FOUNDATION STIFFNESS ESTIMATES AND EARTHQUAKE RESISTANT STRUCTURAL DESIGN

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ABSTRACT :

The significance of the stiffness and strength of soil beneath a shallow foundation with respect to the earthquake performance of shallow foundations is illustrated. Within the context of performance based design it is proposed that instances of failure during the course of an earthquake are of little significance provided the residual displacements are acceptable. These conclusions are obtained from two entirely different methods of modeling of the foundation response – each is shown to give similar results. One method is to use a macro-element model to develop a simple four degree of freedom model and the other uses structural analysis software which represents the shallow foundation as a bed of springs. The paper emphasizes that sophisticated numerical modeling of earthquake response is of little value without comparable site investigation effort.

KEYWORDS: Earthquake resistant shallow foundation design, LRFD, macro-element, spring bed shallow foundation model, design recommendations, residual displacement.

1. INTRODUCTION

There are two facets to the earthquake resistant design of shallow foundations. One consists of modelling the interaction of the foundation with the underlying and surrounding soil as well as the interaction between the foundation and the structure which it supports. The aim of this part of the design process should be to develop a unified computational model of the structure-foundation system, the calculated response of which can inform the decisions of the designer. The other facet is obtaining realistic values for the parameters that specify the soil behaviour.

The intention of this paper is to investigate the earthquake response of a single degree of freedom structure supported on a rigid foundation block resting on a deposit of saturated clay. The main emphasis will focus on the effects of decisions about the stiffness and strength of the soil. Two approaches will be used to represent the interaction between the foundation block and soil. One approach, broadly following the Federal Emergency Management Agency (FEMA) documents FEMA 273 (1997), FEMA 356 (2000), and FEMA 440 (2005), will represent the soil beneath the foundation block as a bed of independent springs. The other will use a macro-element approach, similar to that of Paolucci (1997) and Cremer et al (2001). For both of these models we can calculate nonlinear response and so assess the displacements which may accumulate by the end of the earthquake. From the perspective of performance based design, limit(s) on the amount of permanent residual displacement may be used as one of the criteria for satisfactory performance.

In representing the soil stiffness and strength we have to allow for two aspects of soil behaviour. First, there is the well known degradation of soil stiffness with strain amplitude. Second, there is the natural point to point variability of the soil properties within the soil layers at the site of the foundation. For the first of these we use the recommendations given in Part 5 of Eurocode 8 (CEN 2003) and in the FEMA documents. The variability is handled using the suggestions given in the FEMA 273 document.

We compare the calculated earthquake responses of single degree of freedom structures supported on a rigid foundation block, the natural period of the structure being set at either 0.5 seconds or 1.0 seconds. Because of space limitations we discuss results obtained with only one earthquake record, that for El Centro 1940 N-S. The horizontal components of this record are approximately "symmetrical", i.e. there is no bias in the accelerations to one side. Furthermore, despite the fact that this is a much used record it has the advantage that the response spectrum is of a similar shape to the shallow soil site spectrum given in NZS 1170.5 (2004). We scaled the record to a PGA of 0.1g, 0.3g and 0.45g.

2. STRAIN DEPENDENT SOIL STIFFNESS BENEATH THE FOUNDATION

It is well known that with increasing severity of earthquake motion larger shear strains are induced in the soil profile with a consequent reduction in the soil stiffness. This is a complex phenomenon involving nonlinear soil stress-strain behaviour. A simplified approach is to assume the soil behaves "elastically" but with a reduced soil modulus, the reduction from the small strain elastic shear modulus of the soil being a function of the peak ground acceleration of the earthquake. This idea was first proposed in early drafts of Part 5 of Eurocode 8, and has been adopted in the various FEMA documents. Some of these recommendations are summarised in Tables 1 and 2, from which it is apparent that the FEMA 440 (2005) recommendations are quite close to those of EC8 Part 5 (2003). Note that the EC8 document also gives suggestions for the soil damping.

Both Tables 1 and 2 are based on the small strain stiffness of the soil. This can be estimated from geophysical methods of site investigation which gives the shear wave velocity of the soil layers. The small strain shear modulus of the soil is obtained from the shear wave velocity. This is not usually the stiffness needed for foundation design as it is an upper bound. However, the methods of determination have the advantage that no correlations are involved, such as those between penetration resistance and soil stiffness.

FEMA 273 and 440 consider the effect of natural variability of the soil; they suggest that upper and lower bounds should be used for the soil stiffness which are twice and one half of the best estimate of the soil stiffness when allowance is made for the effect of earthquake PGA (using, for example, Tables 1 and 2). The documents point out that geotechnical reports will often provide a conservative assessment of soil stiffness and strength. It is not inconceivable that what is supplied in the geotechnical report is equivalent to the above lower bound on the actual soil stiffness. The FEMA documents emphasise that the basis of recommendations given in geotechnical reports needs to be clearly stated, because, as our results below show, ground stiffness has a very significant effect on foundation response to earthquake shaking. A further attraction of geophysical methods of investigating soil stiffness is that the methods give a representative value for a large volume of soil and include inherent variability.

FEMA 273 also proposes a similar four-fold variation between upper and lower bounds on the foundation bearing strength. Once again the reserve of strength assumed by the authors of the geotechnical report needs to be known. One commonly used criterion for the design of shallow foundations under static loading is that about one third of

Table 1 Degradation of soil stiffness with ground acceleration from EC8 Part 5 (2003)

Ground acceleration ratio, αS	Damping ratio	V_s/V_{smax}	G/G_{max}
0.10	0.03	0.90 (±0.07)	0.80 (±0.10)
0.20	0.06	0.70 (±0.15)	0.50 (±0.20)
0.30	0.10	0.60 (±0.15)	0.36 (±0.20)

Table 2 Shear wave velocity reduction factors from FEMA 440 (2005)

Peak ground acceleration (g)	0.10	0.15	0.20	0.30
Shear wave velocity reduction factor	0.90	0.80	0.70	0.65

the bearing strength is mobilised. With the FEMA guideline one would then check the seismic design for a static mobilisation of two thirds and one sixth of the actual vertical bearing strength of the foundation. Tables 1 and 2 estimate the strain dependent change is soil stiffness during earthquake motion but, since we are dealing with clay, there is no earthquake induced decrease in the undrained shear strength. In fact, because we are dealing with rapid loading, the undrained shear strength of the clay may be higher during the earthquake than under static conditions; this is not considered herein.

3. FOUNDATION CONFIGURATIONS

Using the above recommendations we investigate the following foundation configurations. We have a structure of mass 1500 tonnes supported on a surface foundation 10 m square with mass 500 tonnes (this is broadly reminiscent of a bridge pier on a shallow foundation). The natural period of the structure alone is set to either 0.5 or 1.0 seconds. The undrained shear strength of the ground is 100 kPa, and the small strain shear modulus is 50 MPa. The earthquake records are scaled so that the PGA is 0.1g, 0.3g and 0.45g. The size of the foundation is such that under static conditions it is supporting a vertical load of about 20,000 kN at a bearing pressure of 200 kPa, ie about one third of the static bearing strength is mobilised (for a square surface foundation on saturated clay the ultimate bearing capacity is about $6s_u$).

Table 3 gives the bearing strength as well as the soil stiffnesses adopted for both the upper and lower bounds suggested in the FEMA 356 document; we assumed that the stronger soil is correlated with the stiffer material. The stiffness reduction factors given in Tables 1 and 2 are used to get the representative soil shear modulus for the 0.1 and 0.3 g PGA values. Tables 1 and 2 do not cover the 0.45g PGA, so the values given in Table 3 are “extrapolated” from those for the lower PGA values.

Table 3 Upper and lower bounds on soil stiffness and foundation bearing strength

	PGA	0.1g	0.3g	0.45g
G (G_o = 50MPa)	Upper bound	$2.0 \times 0.9^2 \times 50$ = 81 MPa	$2.0 \times 0.6^2 \times 50$ = 36 MPa	12 MPa
	Lower bound	$0.5 \times 0.9^2 \times 50$ = 20 MPa	$0.5 \times 0.6^2 \times 50$ = 9 MPa	3 MPa
Bearing strength	Upper bound	$Q_u = 6V$	$Q_u = 6V$	$Q_u = 6V$
	Lower bound	$Q_u = 1.5V$	$Q_u = 1.5V$	$Q_u = 1.5V$

V = vertical static load on the foundation

4. THE SPRING FOUNDATION MODELS

Computer modelling with a shallow foundation supported on a bed of springs was undertaken using Ruaumoko (Carr 2004), a nonlinear dynamic structural analysis program. Both yielding of the foundation springs and uplift (i.e. detaching of some of the springs from the foundation block) can be modelled. (Yielding of beams and columns in the structure can be included, but this feature is not required in the current work.) Various foundation and structural characteristics were investigated to demonstrate effects on the behaviour of the whole system. The purpose of using a software package such as Ruaumoko, which is intended for structural analysis, is to investigate what can be achieved with existing facilities and also to develop an environment to enhance communication between structural and geotechnical specialists.

FEMA 356 suggests that if the foundation is rigid then the expressions for foundation vertical, horizontal and rotational stiffness given by Gazetas (1991) be used. This means that the foundation is modelled for elastic interaction only. In addition FEMA 356 suggests that if the foundation is not rigid then discrete vertical springs may be used, but with a concentration of spring stiffness towards the edge of the foundation for a distance one sixth of the foundation width from each edge. The document says quite clearly that this is for foundations that are not rigid,

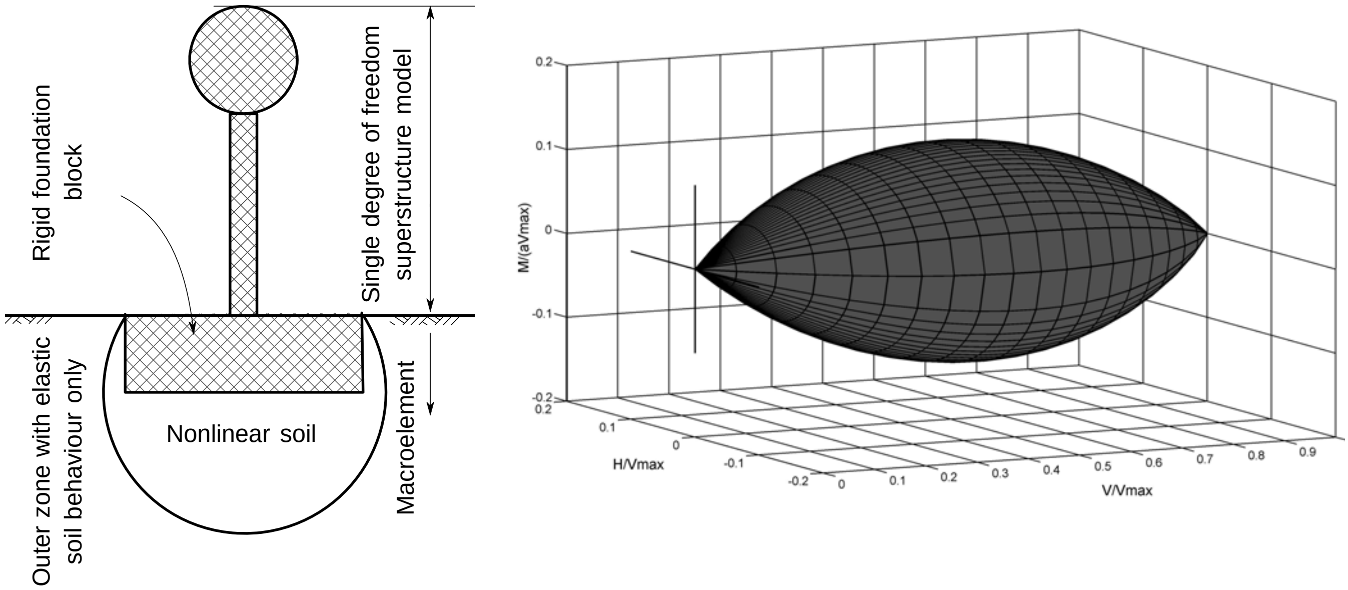

Figure 1 Macro-element concept (left), and bearing strength surface used as a yield surface for the macro-element mode (right).

but with such a ratio between spring stiffness (in effect the reaction pressure) between the middle part and edge we are dealing with a foundation that is very stiff relative to the soil. The values of the spring stiffnesses are set so that the vertical stiffness of the footing is correct. The problem with using independent springs to represent foundation stiffness is that if one sets the overall vertical stiffness correctly (that is equal to the vertical stiffness given by the relevant Gazetas expression) then the rotational stiffness is too low, Pender (2006), Wotherspoon (2007). Herein we have dealt with this difficulty by adding additional rotational springs to the footing when using spring-bed model in Ruaumoko. Since the shallow foundation carries a constant vertical load the actions applied during the earthquake are constrained to a vertical section through the bearing strength surface. When the moment and shear applied reach the boundaries of this constant vertical load section of the bearing strength surface, all the foundation springs are assumed to yield and the stiffnesses are reduced to small values. When the direction of loading is reversed, the spring stiffnesses revert to their original elastic values. Note that this aspect of our modelling follows an approach which is different to that recommended in the FEMA 356 document.

5. THE MACRO-ELEMENT MODEL

A macro-element, Fig. 1, is a type of foundation element that can be used to represent the way that a shallow foundation and the underlying soil interact in response to general dynamic motion. The macro-element provides a single computational entity to represent all the response (elastic, bearing failure, and post-failure plastic) between the foundation and the underlying soil. A bearing strength surface, see Fig. 1, represents all possible combinations of vertical, horizontal, and moment loading that will cause bearing failure of the foundation. This surface is used as a yield locus in the maco-element implemented in this paper. The macro-element has three degrees of freedom corresponding to the rotation, and the horizontal and vertical displacements of the foundation. The macro-element model used for the modeling discussed in this paper is broadly similar to that of Paolucci (1997) and Cremer et al (2001), further description is given by Toh (2008). At small levels of excitation the soil-foundation interaction is represented by three spring-dashpots, one for each degree of freedom. The stiffness and damping values were set to the same values as used in the Ruaumoko model. The values were calculated from the expressions given by Gazetas (1991). In the macro-element, foundation response is assumed to be linear elastic when the actions lie within the bearing strength surface, and plastic when the actions reach the bearing strength surface (that is, when bearing failure is occurring).

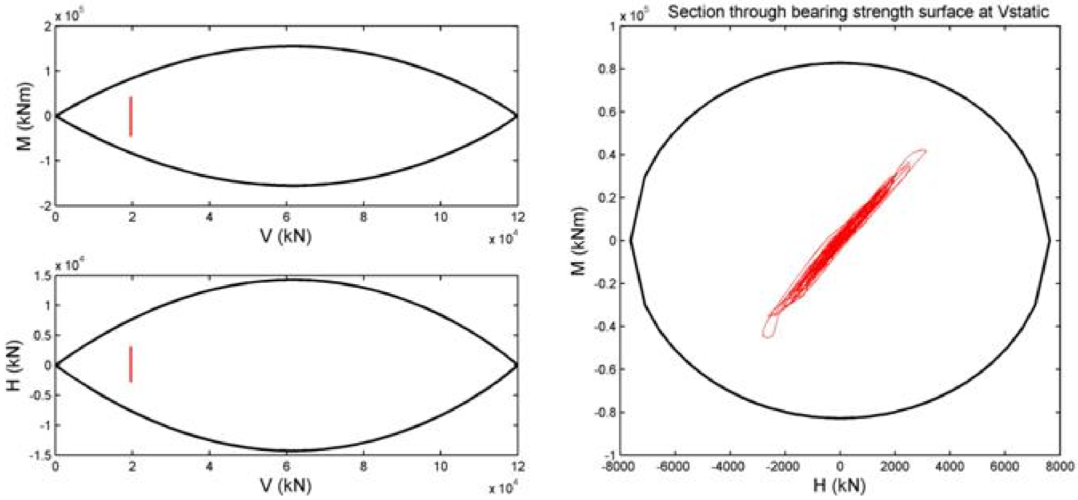

Figure 2 Action path contained within the yield locus/bearing strength surface for elastic response of the system.

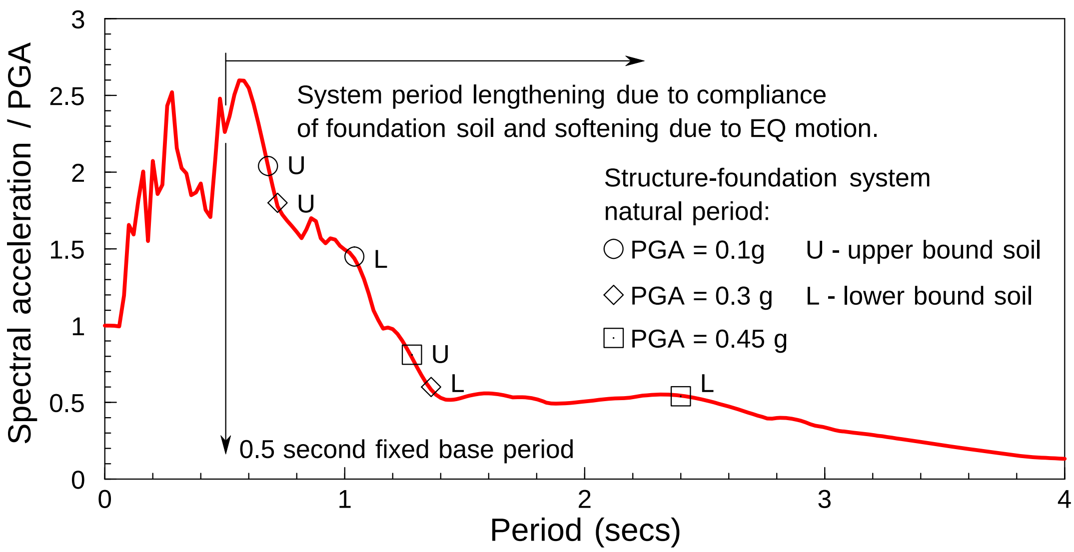

Figure 3 Normalised El Centro 1940 N-S ground motion response spectrum showing the effect of the upper and lower bound soil stiffnesses from Table 3 on a system with a fixed base period of 0.5 sec.

6. RESULTS

Initially the compatibility of the Ruaumoko and macro-element model was checked by running elastic only models. A further check showed that the same elastic results were obtained for the same structural configuration using commercially available structural analysis software. Figure 2 shows an example of the elastic response when the foundation action path lies within the bearing strength surface. The slope of the trace in the M-H plane is mainly a consequence of the height of the structural mass (500 tonnes) above the foundation level (15 m in the calculations in this paper).

As explained above we used the El Centro 1940 north south record, scaled to PGAs of 0.1g, 0.3g and 0.45g. The 5% damped response spectrum for the recorded motion is shown in Fig. 3. Two values for the fixed base natural period of the structural part of the model were used: 0.5 seconds and 1.0 seconds. The effect of the compliance of the soil beneath the foundation and the softening of the soil with increasing PGA is illustrated in Fig. 3. For periods greater than about 0.5 seconds the spectral acceleration values fall. The effect of foundation compliance is to lengthen the natural period of the structure-foundation system thus moving it away from the higher spectral accelerations.

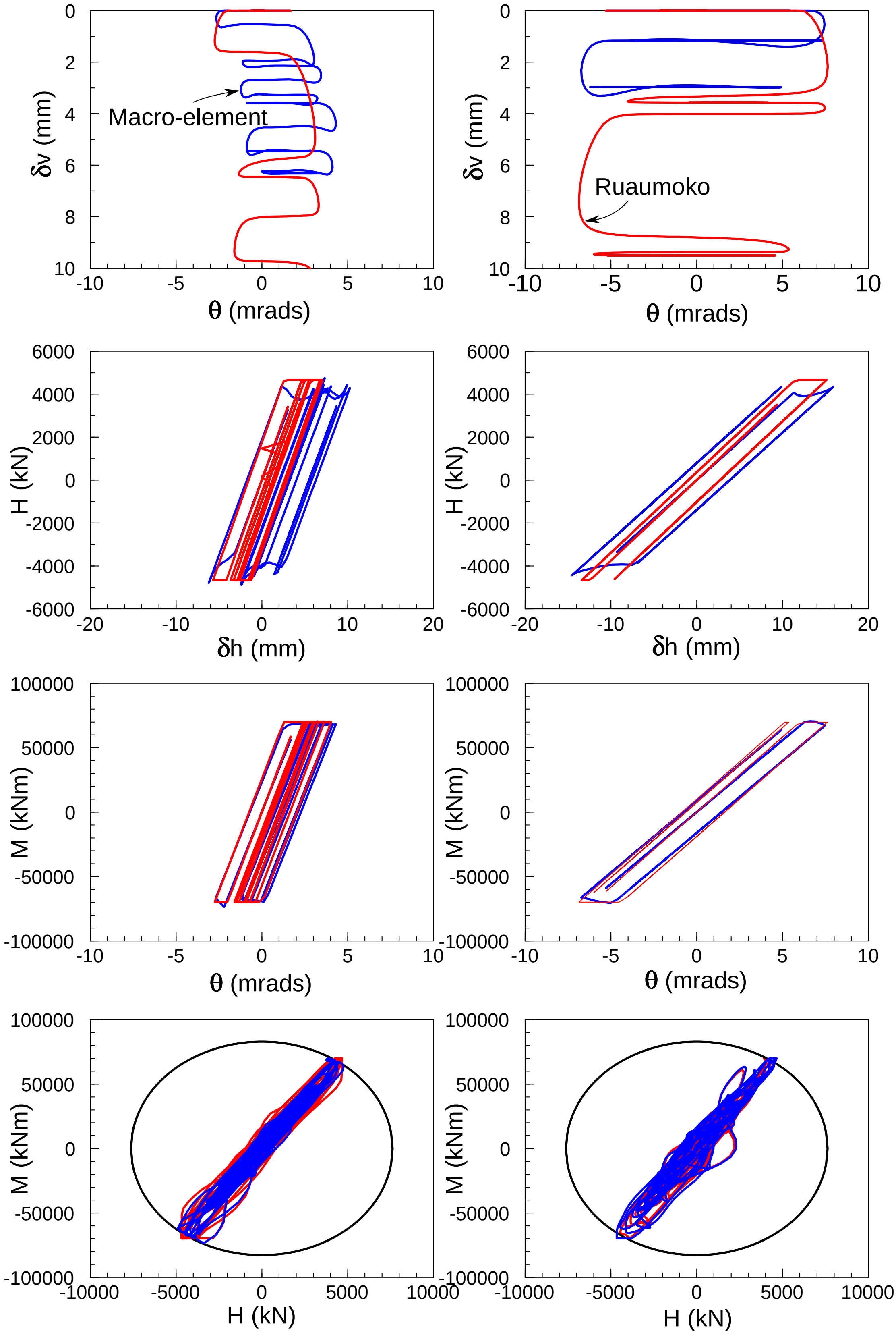

Figure 4 Comparison of the calculated response of the Ruaumoko and macro-element models. Left PGA 0.30g and soil shear modulus 36 MPa, right PGA 0.45g and soil shear modulus 12 MPa.

Table 4 Residual displacements (upper bound soil stiffness and strength)

PGA (g)	0.1		0.3		0.45	
	Ruaumoko	Macro-ele.	Ruaumoko	Macro-ele.	Ruaumoko	Macro-ele.
δ_v (mm)	0.0	0.0	20.0	6.3	9.0	3.0
δ_h (mm)	0.0	0.0	1.3	2.0	0.5	0.7
θ (radians x 10^{-3})	0.0	0.0	1.3	5.5	2.7	2.1

For the runs with a PGA of 0.1g there was no yielding and so the system response was elastic. For the calculations with the PGA set to 0.3g and 0.45g there were instances of yielding during the earthquake excitation. The outputs from two sets of these results are presented in Fig. 4. The left hand side of the diagram is for a PGA of 0.3g and the right hand side for a PGA of 0.45 g. For both these sets of calculations upper bound soil stiffness and strength parameters were used. From top to bottom the four diagrams are: vertical displacement versus foundation rotation (top), horizontal shear on the foundation versus horizontal displacement, moment applied to the foundation versus foundation rotation, and moment versus horizontal shear (bottom). The bottom two diagrams in Fig. 4 show how the action paths for the foundation are contained within the bearing strength surface (yield locus) when yielding (bearing strength failure) occurs, thereby indicating satisfactory calculations with both the macro-element and Ruaumoko models.

Apart from the vertical displacements, it is apparent from Fig. 4 that the two models give comparable results, despite the very different computational processes used for each. The reason that the Ruaumoko model gives more settlement for the foundation is because the method used to represent yielding was simply to reduce the stiffness of all the springs abruptly. In the case of the macro-element, proper calculations with a plastic constitutive relationship are made. The slopes of the load deformation paths in the middle four plots in Fig. 4 reflect the decrease in stiffness of the ground as the PGA increases.

The fact that yielding occurs (that bearing strength failure of the foundation is induced) means that the earthquale motion takes the foundation well beyond the region that would be permitted by conventional LRFD ultimate limit state design (in New Zealand this restricts mobilization of bearing strength to a surface based on 50% of the ultimate vertical bearing strength). This being so, it is of interest to consider the residual deformations of the foundation after the passage of the earthquake; this information is presented in Table 4. As explained above there was no yielding when the PGA was set to 0.1g, so for this case there is no residual deformation as the system remains elastic throughout the earthquake motion. For the other two PGA values there are residual displacements at the end of the earthquake; that is there are permanent displacements of the foundation. However, given the size of the foundation block, 10 m square, these displacements are quite modest.

This observation suggests that mobilsation of the bearing strength of shallow foundations during the passage of an earthquake may not have catastrophic or even serious consequences, and that the residual displacement of the foundation following the earthquake may be a useful criterion for satisfactory design. Before such a suggestion can be made more definite though, calculation with a much wider range of earthquake records will be needed, particularly with earthquake motions that have the horizontal acceleration biased to one side.

However, there is one further aspect of the calculations done herein that is also very important. This is the effect of soil stiffness on the natural period of the system. Figure 3 shows how the use of the lower bound soil shear modulus suggested in FEMA 356 leads to considerable lengthening of the system period. The effect of this is that the system is moved into a region of low spectral acceleration. However, if the real stiffness is at the upper end the FEMA range then the foundation is subject to larger actions. Studies of recorded earthquake response of instrumented buildings during earthquakes by Stewart et al (2002) et al show that period lengthening due to soil-structure interaction effects is quite modest for the majority of cases. For this reason our calculations focused on the upper bound stiffnesses of the soil beneath the foundation. It seems to us that the lower bound suggested in FEMA 356 is unrealistic (although one has to admit that the FEMA recommendation was intended to cover the case with little of no site investigation). The obvious conclusion following from this is that there is no substitute for thorough site investigation so that the actual stiffness of the soils at the site can be specified within much finer limits that the factor of four suggested by the FEMA document. In this light the earlier comments about the potential of geophysical methods is worthy of note. These thoughts also highlight the need for better communication between the geotechnical and structural engineering teams. Perhaps the way forward is for the geotechnical people to be more involved in the aseismic design process rather than simply providing a geotechnical report.

7. CONCLUSIONS

We have reached the following conclusions:

- Good estimates of the strength and stiffness of the soil beneath and adjacent to shallow foundations is essential for meaningful modeling of soil-structure-foundation interaction; without this any effort expended on sophisticated numerical modeling is of little value.
- The macro-element approach has been found to be a convenient way of representing nonlinear soil structure interaction giving results of considerable interest, but requiring only modest computational resources.
- Ruaumoko gives results close to those for the macro-element model provided the yielding is "calibrated" to occur when the foundation moment intersects the bearing strength surface.
- The examples calculated show that although the static actions applied to the foundations satisfy LRFD requirements, actions during earthquake do not, yet residual displacements were found to be minimal.
- It is suggested that performance based design of shallow foundations could be based on post-EQ residual displacement rather than mobilization of a fraction of the bearing strength as in the conventional LRFD or partial factor approaches.

REFERENCES

Carr, A. J. 3D RUAUMOKO: inelastic three-dimensional dynamic analysis program, University of Canterbury - Department of Civil Engineering, Christchurch, NZ, 2004.

Comité Européen de Normalisation Eurocode 8, part 1: Geotechnical Design, General Rules., Draft 6, January 2003.

Cremer, C. and Pecker, A. and Davenne, L. (2001) "Cyclic macro-element for soil structure interaction: material and geometrical nonlinearities." *International Journal of Analytical and Numerical Methods in Geomechanics*, **25(13)**: 1257-1284.

Federal Emergency Management Agency (1997) "Guidelines for the Seismic Rehabilitation of Buildings - FEMA 273". Washington D. C.

Federal Emergency Management Agency (2000) "Prestandard and Commentary for the Seismic Rehabilitation of Buildings - FEMA 356". Washington D. C.

Federal Emergency Management Agency (2005) "Improvement of Nonlinear Static Seismic Analysis Procedures - FEMA 440". Washington D. C.

Gazetas, G. Foundation Vibrations. In *Foundation Engineeing Handbook*, H. Y. Fang, Ed. Van Nostrand, 553-593, 1991.

Standards New Zealand (2004) "NZS 1170.5:2004 Structural design actions Part 5: Earthquake Design Actions – New Zealand". Wellington.

Paolucci, R. (1997) "Simplified evaluation of earthquake-induced permanent displacements of shallow foundations". *Journal of Earthquake Engineering*. **1(3)**, 563-579, 1997.

Pender, M. J., Wotherspoon, L. M., Ingham, J.M. and Carr, A. J. (2006) "Shallow foundation stiffness: continuous soil and discrete spring models compared", Annual Conference NZ Society for Earthquake Engineering, Napier.

Standards New Zealand (2004) "NZS 1170.5:2004 Structural design actions Part 5: Earthquake Design Actions – New Zealand". Wellington.

Stewart, J. P., Seed, R. B. and Fenves, G. L. (2001) "Seismic soil-structure interaction in buildings. II: Empirical findings". Journal Geotechnical and Geoenvironmental Engineeering, **125(1)**, 38 – 48.

Toh, J. C. W. (2008) "Performance based aseismic design of shallow foundations", ME thesis, University of Auckland.

Wotherspoon, L. M. (2007) "Integrated modeling of Structure-Foundation systems", PhD thesis, University of Auckland.