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RANDOM VIBRATION THEORY BASED SEISMIC SITE RESPONSE ANALYSIS

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ABSTRACT :

In this paper, an alternative approach for seismic site response analysis is presented. This approach is based on Random Vibration Theory (RVT) and works in frequency domain. With this approach, the need for time history generation is eliminated and the responses are computed at the selected confidence level. This approach follows the SHAKE theoretical framework and is based on closed form solution of one-dimensional wave propagation. The main difference with the commonly used approach in the program SHAKE are as follows:

- The calculation starts by computing the power spectral density (PSD) function of the design acceleration response spectrum.
- Following the computation of frequency domain transfer function for the repose of the interest, the PSD of the response is obtained. The peak factors for the PSD responses are computed and the statistical means of the maximum responses are computed. The maximum shear strain in each soil layer is used to iterate on soil properties using the strain-dependent soil properties of each layer based on the equivalent linear method until convergence is reached for all soil layers.
- After the final iteration on soil properties, the maximum responses of interest such as acceleration response spectra, peak shear stress and strains are obtained.
- This approach does not require time history as input and the design response spectra can be used directly in the analysis.

This approach has been implemented in a new computer program named P-SHAKE and has been used successfully in recent projects. This paper provides the theoretical basis of the approach and presents a comparison of the P-SHAKE and SHAKE results and describes the advantages and limitations of the new method.

KEYWORDS: Site Response Analysis. Random Vibration Theory. P-SHAKE

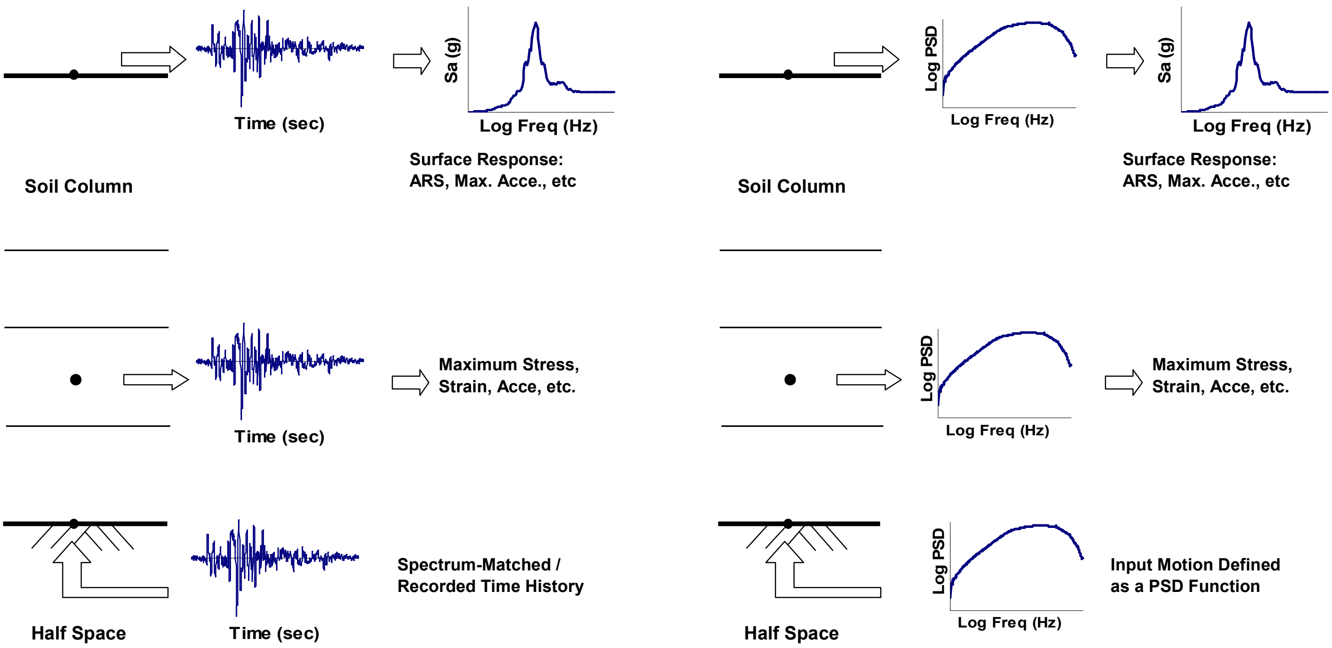
INTRODUCTION

For most critical structures, seismic design motions are developed by generating the rock motions using probabilistic seismic hazard analysis (PSHA) and propagating the rock motions through the soil column to include the local site effects. The latter part of the process is commonly referred as the site response analysis. In the current engineering practice, most site response analysis studies were performed using the well-known program SHAKE (Schnabel et al 1972, Idriss and Sun 1992) and its linear and non-linear variations. The analysis approach utilized in SHAKE and its variations is shown schematically in Figure 1(a). In this approach, an acceleration time history is generally required as the input motion. The time history is typically generated by matching the rock motion target spectrum obtained from the seismic hazard analysis. It is well known that using several time histories, in spite of all matching the same target spectrum, results in a range of amplified ground motions. This is mainly due to the fact that the phasing and energy characteristics of the time history play a significant role on soil column responses, particularly for site conditions where soil nonlinearity becomes important (e.g., Ostadan et al, 1996). Recognizing this effect, typically a suite of time histories e.g. 30 – 60 time histories, all matching the same rock target spectrum are generated using different recorded time histories considering the seismic setting and geological condition of the project site. Selection of such a large suite of time histories where limited recorded motions are available (e.g. Eastern U.S.) is very challenging and often involves modifying the motions from other regions to the project site.

This paper presents an alternative approach for conducting seismic site response analysis which eliminates the need of time-history generation. This approach follows the SHAKE theoretical frame work but using random vibration theory (RVT) formulation for input motion and soil column analysis. This new approach follows three basic steps:

- The input target rock response spectrum is first converted to a power spectrum density (PSD) function.
- The PSD of responses in the soil column are computed based on the input PSD and the transfer functions of the site. The statistical means of the maximum shear strains and effective strains are obtained based on the PSD, and the process is repeated until the strain-compatibility is reached over the entire soil column.
- The PSDs and the statistical means of the maximum responses of other required quantities, such as the acceleration response spectra and maximum accelerations, are computed once convergence on soil properties has been reached.

Figure 1(b) shows schematically the new approach.



(a) Time History Analysis (SHAKE) (b) RVT Based Analysis (This Paper)

Figure 1 Different Approaches for Site Response Analysis

The basic theory of the SHAKE program is well known and well documented. Thus it is not repeated herein for sake of conciseness. The following section presents only the formulations unique for this new approach.

THEORY

Converting an Acceleration Response Spectrum to a Power Spectrum Density Function

It is well known from basic RVT theory (e.g., Der Kiureghian, 1983) that the following relation exists

$$S_d(\omega) = H^2(\omega) S_a(\omega) \tag{1}$$

where $S_d(\omega)$ is the relative displacement PSD, $S_a(\omega)$ is the acceleration PSD, and $H(\omega)$ is the transfer function between displacement response and absolute acceleration input of a single degree of freedom oscillator with frequency ω_o and damping ξ

$$H^2(\omega) = \frac{1}{(\omega_o^2 - \omega^2)^2 + 4\xi^2\omega_o^2\omega^2} \tag{2}$$

The mean of the maximum relative displacement response of the oscillator (definition of a mean relative displacement response spectrum) is given by:

$$D = \frac{p}{\sqrt{\lambda_0}} \tag{3}$$

Where p is a peak factor, and λ_0 is the zero moment of the response defined in Equation (6). Following Davenport (1964)

$$p = \sqrt{2\ln \nu(0)\tau} + \frac{0.5772}{\sqrt{2\ln \nu(0)\tau}} \tag{4}$$

$\nu(0)$ is the mean zero crossing of the response between 0 and τ and equal to:

$$\nu(0) = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}} \tag{5}$$

τ is taken as the strong motion duration of the earthquake

The moments of the response are defined as the following

$$\lambda_n = \int_0^\infty \omega^n S_d(\omega) d\omega \tag{6}$$

$n = 0, 1, 2$ for the zero (λ_0), first (λ_1), and second (λ_2) moments of the response.

Following Igusa and Der Kiureghian (1983) and Venmarcke (1972), $\nu(0)$ can be adjusted with the parameter δ, where

$$\delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0\lambda_2}} \tag{7}$$

The steps to calculate the acceleration power spectral density function from a given acceleration response spectrum are as follows.

1. Convert the acceleration response spectrum $RS_a(\omega)$ to a relative displacement response spectrum $RS_d(\omega)$,
2. Assume an initial acceleration power spectral density function $S_{a,0}(\omega)$
3. With the assumed $S_{a,0}(\omega)$ and the relations given above, calculate the mean of the maximum relative displacement response for all the frequencies defining the response spectrum. This will be a new relative displacement response spectrum $RS_{d,1}(\omega)$.
4. Calculate the ratio $R(\omega) = RS_d(\omega)/RS_{d,1}(\omega)$.
5. Correct the assumed acceleration power spectral density function $S_{a,0}(\omega)$ by $R^2(\omega)$ to calculate a new acceleration power spectral density function $S_{a,1}(\omega)$
6. Iterate from step 3 to step 5 until the desired accuracy is reached in the calculation of the displacement response spectrum.

Determine the Mean of Maximum Responses

Having the acceleration PSD $S_a(\omega)$ of the input motion and the transfer function between the input and any desired response $H_r(\omega)$, which is calculated following the normal SHAKE procedure, the steps to calculate the mean of the maximum response are the following:

1. Calculate the PSD of the desired response

$$SR(\omega) = H_r^2(\omega) S_a(\omega) \tag{8}$$

2. Calculate the moments λ_0, λ_1, λ_2 of the response

$$\lambda_n = \int_0^\infty \omega^n SR(\omega) d\omega \tag{9}$$

3. Calculate the peak factor p with these moments as described in Step 1

4. Calculate the mean of the maximum response

$$M_R = \frac{p}{\sqrt{\lambda_0}} \tag{10}$$

Where $\boldsymbol{p}$ is the peak factor for the desire response, following the same procedure outlined in Equations (4) through (7) but with the response PSD in Equation (8)

NUMBERICAL EXAMPLE

The above procedure is coded in a computer program P-SHAKE (Bechtel, 2006). The following numerical example illustrates compatibility of the P-SHAKE results with the SHAKE analysis results.

A 150-ft deep soil profile consisting of sand and clay layers overlaying half-space is being analyzed in this example problem. The site model and shear wave velocity profile are shown in Figure 2. The strain-dependent properties for both sand and clay layers are shown in Figure 3.

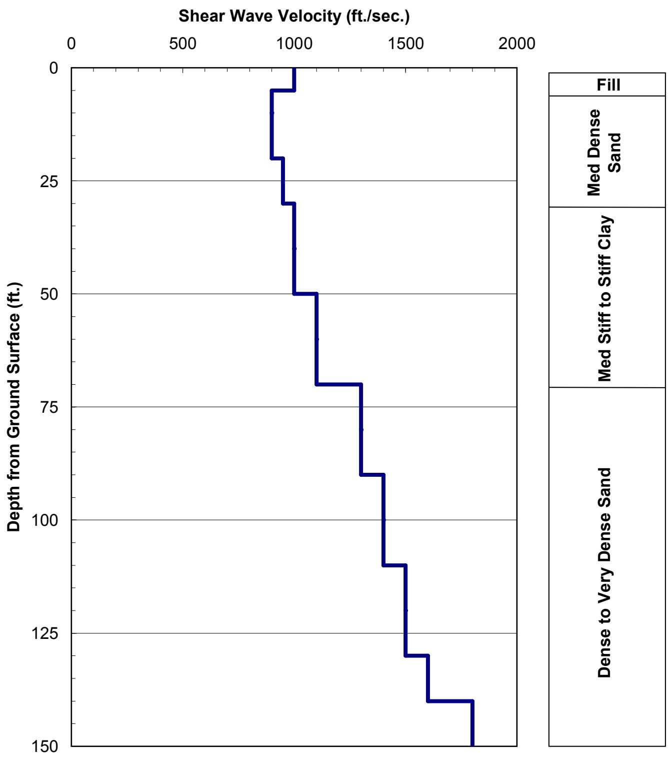


Figure 2 Soil Profile

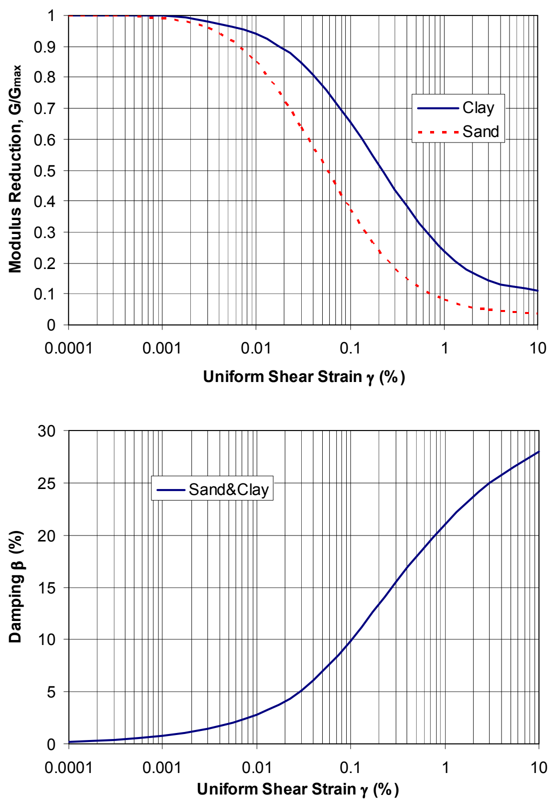
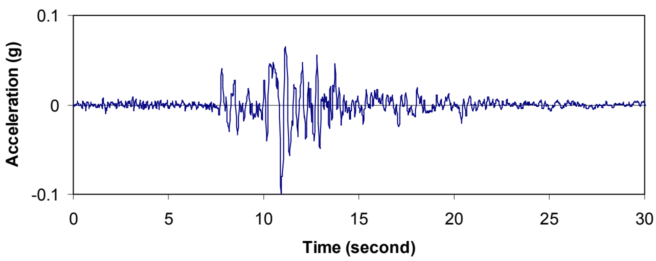
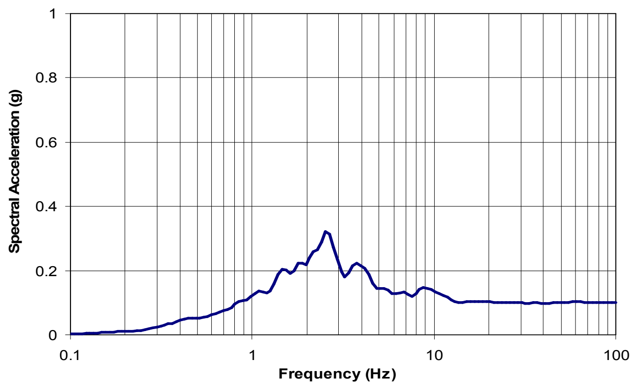


Figure 3 Modulus Degradation and Damping Relations vs. Shear Strains

The input motion used in the example problem is the recorded EW component time history at Diamond Heights during the 1989 Loma Prieta Earthquake, scaled to a peak acceleration of 0.1g. For SHAKE analysis, the scaled time history is used as the input motion, Figure 4(a). For P-SHAKE analysis, the 5%-damped acceleration response spectrum of the time history is used as the input motion, Figure 4(b). The input motion is specified at top of the half-space as outcrop motion.



(a) Acceleration Time History



(b) 5% Damped Response Spectrum

Figure 4 Input Motion for the Example Problem

Figure 5 shows the maximum shear strains developed in the soil profile after convergence is reached on soil properties. Also depicted in the figure are the strain-compatible shear wave velocity and soil damping profiles from both SHAKE and P-SHAKE analyses. It is observed that the shear strains developed in P-SHAKE analysis are slightly smaller than in SHAKE analysis, but the overall comparison, especially in terms of shear wave velocity, is minimal.

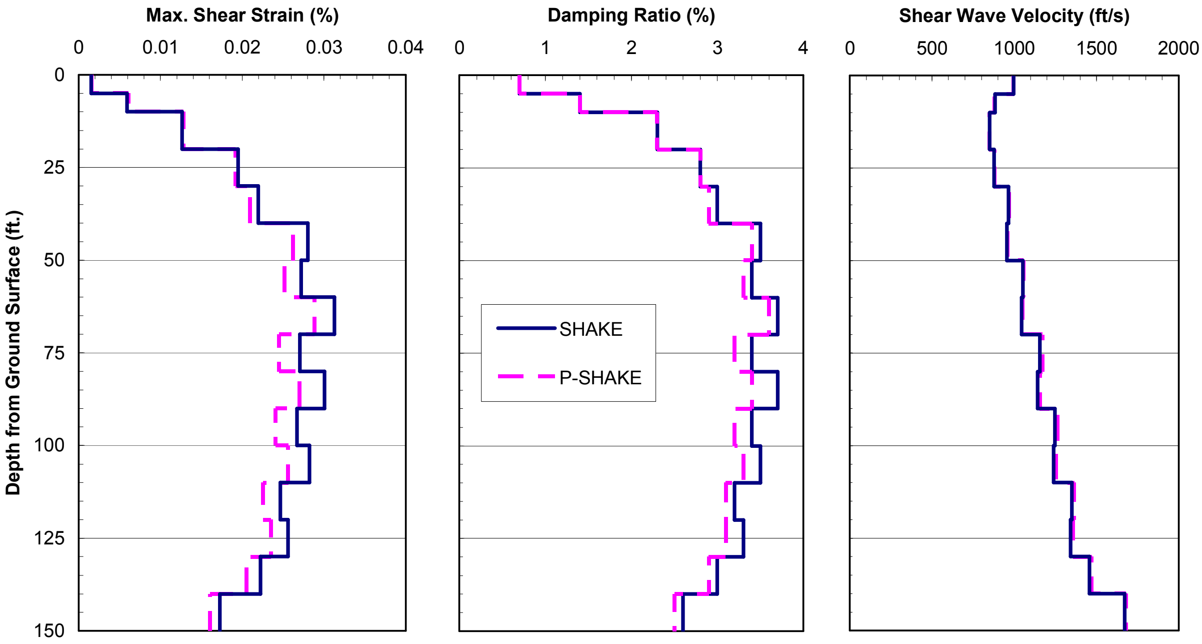


Figure 5 Comparison of Maximum Shear Strains and Strain-Compatible Soil Property Profiles

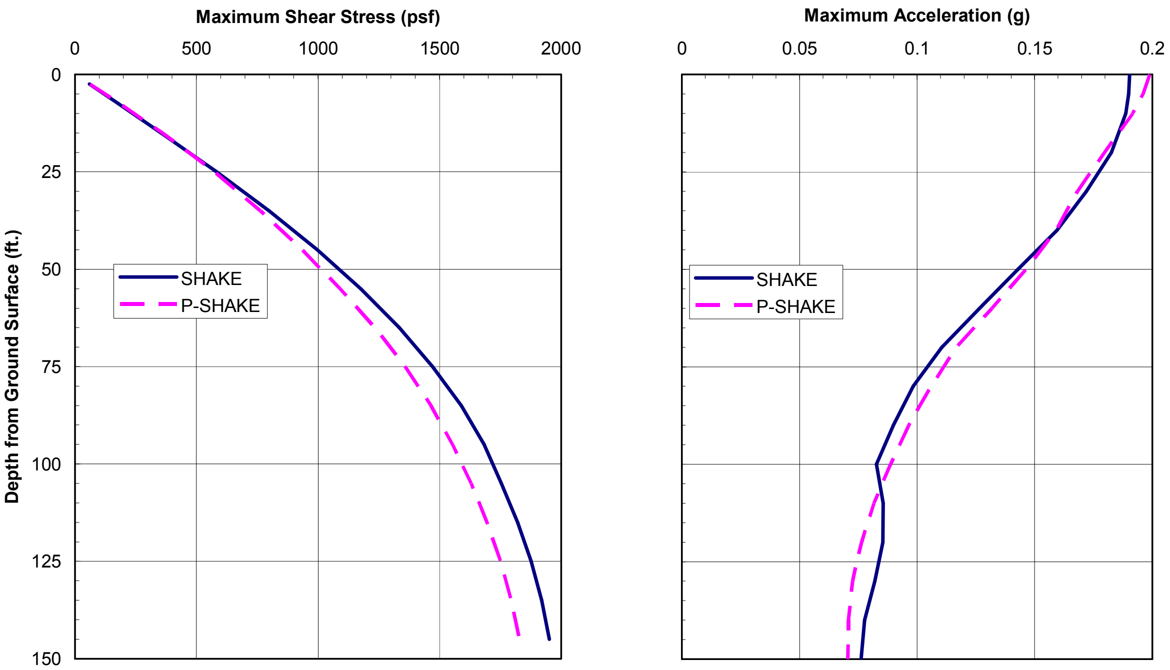


Figure 6 Comparison of Maximum Shear Stress and Maximum Acceleration Profiles

Figure 6 shows the calculated maximum shear stress and maximum acceleration profiles from both SHAKE and P-SHAKE. Figure 7 compares the 5% damped acceleration response spectra at the ground surface level, and Figure 8 shows amplification functions between motions at ground surface and at the rock base for both in-column and outcrop motions. These results show very good to excellent agreement between the two solutions.

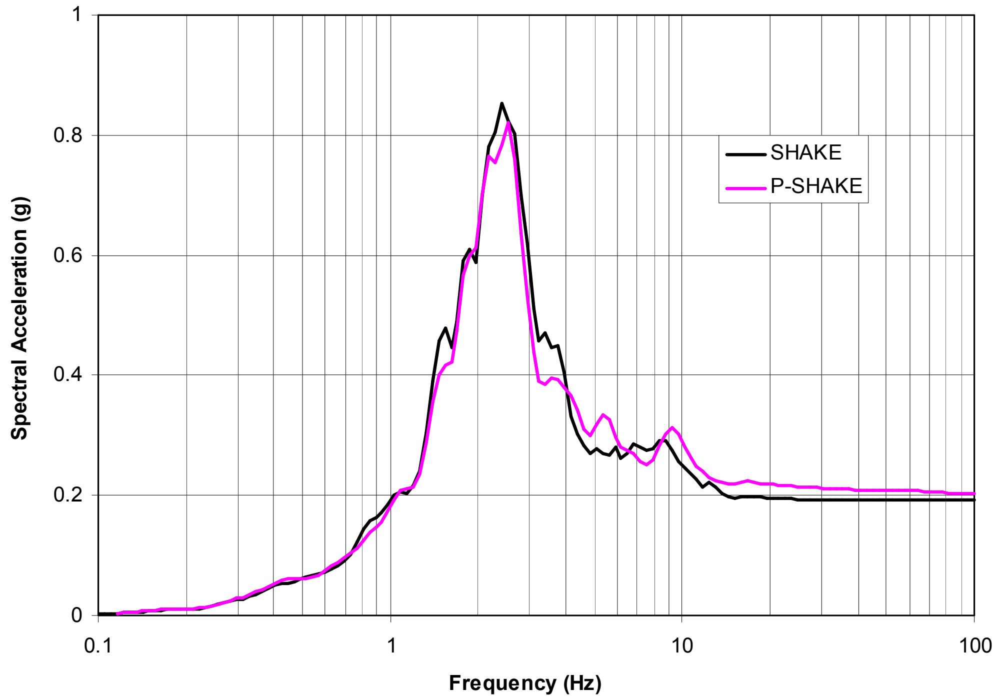


Figure 7 Comparison of 5% Damped Acceleration Response Spectra at Ground Surface

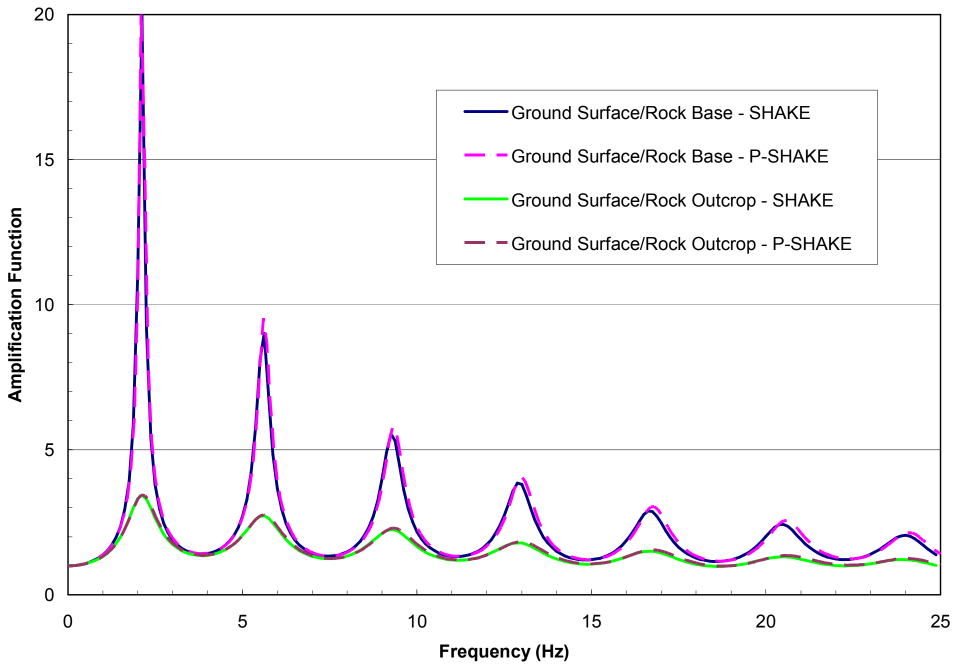


Figure 8 Comparison of Amplification Functions Between Motions at Ground Surface and at Rock Base

In addition to the example above, numerous other soil profiles with multiple time histories have been tested to show that P-SHAKE and SHAKE results are in good agreement, but not presented here due to the space limit. The computer program P-SHAKE is now widely used for major Bechtel projects.

CONCLUSIONS

An alternative approach for seismic site response analysis is proposed in this paper. This approach is based on the random vibration theory and works within the formulation of the computer program SHAKE. In this approach, the design input motion is characterized by the design response spectrum directly, and all responses of interest are calculated as the statistical averages. This approach avoids the difficulties associated with generating multiple spectrum-matching input time histories and is most suitable with the current approach of using a suite of randomized soil profiles for soil amplification.

Numerical examples show that the results computed by the new approach are essentially the same as the results computed by the SHAKE program. Thus, all practical experiences and empirical relationships built upon SHAKE are still applicable.

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