AN INVESTIGATION ON SEISMIC ANALYSIS OF SHALLOW TUNEELS IN SOIL MEDIUM

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ABSTRACT :

Buried tunnels with circular cross section, such as urban undergrounds are very popular in transportation engineering. The seismic behavior of such structures during operation is of importance. There exist different closed form solutions, such as Penzien and Wang methods for seismic evaluation of shallow tunnels. In the present research, these analytical methods are employed to analysis tunnel lining forces, constructed in soft to hard soil with different mechanical properties and constant shear strain. A numerical model, based on finite element method, was then developed to compare the analytical and numerical method outputs when the tunnel is under seismic loads. For this purpose seven types soil, soft soil to very dense sand, was selected and analysis was carried out. The analysis shows relative error in analytical methods. Also it is shown that with increasing soil stiffness, the induced circular stress in lining is reduced

KEYWORDS: Buried tunnel, Closed form solution, Shear strain, Seismic analysis

1. INTRODUCTION

The buried structures depend on geometry, depth of buried and other geotechnical and seismic parameters behave in different manners during an earthquake. The researches indicate buried structures experience a lower rate of damages than surface structures. Nevertheless, the buried structures also faced significant damages in recent large earthquakes. When designing such structure under earthquake conditions, seismic parameters and physical and mechanical properties if soil play important role. In this study a specified circular tunnel was analyzed, using various analytical solutions based on work of Wang (1993) and Hashash (2001 & 2005) for various soil groups.
Wang in 1993, based on closed form solutions recommended by Peck et al (1072), proposed modified analytical equations in terms of axial force, bending moments and displacements under external loading conditions. Penzien and Wu (1998) developed similar analytical solutions for thrust, shear, and moment in the tunnel lining due to racking deformation. Later, Penzien (2000) provided a complementary analytical procedure for racking deformation evaluation of rectangular and circular tunnels. Recently, Hashash in 2001 and 2005 with comparison these two methods, found out that the calculated forces and displacements are identical for the full-slip assumption, however, Penzien's solution results in much lower estimation of maximum thrusts compared to Wang's solution for the no-slip assumption. This difference is also reported by Park (2006).
In the present investigation a series of numerical analyses were performed, using finite element code PLAXIS (PLAXIS-B.V., 2002) and outputs were compared with result of analytical solutions. On the other hand sensitivity analysis on the mechanical parameters of soil was performed and the effect of various soil properties and maximum shear strain was evaluated on the tunnel response.

2. ANALYTICAL SOLUTION

The response of a lining of buried circular tunnel against the earthquake is a function of the compressibility and flexibility ratios of the structure, the in-situ overburden pressure and at-rest coefficient earth pressure, K_0, of soil. The trust, bending moment and deformations can be calculated using closed form solution [Peck et al., 1972]. The closed form solutions, reported by Wang and Penzien are based on slip between soil and tunnel lining.

2.1 Wang formulation

Wang (1993) proposed the solutions for the maximum thrust and moment in the lining due to the equivalent static ovaling deformations. The solutions for the full-slip condition at the soil lining interface, to evaluate thrust and moment due to seismic loading can be expressed as:

$$\frac{\Delta d_{lining}}{d_{lining}} = \frac{1}{3} K_1 F \gamma_{\max} \tag{2.1}$$

$$T_{\max} = \frac{1}{6} K_1 \frac{E_m}{(1-\nu_m)} r \gamma_{\max} \tag{2.2}$$

$$M_{\max} = \frac{1}{6} K_1 \frac{E_m}{(1-\nu_m)} r^2 \gamma_{\max} \tag{2.3}$$

Where υ_m, r and F are Poisson ratio of soil, radius and flexibility ratio of tunnel respectively.

$$K_1 = \frac{12(1-\nu_m)}{2F+5-6\nu_m} \tag{2.4}$$

$$F = \frac{E_m(1-\nu_l^2)r^3}{6E_l I(1+\nu_m)} \tag{2.5}$$

Where E_m, E_l and υ_l are modulus of elasticity of soil, modulus of elasticity and Poisson ratio of lining respectively. The maximum shear strain in the medium for both constant shear strain and variable shear strain may be obtained as:

$$G_m = \frac{E_m}{2(1+\nu_m)} \tag{2.6}$$

$$C_m = \sqrt{\frac{G_m}{\rho_m}} \tag{2.7}$$

$$\gamma_{\max} = \frac{Vs}{C_m} \tag{2.8}$$

Where $\rho_m = \gamma_s / g$ and G_m is shear modulus. Full-slip assumption under simple shear deformation, however, may cause significant underestimation of the maximum thrust. It has been therefore, recommended to evaluate $T_{\max}$ by the following equation for the for full-slip assumption case [Wang, 1993]:

$$T_{\max} = \pm K_2 \tau_{\max} r = \pm K_2 \frac{E_m}{2(1+\nu_m)} r \gamma_{\max} \tag{2.9}$$

Where $\tau_{\max}$ is maximum shear strain and K_2 is:

$$K_2 = 1 + \frac{F[(1-\nu_m)-(1-2\nu_m)C] - \frac{1}{2}(1-2\nu_m)^2 + 2}{F[(3-2\nu_m)+(1-2\nu_m)C] + C[\frac{5}{2} - 8\nu_m + 6\nu_m^{\ 2}] + 6 - 8\nu_m} \tag{2.10}$$

$$C = \frac{E_m(1-\nu_l^2)r}{E_l t(1+\nu_m)(1-2\nu_m)} \tag{2.11}$$

2.2 Penzien formulation

Penzien & Wu (1998) and Penzien (2000) developed similar analytical solutions for the thrust moment and shear in tunnel lining. For full-slip condition at the soil-lining interface we have:

$$\Delta d_{lining}^{n} = R^n \Delta d_{free-field} \tag{2.12}$$

$$T(\theta)=\frac{12E_l I\Delta d^n{}_{lining}}{d^3(1-\nu_m{}^2)}Cos2(\theta+45) \tag{2.13}$$

$$M(\theta)=\frac{6E_l I\Delta d^n{}_{lining}}{d^3(1-\nu_m{}^2)}Cos2(\theta+45) \tag{2.14}$$

$$V(\theta)=\frac{24E_l I\Delta d^n{}_{lining}}{d^3(1-\nu_m{}^2)}Sin2(\theta+45) \tag{2.15}$$

Where R^n and θ are inertial moment and wave incident angel respectively, i.e.

$$R^n=\frac{4(1-\nu_m)}{\alpha^n+1} \tag{2.16}$$

$$\alpha^n=\frac{12E_l I(5-6\nu_m)}{d^3 G_m(1-\gamma_l^2)} \tag{2.17}$$

The preceding equations take the following form for the no-slip condition at the soil-lining interface,:

$$\Delta d_{lining}=R\Delta d_{free-field} \tag{2.18}$$

$$T(\theta)=\frac{24E_l I\Delta d_{lining}}{d^3(1-\nu_l{}^2)}Cos2(\theta+45) \tag{1.19}$$

$$M(\theta)=\frac{6E_l I\Delta d_{lining}}{d^3(1-\nu_l{}^2)}Cos2(\theta+45) \tag{2.20}$$

$$V(\theta)=\frac{24E_l I\Delta d_{lining}}{d^3(1-\nu_l{}^2)}Sin2(\theta+45) \tag{2.21}$$

3. COMPARISON BETWEEN ANALYTICAL SOLUTIONS AND NUMERICAL ANALYSIS FOR THE CASE OF CONSTANT SHEAR STRAIN

As stated above, there exists a significant difference between analytical and numerical solution in case of no-slip assumption. Here, a numerical model, called origin one, developed to compare analytical and numerical results.

3.1 Numerical model development

In order to evaluate analytical solutions for seismic-induced ovaling deformation of the lining in a circular tunnel, the numerical analysis has been performed, using PLAXIS Code. The numerical analysis is based on the following assumptions:
1) Plane-strain condition exists.
2) Soil and tunnel lining behave linearly and elastically.
In this model the maximum shear strain is converted to ovaling deformation and these deformations are applied to the model. It should be noted that in PLAXIS, only no-slip condition between the tunnels lining and ground can be simulated. A 15 node element and also a fine mesh in geometry of the model were used to obtain higher accuracy.

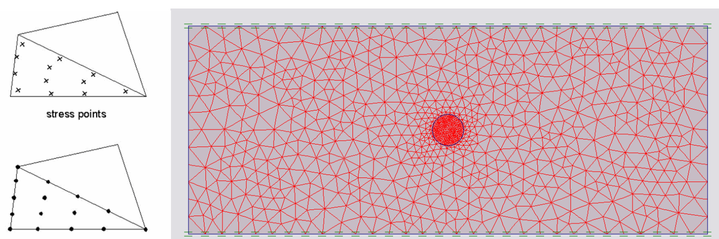

Figure 1 Typical modeling mesh and 15 node element

For more accuracy the meshes around the tunnel are more refined. Figure 1 shows a typical modeling mesh used in this study. Now, the ovaling deformations were applied at the two vertical boundaries of the model to simulate pure shear condition. This shear loading is also applied at horizontal boundaries so that the half of the ovaling deformation in positive sign is applied at the top and other half this deformation with negative sign is applied at the bottom boundaries. The deformation contours due to the applied shear force is shown in figure 2. These contours show the uniformly distributed deformations, indicating pure shear.

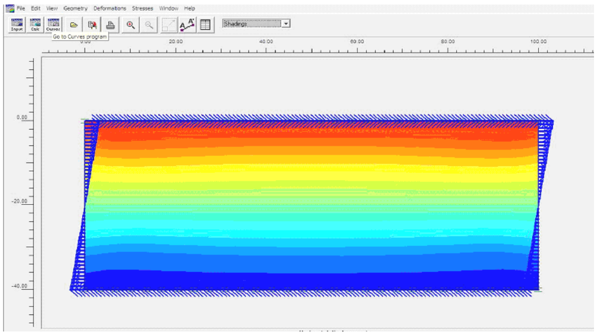

Figure 2 Deformation contours in final analysis

The induced ovaling deformations in the ground can be calculated in two cases, namely perforated ground and non perforated ground. In the case of non-perforated ground, the maximum diametric deformation is a function of maximum free-field shear strain only. In the case of perforated ground, however, it is further related to the Poisson's ratio of the medium. The diametric strains can be stated by Eqn. 3.1 and 3.2.

$$\frac{\Delta d_{non-perforated}}{d} = \pm \frac{\gamma_{\max}}{2} \tag{3.1}$$

$$\frac{\Delta d_{perforated}}{d} = \pm 2\gamma_{\max}(1-\nu_m) \tag{3.2}$$

Now, based on the closed form solution and also considering the geometry and physical properties of the original tunnel including d=6.6 m, $\gamma_{\max} = 0.00622$ and $\nu_m = 0.32$ from original tunnel, the diametric deformation can be calculated, i.e.

None perforated ground: $\frac{\Delta d_{non-perforated}}{d} = \pm \frac{\gamma_{\max}}{2}$ $\quad \Delta d_{non-perforated} = \pm \frac{0.00622}{2} \times 6.6 \Rightarrow \Delta d_{non-perforated} = 0.0205\ m$

Perforated ground: $\frac{\Delta d_{perforated}}{d} = \pm \gamma_{\max}(1-\nu_m)$ $\quad \Delta d_{perforated} = \pm 2 \times 0.00622(1-0.32) \times 6.6 \Rightarrow \Delta d_{perforated} = 0.0558\ m$

It is obvious that the dimensions of the model, i.e. the height and width of the media around the tunnel can affect the result. To verify the model, models with various heights and widths were analyzed and results compared with analytical solution. The model is shown in figure 3 and the results are gathered in table 1.

Table 1 The variations of the diameter in non-perforated and perforated ground

Model Dimension	30 × 30	20×20	20×30	30×50	30×60	40×60	40×80	40×90	40×100
$\Delta d_{non-perforated}$	0.02127	0.02612	0.02252	0.02243	0.02197	0.02265	0.02198	0.02168	0.02143
$\Delta d_{perforated}$	0.05086	0.04499	0.05040	0.05535	0.05438	0.05541	0.05629	0.05552	0.05498

From table 1, it can be observed that the model with 40 meter height and 100 meter width produces the least error which can be calculated as follow:

None perforated ground $\quad Error = (0.02143 - 0.0205)/0.02143 \times 100 \approx 3\%$

Perforated ground $\quad Error = (0.05498 - 0.0558)/0.0558 \times 100 \approx -1.9\%$

It can be seen the error between analytical and numerical methods (model 40×100) is less than 3%, indicating identical deformation for analytical and numerical methods.

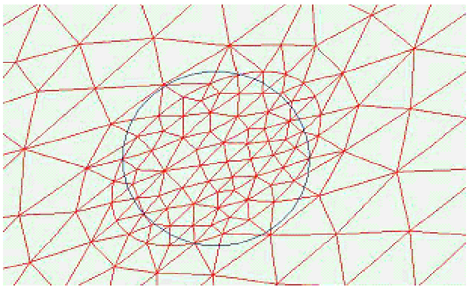

Figure 3 The created model in PLAXIS for non-perforated condition

3.2 The result of analysis for original tunnel

The specifications of the reference tunnel used in this study are based on Tabriz Underground tunnel with 15 m depth from ground level and 3.3 m radius. Other specifications are γ_m=20.5 kN/m^3, E_m=27167 kN/m^2, ν_m=0.32, t=0.3 m, E_l=2.48×10^7 kN/m^2, ν_l=0.2 a_{max}=0.35g. Also the internal forces in the lining caused by changes in shear deformation must be studied. Employing equations 2.6 to 2.8, the calculated shear strain is 0.0062 which can be applied to the numerical model after inverting to the shear deformation. The analysis results, based on Penzien, Wang and numerical method, which are shown in figure 4 indicate that the shear forces and bending moments calculated by analytical and numerical method are well coincide but axial forces calculated by Wang formula, especially in maximum θ shows high differences.

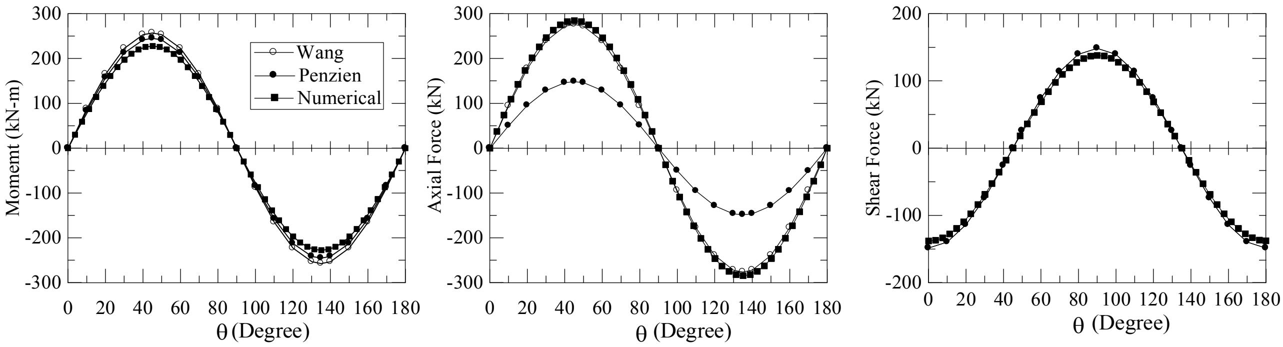

Figure 4 Comparison of numerical analyses and analytical solutions for tunnel–ground interaction

3.3. The analysis results for classified soils

To investigate the soil strength parameters on axial forces, shear forces and bending moment of lining, classified soils with different elastic modulus and Poisson ratio were considered. Table 2 summarizes the upper and lower limits of elastic modulus and Poisson ratio for different soils including clay, silt, sand and gravel. The elastic modulus varies from 2 to 500 mPa and Poisson ratio from 0.2 to 0.4. To investigate the soil strength parameters changes, the maximum shear strain was considered constant and equal to γ_{max}=0.002206 which is an average value for soft to hard soils.

Table 2 Specifications of classified soils

		Upper limit of Soil parameters			Lower limit of Soil parameters		
Model	Soil type	ν_m	E_m (kN/m^2)	γ (kN/m^3)	ν_m	E_m (kN/m^2)	γ (kN/m^3)
S1	Soft clay	0.4	5180	16	0.4	2070	16
S2	Medium clay	0.4	10350	16	0.4	5180	16
S3	Hard clay	0.4	24150	18	0.4	10350	16
S4	Dense sand	0.35	55200	20	0.35	34500	18
S5	Sand & gravel	0.3	172500	20	0.3	69000	20
S6	Dense sand & gravel	0.25	300000	22	0.25	200000	22
S7	Highly dense sand	0.2	500000	22	0.2	400000	22

The analysis results, using Penzien, Wang and numerical method are shown in table 3. In this table T, V and M are axial force, shear force and bending moment of lining respectively. Subscripts P, W and N refer to Penzien, Wang and numerical method.

Table 3 The closed form and numerical method analysis results

Model	Penzien (2000)			Wang (1993)		Numerical method		
	T_P (kN)	V_P (kN)	M_P (kN.m)	T_W (kN)	M_W (kN.m)	T_N (kN)	V_N (kN)	M_N (kN.m)
S1	8.3	8.3	13.7	8.8	14.6	5.57	8.04	13.25
	18.1	18.1	29.9	21	31.7	12.66	17.53	28.9
S2	18.1	18.1	29.9	21	31.7	12.66	17.53	28.9
	29.8	29.8	49.2	39.7	51.6	22.22	28.86	47.58
S3	29.8	29.8	49.2	39.7	51.6	22.22	28.86	47.58
	49.1	49.1	81.1	86.8	84.8	43.21	47.67	76.56
S4	57.1	57.1	94.2	121.1	97.7	55.54	55.22	90.98
	66.5	66.5	109.8	184.8	112.7	77.22	64.11	105.6
S5	74.4	74.4	122.7	238.7	126.2	96.86	71.11	117.1
	92.9	92.9	153.2	603.4	155.6	213.64	87.06	143.5
S6	94.5	94.5	155.9	695.1	158	241.92	88.46	154.4
	98	98	161.7	1022.8	163.3	343.33	91.55	150.4
S7	106.5	106.2	175.2	1428.7	176.7	481.38	97.72	160.5
	107.1	107.5	177.4	1760.8	178.6	589.44	98.82	162.3

Considering analysis results, the calculation error between analytical and numerical methods are given in table 4. In this table ΔT, ΔV and ΔM are the magnitude of error in axial force, shear force and bending moment of between closed form solution and numerical method. Subscripts P-N and W-N refer to Penzien and Wang and numerical method.

To compare the errors for different methods, the results are illustrated in figure 5. It can be seen that with increasing flexibility ratio, i.e. increasing soil stiffness, the error magnitude increases, implying that for soft clay to dense sand the error between analytical and numerical methods will be less than 10% which may be considered satisfactory. For had soils or soil with flexibility ratio higher than 4 closed form solution lead to conservative results.

Table 4 The magnitude of error between closed form solutions and numerical method

Model	Penzien (2000)			Wang (1993)	
	ΔT_{P-N} (%)	ΔV_{P-N} (%)	ΔM_{P-N} (%)	ΔT_{W-N} (%)	ΔM_{W-N} (%)
S1	-12	-4.1	-4.1	-5.5	2.6
	-21.5	-2.1	-2.1	-4.6	3.7
S2	-21.5	-2.1	-2.1	-4.6	3.7
	-37.8	0.2	0.2	-3.7	4.8
S3	-37.8	0.2	0.2	-3.7	4.8
	-80.8	1.8	5.9	-2.3	10
S4	-114.2	7.4	7.5	-1.8	10.8
	-181	9	9.3	-1.2	11.6
S5	-223.6	12.7	12.9	-0.9	15.3
	-584.8	17.5	18.3	0.1	19.5
S6	-634.3	17.7	18.5	0.2	19.6
	-940.4	17.6	19	0.3	19.7
S7	-1236	20.3	22	0.6	22.7
	-1518	20.3	22.1	0.9	22.7

4. COMPARISON BETWEEN ANALYTICAL SOLUTIONS AND NUMERICAL ANALYSIS FOR THE CASE OF VARIABLE SHEAR STRAIN

In this section shear strain is not held constant, i.e. based on the soil properties shear strain is calculated and its real value is then considered in the analysis process.

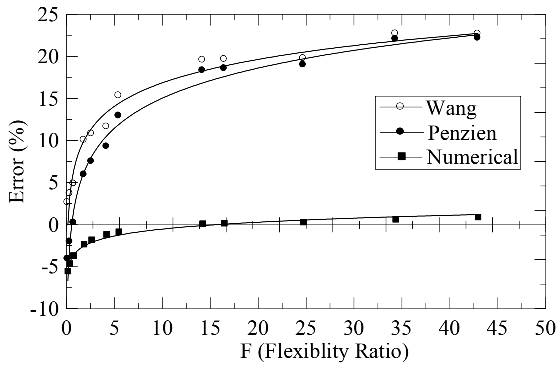

Figure 5 Existing errors between analytical and numerical methods

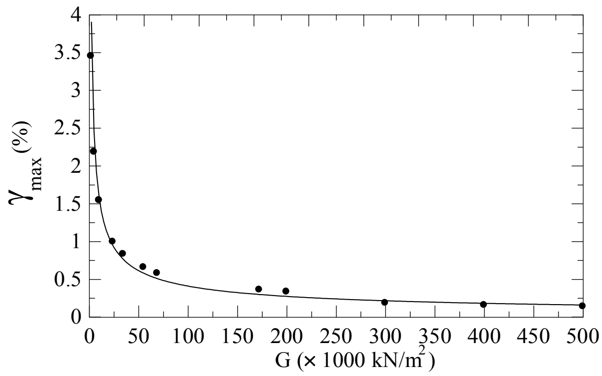

Figure 6 Variations of maximum shear strain with shear modulus

4.1. Closed form solution methods

The analysis reviewed in preceding sections was based on that the maximum shear strain (γ_{max}), similar to other structural and earthquake parameters is constant. It was noted in section 3.2 that the maximum shear strain is dependent to parameters such as shear wave velocity and maximum mass velocity in the media. The shear wave velocity, Vs, is dependent to the maximum velocity and acceleration, which is in turn dependent to the elasticity modulus and Poisson ratio. If, therefore, E and ν vary, shear modulus, G and C_m vary as well. With varying C_m, maximum shear strain (γ_{max}), will vary. In this method the earthquake entry is the maximum acceleration of earth movement, which may be calculated using ground level acceleration and soil layers specifications. In this section based on maximum acceleration at the ground level assumption to be a_{max}=0.35g and also soil layer specifications, γ_{max} can be calculated. Figure 6 illustrates the variations of γ_{max} with elasticity modulus. It can be seen that with increasing elasticity modulus, γ_{max} increases initially sharply, but remaining roughly constant with higher values of elasticity modulus.

The closed form solution with constant maximum shear strain, presented in the previous section, can now be applied for an earthquake with specified acceleration gravity for which the maximum shear strain is calculated with respect to different soil specifications. Here, the analysis was carried out for Wang's axial force and Penzien's bending moment and shear force which the results are shown in figure 7.

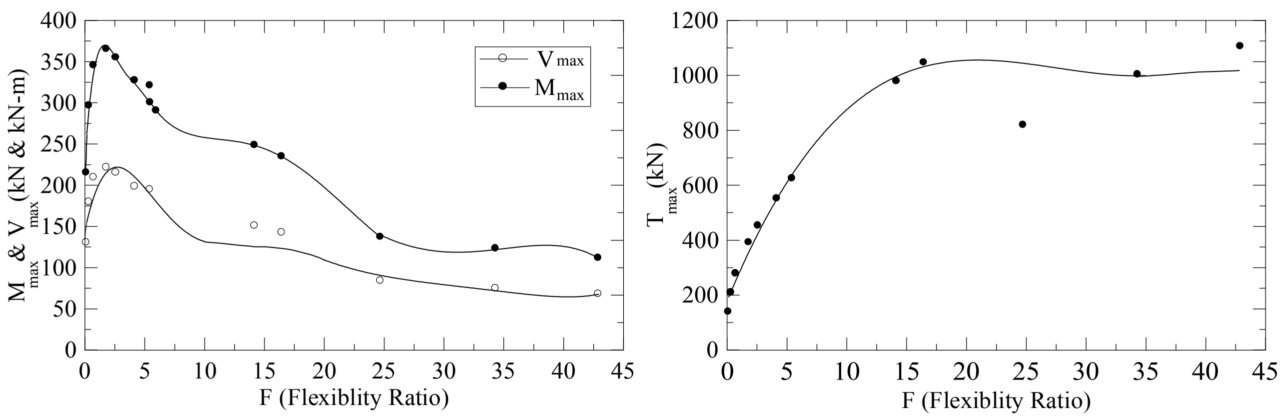

Figure 7 Variations of analytical results with flexibility ratio

CONCLUSIONS

Based on this research considering the original tunnel the following points can be concluded:

- With comparisons closed form solution with numerical method, there is no significant difference between numerical and Wang method in axial force calculation, however Penzien method leads to underestimation axial force in no-slip condition.
- Since the analytical methods are not able to evaluate shear strain distribution it is recommended to evaluate maximum shear strain based on seismic time history maximum shear, and then employ this value in presented in analytical methods.
- It was observed that the bending moment and axial force variations depend on the flexibility ratio, it is, therefore, recommended stress distribution to be considered for analysis and design of lining of the tunnels.

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