. (in Chinese)

Pitilakis KD, Anastassiadis A and Raptakis D (1992), "Field and laboratory determination of dynamic properties of natural soil deposits," *Proc. 10th World Conference on Earthquake Engineering*, Balkema, Rotterdam, 1275-1280.

Seed HB, Wong RT, Idriss IM and Tokimatsu K (1986), "Modulus and damping factors for dynamic analysis of cohesionless soils," *Journal of the Soil Mechanics and Foundations Division*, *ASCE*, **112**(11): 1016-1032.

Yu PJ, Liang YX and Qin WQ (1988), "Dynamic modulus and damping ratio of disturbed sands," *Chinese Journal Geotechnical Engineering*, **10**(4): 57-63. (in Chinese)

Zen K and Higuchi Y (1984), "Prediction of vibratory shear modulus and damping ratio for cohesive soils," *Proc. 8th World Conference on Earthquake Engineering,* Vol. 3, San Francisco, 23-30.