LARGE DEFORMATION ANALYSIS OF PILE FOUNDATION USING MESHLESS METHOD

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ABSTRACT :

Considering on transmogrification of pile foundation under load, the basis function equations of the element-free Galerkin method were built, and a meshless method based on large deformation was discussed. Following this theory, a nonlinear program is made as well. In order to verify the theory and the program, a pile foundation loading test of Maocaojie Bridge in Hunan Province is chosen to validate calculation data. The result shows that the calculation results fit close to survey data on-the-spot. It illuminates that the method presented in this paper can be applied in numerical simulations on the consideration of large deformation rationally and effectively. Moreover, it also indicates that due to the strength of surrounding soil, the horizontal carrying capacity of pile will be varied widely.

KEYWORDS: large deformation; nonlinear analysis; pile foundation; meshless method

1. INTRODUCTION

Traditional finite element methods depend on grids heavily. When dealing with discontinuity and large deformation, it causes decrease in computation precision and increase in computation time, especially in grids re-built and aberrance. The meshless method, created in 1970's, has the characteristics of needing only node information, but no element information. Currently a few number of meshless methods (such as mainly smoothed particle hydrodynamics (SPH), diffuse element method (DEM), element-free Galerkin method (EFGM), reproducing kernel particle method (RKPM), partition of unity method (PUM), meshless local Petrov-Galerkin method (MLPG), finite point method (FPM), boundary point interpolation method (BPIM) and so on have been presented and discussed.

Generally speaking, the meshless methods can be used in any fields that finite element method may do without much difficulties in principle. That is why the numerical strategy of meshless method has already been conducted in engineering areas widely. For example, meshless method is one of effective ways to solve derivation failure by occasion of mesh distortion, when using Lagrange FEM formulation settles the problem of large deformation. Chen and his co-operators, Li and his partners have already applied this method in simulating of rubber's large deformation and procedure of metal forming. In these pieces of essay, the basis function equations of the element-free Galerkin method were built. What's more, a meshless method based on large deformation was discussed by considering pile transmogrification under foundation load. On the base of that, a nonlinear program is made. In order to verify the method and program, a pile foundation loading test of Maocaojie Bridge in Hunan Province is chosen to provide calculation data.

2. MESHLESS METHOD IN PILE'S LARGE DEFORMATION ANALYSIS

Be different from the usual finite element method, in which interpolation is used to establish approximate function, the meshless method obtains superior localized approximation through the computation points in the neighbor domain, and then utilizes the superior localized approximation to solve the overall approximate functions in the solution domain. In order to account the displacement coordination of pile-soil interaction, the meshless method utilizes the computation points to establish approximate function in pile-soil domain.

2.1. General Process of Large Deformation

In meshless method, the approximate plan does not differentiate that the approximate function established in initial configuration or current configuration, because the error between both may be ignored in linear elasticity matter with small deformation. However, it is very necessary that differentiating this one in matter with large deformation.

Supposing that domain Ω_X with boundary Γ_X is occupied in initial state, and that domain Ω_x with boundary Γ_x is occupied after deformation. Then the space coordinate x of certain point X in the object at the moment t may be denoted by following mapping.

$$x = \varphi(X, t) \tag{2.1}$$

Noting that the mapping is in one-to-one correspondence, so the Jacobi determinant $J = \left|\partial x_i / \partial X_j\right|$ is assured significative. The deformation procedure of matter is corresponded with control formula of elastic mechanics discussed above, and described by motion equation and relevant boundary condition and initial condition. Namely.

Equivalent formula: $\sigma_{ij,j} + \bar{f}_i = \rho \ddot{u}_i$ within Ω_x (2.2)

Force boundary condition: $\sigma_{ij} n_j - \bar{t}_i = 0$ within Γ_t (2.3)

Displacement boundary condition: $u_i = \bar{u}_i$ within Γ_u (2.4)

$$u_i(X, 0) = u_i^0(X) \tag{2.5}$$

where ρ is the density of matter; Ω is the given physical strength of domain; $\bar{t}_i$ is the given surface strength; n_j is the outer normal direction cosine of the boundary of Γ_t; $\bar{u}_i$ is the given displacement; u_i^0 is initial displacement; σ_{ij} is Cauchy stress tensor, represented as displacement function depending on concrete material's constitutive relation. Cauchy stress is related with the second Piola－Kirchhoff stress tensor S_{mn} by following formula.

$$\sigma_{ij} = \frac{1}{J} F_{im} S_{mn} F_{nj} \tag{2.6}$$

where, F_{ij} is deformation gradient tensor. For superelastic material, the second Piola－Kirchhoff stress tensor may also be present by strain energy density function W and Green- Lagrange strain tensor E_{ij}.

$$S_{ij} = \frac{\partial W}{\partial E_{ij}} \tag{2.7}$$

Equivalent variation of the above equation may be described as.

$$\int_{\Omega_x} \delta u_i \rho \ddot{u}_i d\Omega + \int_{\Omega_x} \delta u_{i,j} \sigma_{ij} d\Omega - \int_{\Omega_x} \delta u_i f_i d\Omega - \int_{\Gamma_t} \delta u_i \bar{t}_i d\Gamma = 0 \tag{2.8}$$

The incremental iteration form of the above equation should be adopted, dealing with nonlinear problems. Considering the arbitrariness of variation, the discretization equation of space domain can be gotten by substituting approximate function of meshless method into the former formula. The matrix form is as follows.

$$M\Delta\ddot{d} + K\Delta d = \Delta f \tag{2.9}$$

where M and K are mass matrix and stiffness matrix respectively; Δf is the difference of external force and internal force in node.

2.2. Equivalent Equation of Large Deformation in Pile

The performing behavior of pile under load is complicated. The flexural deflection of pile body may be produced under load, and along with the development of flexural deflection, the resistance of soil surrounding the pile may be generated by extrusion of soil. This resistance prevents the further development of flexural deflection. All of these compose a complicated system of soil-pile interaction. The relationship between the force and the displacement is not linear anymore. The really condition of stress and deformation may be reflected correctly on the consideration of the non-linear relationship. The research of FEM is based on the assumption of small deformation in classical mechanics in the past. However, when the deformation is large, the assumption of small deformation is unsuitable and cannot reflect the true condition of stress and deformation correctly. Therefore, the meshless method is introduced into the research of large deformation.

Generally, the treatment of large deformation in pile utilizes the updated Lagrange method, which computer iteratively by incremental step in reference configuration at the moment t. Supposed every class of load increment is ΔP_i and the iteration of the i^{th} incremental step is convergence, the linear load value may be extracted from the system tangential stiffness matrix at the beginning of the i^{th} incremental step $[K]_{i-1}$, where all kinds of non-linear factors contributed to pile-soil system stiffness matrix are taken into account during all loading process before the i^{th} incremental step. Then the relevant equation extracted non-linear load value may be described as follows.

$$([K]_{i-1} + [\Delta K]_{\sigma})\{\Delta d\} = \{\Delta f\} \tag{2.10}$$

where, $[K]_{i-1}$ is the system tangential stiffness matrix at the beginning of the i^{th} incremental step; $[\Delta K]_{\sigma}$ is the system geometric stiffness matrix, which express the effect of initial stress on the structural stiffness in large deformation state, that is, the function is related with system load increment ΔP_i calculated from the i^{th} incremental step; $\{\Delta d\}$ is the incremental displacement; $\{\Delta f\}$ is the incremental external load, or physical strength matrix, or surface strength matrix.

Actually, all variable transferred to the initial configuration are calculated in meshless method. The approximate function is established in initial configuration rather than in current configuration, since the simplification of approximate function dose not affect the calculative precision markedly. Thus, the above equation may be simplified as follows.

$$[K]\{\Delta d\} = \{\Delta f\} \tag{2.11}$$

where $[K]$ is the stiffness matrix integrated by every node of soil-pile solution domain based on its weight and physical function in definition domain. Thus, meshless method has a form of equivalent equation, which is similar to FEM and calculated by classical methods such as load method, displacement method and arc-length method.

3. FIELD EXPERIMENT AND CALCULATION ANALYSIS

3.1. Field Experiment

The piles task group of Hunan University carried through a loading test for pile foundation of Maocaojie Bridge in Hunan Province in 2001. The test pile is borehole cast-in-place concrete pile, and the shaft wall is protected by drilling mud. The diameter of pile is 1.0m, and embed depth is 60.0m, which the bottom of pile reaches the sedimentary rock layer slightly weathering. The strength of the concrete in the pile body is C30, the true measurement of the concrete's elastic modulus is 3.47×10^4 MPa, and Poisson's ratio μ is 0.2.

The loading device of experiment adopted the anchor pile of static load test, shown in figure 1 and figure 2. The test adds loads by multi-cycle load. Taking into account the ultimate bearing capacity estimated and behavior of every lifting jack, every class vertical load is 1080 kN and horizontal load is 25 kN. Based on the data measured, the vertical ultimate bearing capacity of pile is 17280 kN and the horizontal ultimate bearing capacity of pile is 325 kN; the scale coefficient of soil foundation measured is 5500 kN/m^4.


Figure 1 Sketch of vertical loading test


Figure 2 Sketch of horizontal loading test

3.2. Calculation Analysis

Taken into account the large deformation of pile under horizontal load, a calculation program is developed with Matlab language. For easy calculation, the pile is simplified as a geometric model shown in Figure 3. The numerical integral all uses 4×4 Gauss integral when compute and the influential radius with minimal error are chosen in computing process. The model of pile underneath the ground is considered as clamped support, the top as free standing. The nodes in model are distributed in the computation domain, and shown in the figure (b). The nodes distribution plan takes 36 (12×3) nodes for example. The computation domain can be divided into 55 (11×5) integral sub-domain, that is, background net, as shown in figure (c).

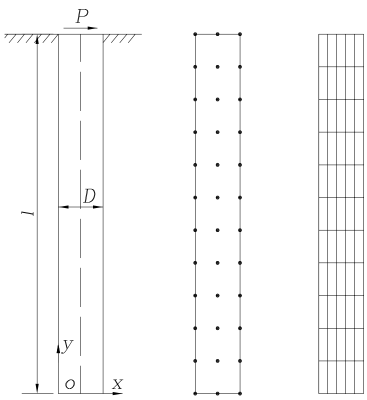

Figure 3 Sketch of the pile foundation

To simulate the process of test, load applied to the pile top is increased gradually by using load increment method. When the horizontal displacement exceeds 6mm, the scale coefficient of soil foundation should reduce to the 0.4 times of original value. And when the stress of pile reaches materials strength, the stiffness of pile should be discount proportionally. Then the curve of load-displacement is obtained and shown in Figure 4 and Figure 5.

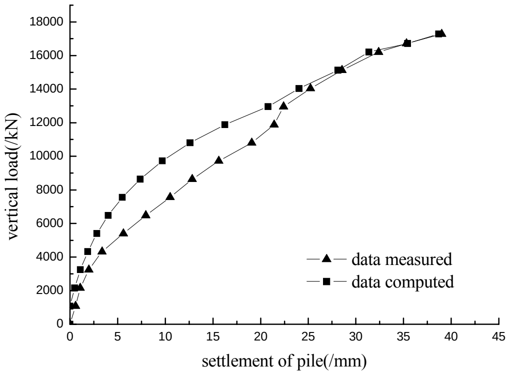

Figure 4 Vertical load-displacement curves

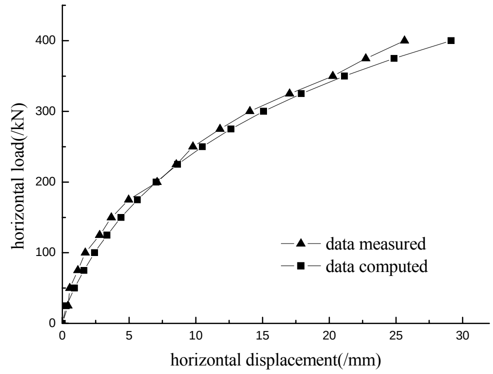

Figure 5 Horizontal load-displacement curves

The vertical load-displacement curves are obtained by measured and meshless method respectively. The calculation results accord with survey data on-the-spot and may be satisfied with the engineering needs. And the results may estimate the bearing capacity and relevant displacement of pile. And the horizontal load-displacement curves are obtained by measured and meshless method respectively. The figure shows that The calculation results accord with survey data on-the-spot.

And the normal stress σ_x of each node in axle wire of pile is calculated and compared by meshless method and exact solution formula. The curve of normal stress is shown in Figure 6. The results show that the calculation precision of meshless method with non-linear analysis is high and may be satisfied with the engineering needs.

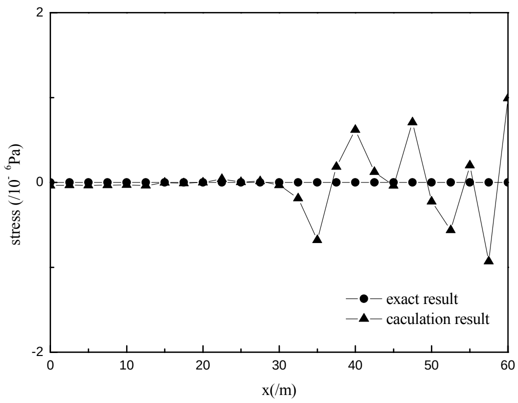

Figure 6 Calculation results of normal stress

These results show that the settlement of pile increases sharply when the vertical load is up to 17280 kN. The concrete pile is compressed failure and the pile is buckling. The horizontal displacement on the top of pile increases with addition of horizontal load, and the surrounding soil moves continually. Meanwhile, these mean that the soil is pressed and bears the loads with pile body. Along with increasing of deformation, the soil begins to failure and the bearing capacity decreases also. The soil failure is earlier than materials failure. The strength and intenerating of soil may influence the horizontal carrying capacity of pile greatly.

4. CONCLUSION

Consideration on large deformation, a non-linear analysis of pile under horizontal load is carried out by using meshless method. Following conclusions can be obtained.

(1) The meshless method, which has enormous advantages in solving large deformation problem, is conducted to investigate the complicated bearing mechanism of pile under load. On the base of that, the equilibrium equation of pile is obtained. The form of equation is simple and it's quite suitable to make program calculation.

(2) Taken into account non-linear effect to the pile under load, a calculation program is developed with Matlab language. By using it, the stress condition of nodes is studied under the circumstance that the pile body has a large lateral movement, and a load-displacement process is obtained as well under the same circumstance. These may reflect bearing conditions and deformation characteristics of pile actually.

(3) A horizontal loading test for pile foundation of Maocaojie Bridge on-the-spot is introduced to provide engineering background and calculation data. The calculation results gained by the meshless method show that the load-displacement calculated curve fits close to the measured curve in field. The horizontal displacement of pile top may influence the horizontal bearing capacity of pile. And the control of displacement should be emphasized in design.

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