DYNAMIC PROPERTY IDENTIFICATION OF SOIL USED IN SHAKING TABLE TEST: A COMPARISON OF SEVERAL METHODS

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ABSTRACT :

Fundamental frequency and damping ratio of soil are very important parameters for calculating dynamic responses of underground structures subject to earthquake excitation. In current practice, these dynamic properties are determined either by static experiment or identification based on spectral analysis. The static experiment, however, may not reflect the evolution of soil properties with the vibration amplitude and frequency. Therefore, the viscous damping ratio of silty clay and its frequency used in a series shaking table test are determined in this paper by three commonly used techniques, i.e. the stochastic subspace identification (SSI), frequency domain decomposition (FDD) and empirical mode decomposition (EMD) plus Hilbert transform (HT). Following items are discussed in details: (1) changing characteristics of soil subjected to different excitation types and with different depth (2) selection of calculating parameters in HHT method (3) discrepancies of results identified by the three methods; also the paper explores the suspending problems in dealing with the large damping ratio systems, willing to propose a new definition method of soil damping in terms of elastic and plastic deformation.

KEYWORDS: Shaking table test, empirical mode decomposition, Hilbert transform, stochastic subspace iteration, frequency domain decomposition

1.INTRODUCTION

The dynamic properties of soil are very important parameters in the calculation of underground structure's response under earthquake excitation. Nowadays lots of methods employed to determine soil properties are based on indoor resonant column test, through indoor experiment, shearing wave velocity (dynamic modulus) and damping ratios are obtained, and then the predominant period/frequency could be estimated by empirical formulas. Currently there exits three sorts of apparatus: resonant column apparatus, torsion shear apparatus and triaxial shear apparatus. These indoor experiments are all carried out under small deformation presumption and fail to reflect neither the inputs-dependant varying properties nor the nonlinear development when subject to strong seismic.

Soil properties could also be determined by field measurements of earth impulse. Through some empirical formulas or some spectrum analysis tools soil properties could also be easily estimated. Stroke proposed that the shear velocity could be calculated by analyzing the spectrum of surface waves; Comparing the results of indoor experiment and in situ measurement method, Anastassiadis (1992) estimated predominant period through Fourier Transform and peak-picking of power spectrum; Toshikazu inversely analyzed the soil parameters using genetic algorithm in terms of seismic array. All these analysis based on field measurements could produce good results, however, the strong seismic records are scarce and it costs much more money.

Fortunately the development and applications of modern mode identification methods provide us the feasibility of analyzing soil properties at a different angle. In this paper soil dynamic properties are identified and analyzed through various mode identification methods in connection with soil free field shaking table test. There are mainly three methods involved here: Stochastic Subspace Identification known as SSI, Hilbert-Huang transform known as HHT and Frequency Domain Decomposition known as FDD. Applicability of these three methods are investigated respectively, the variation of identified frequency and damping ratios according to different excitation type and different soil layer depth will be further discussed as well.

2.METHODOLOGY

2.1. HHT

Hilbert-Huang Transform is firstly proposed by Norden Huang in 1998, which combines the empirical mode decomposition (EMD) with Hilbert transform. As a new signal processing method, it shows much advantage over traditional FT or WT methods in dealing with the non-stationary signal. Yang (2003) proposed the frame work of identification method based on HHT, and then he identified the natural frequency and damping ratio of a 76-story model with wind tunnel test. Chen (2004) successfully identified the natural frequency and damping ratio of Tsing Ma bridge under typhoon Victor based on HHT, he also did some theoretical research on closely spaced structures. It is proved that the HHT could be applied to analyze the non-stationary nonlinear signal and identify modal parameters.

Generally speaking, there are four main steps included in the implementation of EMD-HT process for parameter identification. Firstly the modal responses are extracted from the measured response using EMD through a procedure called sifting process; Secondly the RDT (random decrement technique) is applied to each modal response to obtain the free decaying response (FDR); thirdly the Hilbert transform is implemented to each FDR; finally the modal frequency and damping ratio are estimated through least square fit.

For a SDOF system, the displacement response function of the system under impulsive loading is given by:

$$v(t) = A_0 e^{-\xi\omega_0} \sin \omega_d t \tag{2.1}$$

where ω_0 is the natural circular frequency, ξ is damping ratio and ω_d is the damped natural circular frequency, A_0 is a constant. According to Hilbert transform the analytical signal is defined as follows:

$$z(t) = v(t) + i\tilde{v}(t) = A(t)e^{-i\theta(t)} \tag{2.2}$$

where $\tilde{v}(t)$ is the Hilbert transform of $v(t)$, for a SDOF system where damping ratio ξ is much smaller than circular frequency ω_0, thus amplitude $A(t)$ and phase angle $\theta(t)$ could be approximately obtained through Hilbert transform:

$$\ln A(t) = \ln A_0 - \xi\omega_0 t \tag{2.3}$$

$$\theta(t) = \omega_d t - \pi / 2 \tag{2.4}$$

$$\omega_d = \omega_0\sqrt{1-\xi^2} \qquad \xi = \sqrt{1-(\omega_d / \omega_0)^2} \tag{2.5}$$

The damped natural circular frequency ω_d can be estimated by the slope in Eqn (2-4), with the identified ω_d and the slope $-\xi\omega_0$ of the straight line of the decaying amplitude $A(t)$ in a semi-logarithmic scale, the damping ratio ξ can be identified from the function (2-5). Considering that the phase angle $\theta(t)$ will fluctuate around its mean value with the variation of the amplitude, and the small damping presumption will confine the application of HHT method.

Compared with the traditional signal processing method based on WT and FT, HHT bears no limitation of stationary and linear data. Its adaptive property brings much convenience to calculation, not like WT which should first preset the wavelet function. However, HHT method is empirical, incompleteness still exits in its theoretical demonstration, such as the orthogonality. Even if some deficiencies remains, HHT as a powerful signal processing method has been applied widely to sorts of projects.

2.2. SSI

Stochastic subspace iteration (SSI) is a time domain method, firstly proposed by Van Overchee in 1996. Compared with the traditional time domain method, SSI only needs output of system and has a higher frequency resolution, nowadays it is widely applied in the mechanics and civil engineering as a parameters identification technique, however, it should be noticed that the system input must be stationary stochastic process in SSI. Through some orthogonal projection algorithms like QR decomposition and singular value decomposition (SVD), we can easily determine the system model.

Generally speaking, SSI mainly consists of two steps: (1) determining the extended observyability matrix Γ_s or estimating the observable array X_s of the system (2) calculating system matrix. According to the selection of different weighted matrix, there are three means involved in the first step: (1) Unweighted Principal Component algorithm(UPC) (2) Principal Component algorithm (3) Canonical Variant Analysis algorithm（CVA）. In the calculation of this article only CVA method is referred.

In CVA method, extended observablility matrix Γ_i and extended controllable matrix Ω_i are obtained by the decomposition of condition covariance matrix, simultaneously the state series of system is estimated as $\hat{X}_i$. The weighted matrix as follows:

$$W_1 = E[Y_f Y_f^T]^{-1/2} = E[Y_p Y_p^T]^{-1/2} = L_0^{-1/2} \qquad W_2 = I \tag{2.6}$$

The covariance of $W_1 O_i W_2$ now equals:

$$E[W_1 O_i W (W_1 O_i W)^T] = E[L_0^{-1/2} O_i (L_0^{-1/2} O_i)^T] = L_0^{-1/2} L_i L_0^{-1} L_i^T L_0^{-1/2} \tag{2.7}$$

And finally we see that the extended observability matrix is determined as follows:

$$\Gamma_i = L_0^{-1/2} U_1 S_1^{1/2} T \tag{2.8}$$

This algorithm typically forces the use of a larger state space dimension than the two other available algorithms. The reason is its ability to estimate modes with a large difference in energy level. In order to see low excited modes among well-excited modes, it is necessary to force a large state space dimension. CVA algorithm is adopted in this paper.

2.3.FDD

Frequency domain decomposition method actually is an extension of the classical frequency domain approach, it is proposed by Brinker in 2000, in FDD algorithm, singular value decomposition (SVD) is taken to the spectral matrix of the input signals, then the spectral matrix is decomposed into a set of auto spectral density functions each corresponding to a SDOF system, however this process is only valid in the case where the loading is Gauss white noise, and the structure is lightly damped. Compared with traditional frequency domain method, FDD method has higher frequency resolution which is effective for the closely spaced mode cases; therefore it is widely applied in the real time monitoring system of some key projects or aircrafts.

Assuming the power spectrum density (PSD) of the input white noise is $G_{xx}(j\omega)$, and then the output PSD $G_{yy}(j\omega)$ is as follows:

$$G_{yy}(j\omega) = \bar{H}(j\omega) G_{xx}(j\omega) H^T(j\omega) \tag{2.9}$$

After some mathematical operations, the final form of $G_{yy}(j\omega)$ could be simplified through Heaviside partial fraction theorem as follows:

$$G_{yy}(j\omega) = \sum_{k \in Sub(\omega)} \frac{d_k \phi_k \phi_k^T}{j\omega - \lambda_k} + \frac{\bar{d}_k \bar{\phi}_k \bar{\phi}_k^T}{j\omega - \bar{\lambda}_k} \tag{2.10}$$

where $Sub(\omega)$ are a limited number of modes contributing significantly to the PSD, d_k is a constant and ϕ_k is mode shape vectors.

The main steps involved in this algorithm are as follows :(1)calculating the PSD of original signal

(2)taking SVD to the PSD matrix (3)averaging the singular values if multiple data is available (4)point selecting procedure to estimate the modal parameters.

When the singular value is taken from the PSD matrix, the system frequency will be given corresponding to the peak value of the SVD curve, meanwhile the left singular vector is proportionate to mode shape and the right singular vector is proportionate to modal participating coefficient. In the fourth step, the frequency response function(FRF) of each SDOF could be obtained by taking the mode shape and modal participation coefficient as weighted functions, then the traditional time domain or frequency domain method could be applied to identify the frequency and damping ratios.

2.4. Equivalent damping ratio λ

Actually soil frequency and damping ratio identified by the above methods are equivalent linearized results. Soil should be equalized as linear dynamic viscoelastic material in order to do further comparison with the results calculated by soil dynamic constitute model, the following equation calculates the equivalent damping ratio:

$$\lambda = \frac{1}{4\pi}\frac{\Delta W}{W} \tag{2.11}$$

where ΔW is the energy dissipated in a loading cycle, W is the energy reserved in a loading cycle also known as elastic strain energy. It should be paid attention that the equation above is available only if the amplitude of the dynamic strain is small. The hysteresis loop of strain-stress will not close due to the existence of residual strain when the dynamic strain amplitude is large. For convenience the damping ratio produced by Eqn 2-8 is called definition values (Def.) in the following parts.

3. SHAKING TABLE TEST

The shear box used in the test was shown in Figure1; the overall dimension of the box is 3000mm long, 1800mm wide and 1870mm high. The laminar shear box consists of 16steel tuber frames. The space between every two frames is 21mm, where some balls are put inside the guide groove to guarantee the frames could slide laterally in the shaking test.

Figure 1

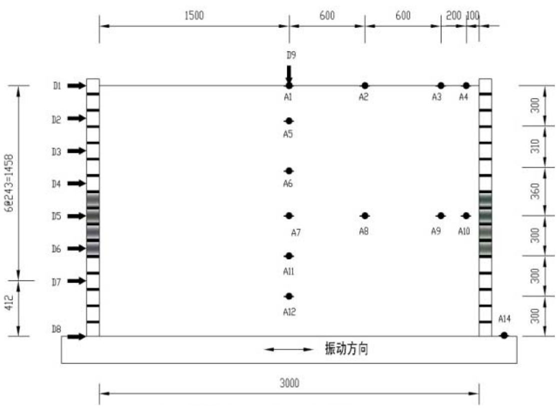

Figure 2

A series of shaking table tests are carried out with different input types, for example: sine wave loading (6Hz, PGA 0.05g, 0.6g and 1.0g), white noise (0-25Hz) and El centro earthquake (PGA 0.05g, 0.1g and 0.4g); As shown in Fig 2, the shaking direction is along the long axis, A1-A19 are accelerometers recording the acceleration response and D1-D8 are displacement gauges recording the displacement of the corresponding soil layer.

4. RESULTS AND DISCUSSION

4.1. Predominant frequency

Predominant frequency identified under different excitation types is listed in table 1; it is found that neither SSI nor FDD can produce reasonable frequency under El centro excitement.

Table 1

Excitation/amplitude	Methods	A1	A7	A12
Sine wave 0.1g/0.6g/1.0g	HHT	6.01/6.01/6.02	6.03/5.97/5.95	5.77/6.00/6.01
	SSI	6.00/6.00/6.01	6.00/6.00/6.01	6.00/6.00/6.01
	FDD	6.01/6.01/6.00	6.01/6.02/6.00	6.01/6.01/6.00
El centro 0.05g/0.1g/0.4g	HHT	7.94/7.24/3.87	7.9/6.95/3.69	7.32/6.44/ NaN
White noise (displacement control)	HHT	6.29	6.54	6.32
	SSI	6.16	6.21	5.67
	FDD	6.00	6.00	6.25

The results under white noise excitation are found around 6Hz. As we know sine wave excitation is the main type in dynamic triaxial test, and steady state sine wave excitation is the common pattern in structural engineering vibration test. The results have nothing to do with the amplitude due to the periodical excitation. The results under El centro excitation present a descending trend, which is identical with the experimental phenomenon. The predominant frequency is varying with the amplitude under earthquake input, and coupled with vibrating process, which is totally different with cases under sine wave and white noise excitation, especially in strong earthquake.

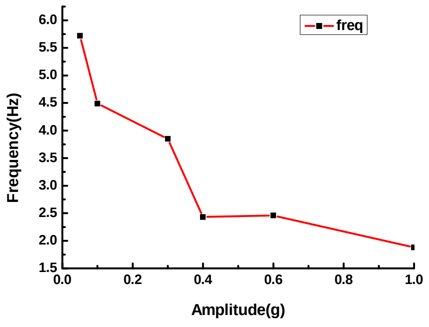

Figure 5

It is found interesting that relationship between excitation amplitude and predominant frequency of soil under El centro earthquake shown in figure 5, where x-axis represents 5 amplitudes from 0.05g to 1.0g, y-axis is the corresponding predominant frequency; it is conspicuous that the predominant frequency is decreasing from 5.73Hz to 1.88Hz with amplitude varying from 0.05g to 1.0g. It is also appealing to see such a sharp decay under 0.3g El centro earthquake, which seems that the soil has entered into nonlinear state.

4.2. Damping ratio

Results are listed in table 2, through dynamic triaxial test, dynamic elastic modulus, shearing modulus and damping ratios of unsaturated soil with different dry density are shown in table 3, where the damping ratio is between 3.01%~11.02%.

Table 2

Excitation/amplitude	Methods	A1	A7	A12
	HHT	1.32/1.43/4	2.63/4.3/4.4	7.11/0.3/0.2
Sine wave	SSI	0.085/0.14/0.11	0.096/0.15/0.14	0.14/0.14/0.14
0.1g/0.6g/1.0g	FDD	2/2/2.08	2/2.04/2.01	2/2.01/2
	Def.	59/98/70	51/10.7/78	12.79/134/42
El centro	HHT	4.66/9.28/6.91	5.4/14.41/11.81	8.19/11.44/NaN
0.05g/0.1g/0.4g	Def.	57/52/29.8	64/68/22.8	2.89/15.97/5.78
	HHT	9.39	6.07	7.09
White noise	SSI	10.41	NaN	9.13
	Def.	36	25.8	132

Table 3

No.	$\rho_d\,(g/cm^3)$	K_c	$\sigma_{3c}\,(kGp)$	$E_{d\max}\,(MPa)$	$G_{d\max}\,(MPa)$	$\lambda_{d\max}\,(\%)$
		1.2	50	7.63	2.73	11.02
1#	1.4	1.2	100	9.27	3.31	3.01
		1.2	150	7.15	2.55	5.01
		1.2	50	12.89	4.60	9.56
2#	1.5	1.2	100	15.20	5.43	7.01
		1.2	150	18.76	6.70	6.01
		1.2	50	14.79	5.28	9.74
3#	1.6	1.2	100	23.75	8.48	9.91
		1.2	150	31.55	11.27	4.92

It is seen in table 2 that damping ratio calculated by definition is unreasonable under sine wave excitation, since the hysteresis loop of shear strain-stress curve under dynamic loading dose not comfort to elliptical assumption, which also reveals the significance of employing identifying method to determine the soil property. FDD fails to produce reasonable results under white noise and earthquake excitation; the results are only 2% under sine wave excitation, which is not in accordance with experimental phenomenon, therefore FDD is not a proper way to deal with the signal in this experiment. The damping ratio obtained by SSI under white noise excitation is reasonable, nevertheless the results is only 0.1% under sine wave excitation. The damping ratio identified by HHT under all three excitation is reasonable, and the results under white noise and earthquake are similar to that produced by dynamic triaxial test.

It is found that the damping ratio is increasing with the soil depth, which comforts to general engineering cognition. Theoretically speaking, HHT is adaptive to deal with nonlinear non-stationary signal, accordingly the results by HHT are more reasonable than other two methods, however the stability of HHT algorithm is greatly influenced by selection of parameters, which still needs further research.

5. CONCLUDING REMARKS

Through the comparing research of these three identifying methods based on shaking table test, some basic conclusions are drawn: (1) predominant frequency of site soil could be identified precisely by

SSI, FDD and HHT under random excitation, results under periodical loading is prominently affected by the loading period, HHT is more reliable under earthquake excitation (2) it is inappropriate to determine the damping ratio of soil through definition. (3) Both SSI and HHT could reflect the real damping ratio under random excitation only if $\xi \leq 10\%$, HHT could be applied under earthquake excitation, and the parameter selection is still a problem. (4) The predominant frequency `and damping ratio will vary with the nonlinear development of soil under strong earthquake and will be coupled with the vibrating process; therefore research on time-varying property of soil should be carried out after soil enters into nonlinear state.

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