

## Application of boundary element method to study the seismic response of triangular hills

A.Razmkhah<sup>1</sup> and M.Kamalian<sup>2</sup> and S.M.A. Sadroldini<sup>3</sup>

<sup>1</sup> Assistant Professor, Faculty of Engineering, Islamic Azad University, South Tehran Branch, Tehran, IRAN  
Email: [ar.razm@iiees.ac.ir](mailto:ar.razm@iiees.ac.ir)

<sup>2</sup> Assistant Professor, Geotechnical Engineering Research Centre, IIEES, Tehran, IRAN

<sup>3</sup> Ph.D. Student of Geotechnical Engineering, Islamic Azad University, Science and Research Branch, Tehran, IRAN

### ABSTRACT :

For gaining insight into seismic response of 2D triangular hills and concluding their 2D amplification pattern in microzonation studies the results of parametric numerical analyses have been processed in both time and frequency domains. To achieve this goal the Direct Boundary Element Method (DBEM) is applied for calculating elastic wave propagation in two-dimensional triangular hills and clear perspectives of the amplification patterns of the hills are presented by considering the frequency-domain responses. It is shown that wavelength and site geometry are the independent key parameters governing the hill's amplification pattern. The effects of two aforementioned parameters are concerned in terms of periodic subintervals and shape ratios.

**KEYWORDS:** topography effect, triangular hill, boundary element, amplification

### 1. INTRODUCTION

Recent destructive earthquakes have revealed marked evidences of the effects of surface topography on ground motion characteristics at a given site. Based on prior observations the amplitude and frequency content of seismic ground motion are key parameters which are mainly affected by local topographical conditions. Prompted by observational and instrumented evidence, the problem of scattering and diffraction of seismic waves by topographical irregularities has been studied numerically with a focus on two-dimensional simulations by different authors but attempts have seldom been made to express the results of parametric studies. Many authors such as Trifunac (1973), Wong and Trifunac (1974), Sanchez-Se'sma and Rosenblueth (1979), and Bard (1982), have investigated the displacement field over topographic features acted upon by oblique and grazing SH plane waves. Boore (1972) modeled the effects of a ridge by using finite differences, Sanchez-Sesma and Campillo(1991) used boundary-element methods, and Bouchon(1973) and Bard (1982) used a discrete-wavenumber method to simulate topographic wave diffraction. In a thorough review of published results, Geli et al. (1988) noted that, in almost all these models, crest-to-base amplification factors of peak acceleration are on the order of two for an isolated ridge. A limited number of examples, which involve more complex numerical simulations, can be found in Bard and Tucker (1985), who investigated the antiplane response of three-dimensional homogeneous ridges. Also, Geli et al.(1988) evaluated the effects of compositional layering and complex topography. They noted a complex pattern of amplitude fluctuations that varied with the location on the ridge and the degree of sediment cover. To expand on this idea, Deng (1991) developed a numerical model, in which the geometry and geology of the configuration are simulated. Recently Kamalian et al (2006) have presented advanced formulation of the time-domain two-dimensional hybrid finite element-boundary element method (FEM/BEM) and have applied to carry out site response analysis of homogeneous and non-homogeneous topographic structures subjected to incident in-plane motions. In their researchs Kamalian et al (2007, 2008) have used direct boundary element method to study the amplification pattern of 2D semi-Sine shaped Valleys and hills Subjected to vertically propagating incident waves. The BEM is a very effective numerical tool for dynamic analysis of linear elastic bounded and unbounded media. The method is very attractive for wave propagation problems, because the discretization is done only on the boundary, yielding smaller meshes and systems of equations. Another advantage is that this method represents efficiently the outgoing waves through infinite domains, which is very useful when dealing with scattered waves by topographical structures. When this method is applied to problems with semi-infinite domains, there is no need to model the far field. In this article, to gaining insight into seismic response of triangular hills a series of

numerical parametric analyses have been conducted by using DBEM technique in the time domain. To this end we used a general purpose two-dimensional nonlinear two-phase BEM/FEM code named as HYBRID. Several examples including site response analysis of half-plane, horizontally layered sites, canyons, alluvial hills and ridge sections subjected to incident *P* and *SV* waves were solved in order to show the accuracy and efficiency of this implemented BE algorithm in carrying out site response analysis of topographic structures.

## 2. METHODOLOGY OF PARAMETRIC ANALYSIS

In all analyses the vertically propagating incident *P*, *SV* waves of Ricker type are adopted as a dynamic excitation. The Ricker type wave equation can be expressed as:

$$f(t) = \left[ 1 - 2 \cdot (\pi \cdot f_p \cdot (t - t_0))^2 \right] e^{-(\pi \cdot f_p \cdot (t - t_0))^2} \quad (1)$$

in which,  $f_p$  and  $t_0$  denote the predominant frequency and an appropriate time shift parameter, respectively. In case of *SV* waves,  $f(t)$  designates the horizontal component of the incident motion while the vertical one is zero, and in case of *P* waves, vice versa. The boundary conditions consisted of the traction free ground surface and the seismic loading was introduced through the term of incident motion. The geometry of the 2D homogenous triangular hill and Ricker type wave time history are demonstrated in Fig. (1), (2) respectively:

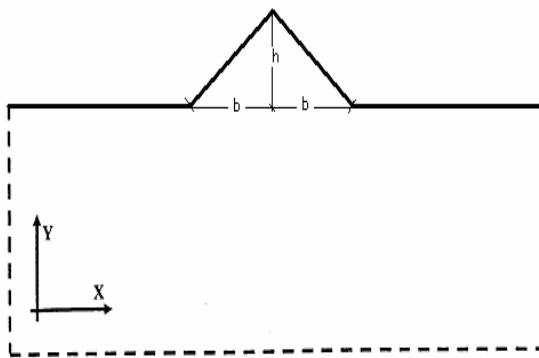


Figure 1 The geometry of 2D hill

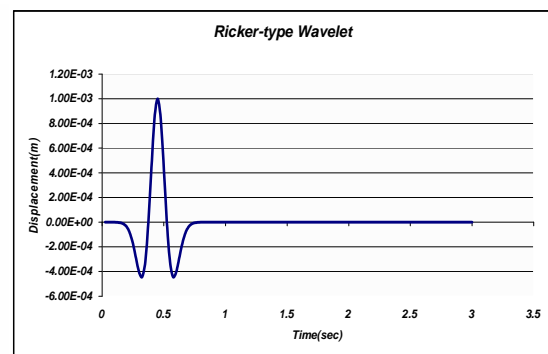


Figure 2 Ricker type wave time history

In order to apply the results of analyses to frequencies and geometrical conditions different from those of this study models, all results were presented in dimensionless forms. To achieve this goal the well known dimensionless period  $T = tc_2/2b$  (or its inverse: the dimensionless frequency), which means physically the ratio of the incident's wave length to the width of the hill and the shape ratio of hill  $SR = h/b$ , are used in which  $c_2$  is wave propagation velocity,  $b$  is the half width of the hill,  $t$  is time and  $h$  is the depth. This study involves a wide range of shape ratios, namely  $SR = (0.1, 0.5, 0.7, 1.0, 1.2, 1.5, 2)$  to consider the effect of geometrical properties on seismic response of 2D hills. Poisson ratio is chosen 0.33 and seismic wave frequency is adopted 3 Hz.

Based on engineering interests, a dimensionless period interval of 0.25 to 8.33 was considered, which corresponds to incident waves with wave lengths of 0.25 to 8.33 times the hill's width. This broad period interval was divided into the following five subintervals: 0.25 to 0.50 (P1), 0.50 to 1.00 (P2), 1.00 to 2.00 (P3), 2.00 to 4.17 (P4) and 4.17 to 8.33 (P5), corresponding to incident waves with very short, short, medium, large and very large wave lengths, respectively. For the sake of simplicity and following the well known concept of average horizontal spectral amplification (AHSA) defined by Borchardt et al (1991) as spectral ratios representing averages over short, intermediate, mid and long period bands, five distinct amplification factors were computed for every point along the hill, by averaging the corresponding amplification curve over each of the above mentioned five period subintervals P1 to P5. The aforementioned average amplifications that are calculated in five periodic subintervals are averaged again to obtain total average amplification factor for each point along the hill.

### 3. GENERAL AMPLIFICATION PATTERN

Figure 3 demonstrates displacement time histories of the points located on a 2D hill with the shape ratio of 1.0 subjected to an incident P and SV wave. The receiving points are arranged within an interval of  $-5b$  to  $5b$  from the hill's center. In the following sections in the case of P (SV) incident wave the vertical (horizontal) and horizontal (vertical) components of ground motion are named as consistent and opposite components respectively. As can be seen at the points located far from the center of the hill, the amplitude of surface displacement is doubled. This is called the free surface effect and is caused by upgoing seismic waves being reflected off the free surface of the earth. Opposite components of motion have no considerable amplification. As the incoming waves impinge to the hill, the diffracted waves travel through the inclined free surfaces until they reach to the crest therefore the displacement time history of the points close to the crest are composed of incoming, reflected, refracted and Rayleigh wave effects. As expected the interference of four aforementioned waves produces the largest value of the motion amplitude in the crest and in its neighboring points. Considering the displacement time histories of the points far from the hill, enable us to discern two kinds of diffracted waves, namely P and Sv-Rayleigh diffracted waves. Since compare to the P waves with higher propagation velocity, SV and Rayleigh diffracted waves have lower velocity of so they arrive in time lag to the points far from the hill and as the distance from the crest increases these two kinds of diffracted waves can easily be distinguished. In more distant points as a consequence of radiation damping the amplitude of diffracted waves reduces to zero.

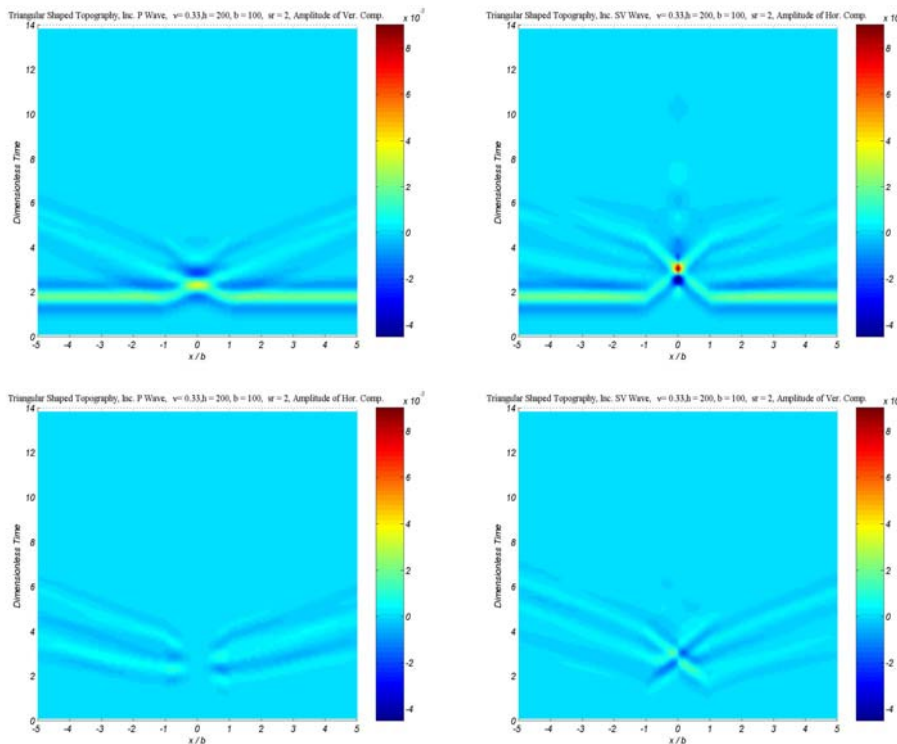


Figure 3 Amplification patterns of a triangular hill with shape ratio of 2 in case of incident SV (right) and P (left) waves (The symbols 'Hrz' and 'Vrt' present the horizontal and vertical components of amplification, respectively.)

### 4. WAVE LENGTH AND SHAPE RATIO EFFECT

Figure 4 demonstrates the amplification patterns of 2D triangular hills subjected to vertically incident P and SV waves, according to their wave lengths (P1 to P5). Only consistent components of motion are shown. As can be seen, irrespective of the shape ratio, the wave length plays a key rule in determining the amplification curve of the hill. In general the maximum amplification factor occurs at the crest, irrespective of the length of the incident wave and of the hill's shape ratio and the amplification curve decays toward the base of the hill. It is

clear that in all periodic subintervals as the shape ratio increases the crest amplification increases but the increasing rate in other points along the hill depends on the wave length and varies across the hill. In other words, increasing the shape ratio (height) of the hill does not necessarily mean intensifying the amplification potential of all points along it. In general, the amplification potential of the hill increases with the shape ratio, although the increasing rate depends on the wave length and varies across the hill.

Considering figure 4 shows that although the amplification patterns of 2D triangular hills subjected to vertically incident P and SV waves seem to be similar in general, but in the case of incident SV waves, larger amplification potentials are observed and unlike the p waves the considerable amplification occurs even at low frequency subintervals.

The variation of the amplification in the crest of the hill with dimensionless period is demonstrated in Figure 5 for SV wave excitation and some different shape ratio. The natural period of the hill in which 1) the amplification takes place in all points on the hill and 2) the peak amplification occurs at the crest of the hill 3) the motion in all points are in phase, can be obtained from Figure 5 for each shape ratio. It can be inferred that as the shape ratio increases the natural period increases (or characteristic frequency decreases). In other word as the shape ratio increases the larger wavelengths of incident seismic waves can be amplified by the hill topography. Figure 5 also indicates that in the dimensionless periods more than natural period of the hill the amplification at the crest of the hill decreases to 1.0. It implies that incident waves possess predominate dimensionless period greater than natural period of the hill will not be amplified by the hill topography.

To apply the 2D study amplification factors calculated by numerical analyses in engineering problems and to modify 1D amplifications, in Figure 6, simple linear relationships are proposed to approximate the 2D/1D average amplifications of vertical and horizontal components of ground motion as a function of shape ratio of the hill. AH and Av represent horizontal and vertical average amplifications respectively. The linear relationships can be expressed as:

$$AH = 0.38(S.R) + 0.8 \quad (2)$$

$$AV = 0.18(S.R) - 0.05 \quad (3)$$

## 5. COMPARISON OF AVERAGE AMPLIFICATION OF TRIANGULAR HILLS WITH SEISMIC CODE PROVISIONS

In spite of significant effect of topography condition on strong ground motion, there are only few building codes which have considered this issue in their procedures. In this section the amplification factors of this article is compared with code requirements. To this end we briefly review some topography effect associated provisions of two seismic codes. For instance the seismic provisions of AFPS 90 incorporate the effect of local topography condition on design ground motion by classifying the possible  $\Delta I$  parameter into three categories and assigning peak amplification factor ( $\tau$ ) for each category within the specified zone. The amplification factor ( $\tau$ ) and  $\Delta I$  are defined in Figure 7 For different conditions. In Figure 7, I and i are angles of first and second slopes of inclined topography structure respectively. European seismic code EC8 is proposing a correction factor, called aggravation factor F for both ridge and cliff type topographies as a function of the height and slope angle. The aggravation factor takes values with an extra 20% increase in anticipated aggravation when a surface soft layer with thickness more than 5m exists. Table (1) compares the amplification factors of triangular hills with specified shape ratios which resulted from 2D numerical analyses and from aforementioned code guidelines. As can be seen, in lower shape ratios the amplifications calculated base on code provisions are conservatively greater than those of 2D numerical analysis results but in larger shape ratios the situation is vice versa.

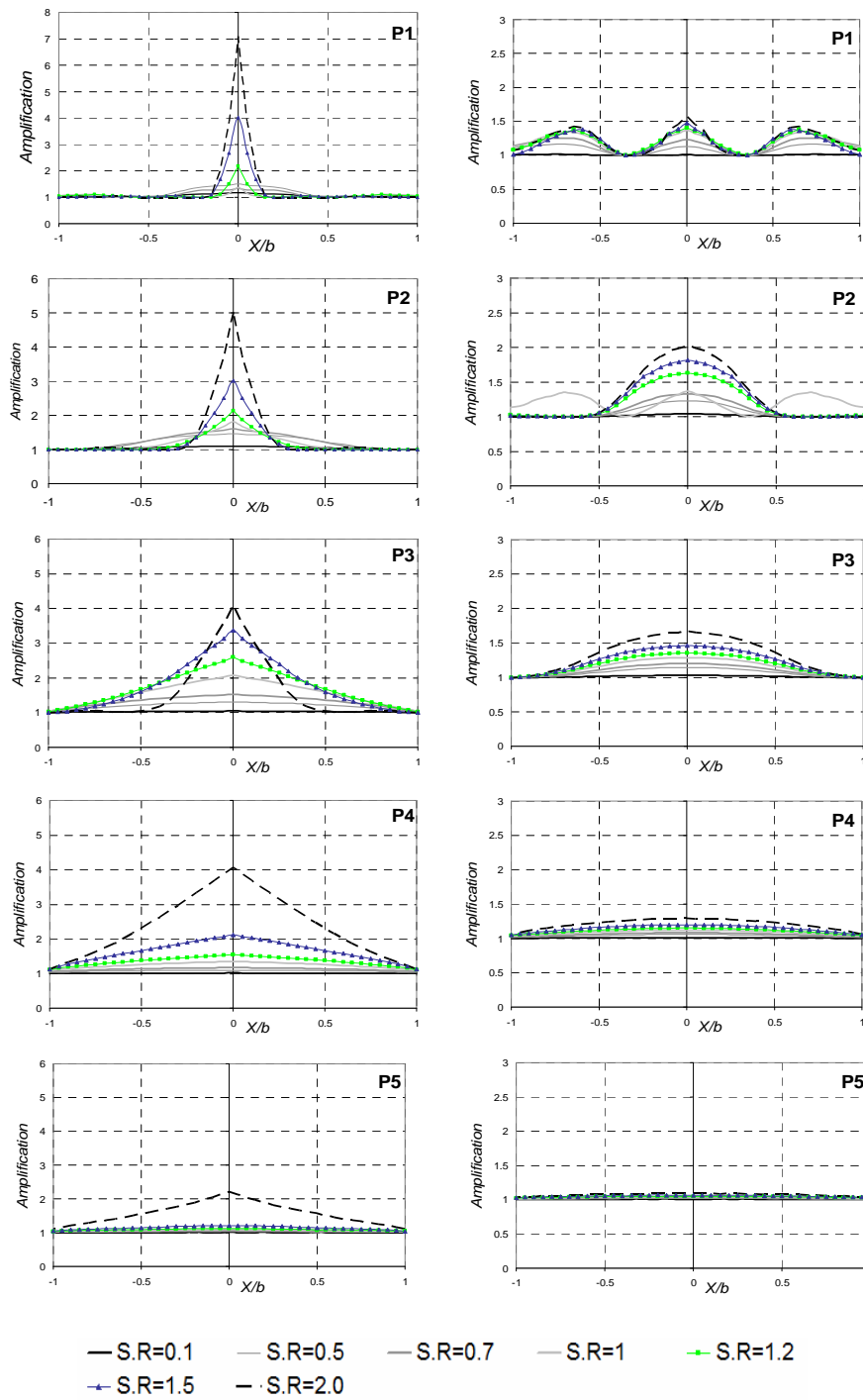


Figure 4 Wavelength effect and shape ratio effect on averaged amplification curves of 2D triangular shaped hills subjected to vertically propagating incident SV (left),P (right) waves.

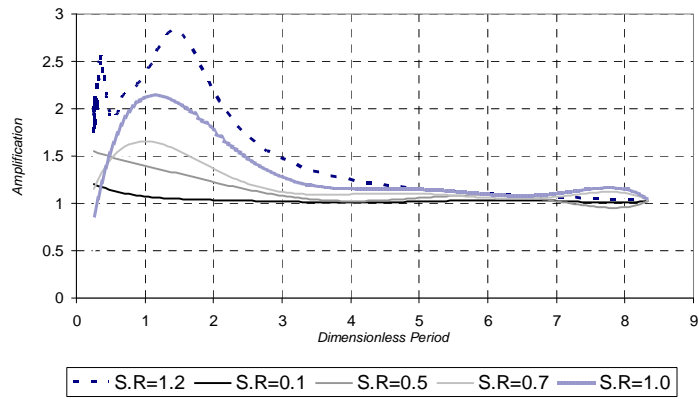


Figure 5 The variation of the amplification at the crest of the hill with dimensionless period for vertically propagating SV wave

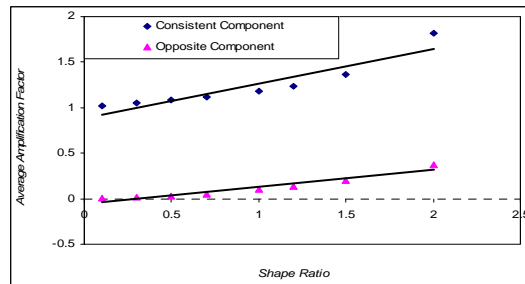
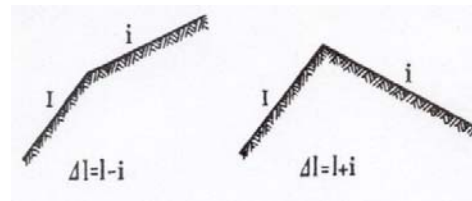


Figure 6 The linear relationship between horizontal and vertical average amplification and shape ratio for SV incident wave

Table 1. Analytical predictions and seismic code provisions for topography amplifications

Shape Ratio	EC8 Aggravation Factor	AFPS Amplification Factor	Numerical Analysis Amplifications
0.1	>1.2	1	1.01
0.3	>1.2	1.16	1.04
0.5	>1.2	1.4	1.07
0.7	>1.4	1.4	1.11
1.0	>1.4	1.4	1.18
1.2	>1.4	1.4	1.24
1.5	>1.4	1.4	1.36
2.0	>1.4	1.4	1.81



For: $\Delta I \leq 0.4$	$\tau = I$
For: $0.9 \leq \Delta I \leq 0.4$	$\tau = I + 0.8(I - i - 0.4)$
For: $\Delta I \geq 0.9$	$\tau = 1.40$

Figure 7-a AFPS seismic provisions for topography Amplification

## 6. CONCLUSIONS

This paper presents clear perspectives of the amplification patterns of 2D homogenous triangular hills subjected to vertically propagating SV and P waves, obtained by an extensive numerical parametric analysis using the time domain BEM. It is shown that:

- The amplification potential of the hill is strongly influenced by the length of the incident wave, by the shape ratio and in a less order of importance by the wave type.
- Every hill has a characteristic period that controls its seismic response. If the incident wave has a predominant period of equal to the characteristic one, all points along the hill show inphase motions and its amplification potential reaches the maximum
- In the case of incident waves with lengths of longer than the width of the hill, where the predominant periods are usually equal to or greater than its characteristic period, the amplification curve finds its maximum at the crest and decrease towards the base of the hill.

## REFERENCES

- Bouchon M. (1973). Effect of Topography on Surface Motion, *Bull. Seism. Soc. Am.*, **63** 615–632
- Boore, D.M. (1972) A note on the effect of simple topography on seismic waves. *Bull. Seism. Soc. Am.*, **62**.
- Trifunac, M.D. (1973). Scattering of plane SH waves by a semi – cylindrical canyon. *Earth. Eng. and Struc. Dyn.*, **1**, pp.267-281.
- Wong, H.L. & M.D. Trifunac, (1974). Scattering of plane SH waves by a semi – elliptical. *Earth. Eng. and Struc. Dyn.*, **3**.
- Sanchez-Sesma, F.J. & E. Rosenblueth, (1979). Ground motion of canyons of arbitrary shape under incident SH waves. *Earth. Eng. and Struc. Dyn.*, **7**.
- H.L. Wong (1982). Effects of surface topography on the diffraction of P, SV and Rayleigh waves, *Bull. Seismol. Soc. Am.*, **72**, 1167–1183,
- Bard, P.-Y., 1982. Diffracted waves and displacement field over two-dimensional elevated topographies. *Geophys. J. R. Astr. Soc.* **71**, pp. 731-760.
- Shah, A.H. & K.C. Wong, (1982). Diffraction of plane SH waves in a half – space. *Earth. Eng. and Struc. Dyn.*, **10**
- Bard, P. Y., and B. E. Tucker (1985). Ridge and tunnel effects: comparing observations with theory, *Bull. Seism. Soc. Am.* **75**, 905–922.
- Geli, L., P.-V. Bard, B. Julien, (1988). The effect of topography on earthquake ground motion: a review and new results. *Bull. Seism. Soc. Am.*, **78**
- Sanchez-Sesma F.J. & Campillo M. (1991). Diffraction of P, SV and Rayleigh waves by topographic features: a boundary integral formulation"; *Bull. Seismol. Soc. Am.* **81**, 2234-2253.[1009]
- Deng, X. (1991). The integrated adjustment of 3-D terrestrial network and GPS observations, *Master's Thesis*,

- Wuhan Technical University of Surveying and Mapping. 800.
- R.D. Borcherdt, C.M. Wentworth, A. Janssen, T. Fumal and J. Gibbs (1991). Methodology for Predictive GIS Mapping of Special Study Zones for Strong Ground Shaking in the San Francisco Bay Region, California, Proceedings of the Fourth International Conference on Seismic Zonation, Stanford, California, 545-552,
- Sanchez-Sesma, F.J. et al., (1993a). An indirect boundary element method applied to simulate the seismic response of alluvial hills for incident P, SV and Rayleigh waves. *Earth. Eng. and Struc. Dyn.*, **22**
- R.D. Borcherdt (1994). Estimates of site-dependent response spectra for design (Methodology and Justification), *Earthquake Spectra*, **10**, 617-653,
- Sanchez-Sesma F.J. & Luzon F.; (1995). "Seismic response of three dimensional alluvial hills for incident P, SV and Rayleigh waves"; *Bull. Seismol. Soc. Am.*; **85**, 269-284
- Alvarez-Rubioa Sonia, Francisco J. Sa'nchez-Sesma, Juan Jose' Benito, Enrique Alarco (2004). The direct boundary element method: 2D site effects assessment on laterally varying layered media (methodology) . *Soil Dyn Earthquake Engng.*; **24**:167-180
- M. Kamalian , (2001). Time Domain Two-Dimensional Hybrid FEM / BEM Dynamic Analysis of Non-Linear Saturated Porous Media, Ph.D. Dissertation, Tehran University
- B. Gatmiri, and M. Kamalian (2002a). Time Domain Two-Dimensional Hybrid FEM / BEM Dynamic Analysis Of Non-Linear Saturated Porous Media, Proc. 2nd Canadian Specialty Conference On Computing In Geotechnique, Winnipeg, Manitoba, Canada, 216-221,
- B. Gatmiri, and M. Kamalian (2002b). Combination of boundary element and finite element methods for evaluation of dynamic response of saturated porous media, Proc. 5th European Conference on Numerical Methods in Geotechnical Engineering, Paris, France, 947-955,
- M. Kamalian, M.K. Jafari, A. Sohrabi, A. Razmkhah & B. Gatmiri; (2006); "Time-Domain Two-Dimensional Site Response Analysis of Non-Homogeneous Topographic Structures by a Hybrid FE/BE Method"; *Soil Dynamic and Earthquake Engineering*; Vol. **26**, No. 8; 753-765.
- Mohsen Kamalian, Behrouz Gatmiri, Abdollah Sohrabi-Bidar and Ali Khalaj, (2007). Amplification pattern of 2D semi-sine-shaped valleys subjected to vertically propagating incident waves COMMUNICATIONS IN NUMERICAL METHODS IN ENGINEERING Commun. Numer. Meth. Engng **23**:871-887
- M. Kamalian, M.K. Jafari, A. Sohrabi Bidar & A. Razmkhah; (2008); "Seismic Response of 2D Semi-Sine Shaped Hills to Vertically Propagating Incident Waves: Amplification Patterns and Engineering Applications", *Earthquake Spectra*, Vol. **24**,
- Association Française du Genie Parasismique (AFPS). Recommandations de la AFPS. Paris: AFPS. 183 pp., (1990)