FIXITY OF PILE FOUNDATIONS IN SEISMICALLY LIQUEFIED SOILS FOR BUCKLING CALCULATIONS – AN EIGENVALUE ANALYSIS

A. A. Kerciku[1], S. Bhattacharya[2], H. J. Burd[3] and Z. A. Lubkowski[4]

[1] *Buildings' Structural Engineer, Arup, UK*
[2] *University Lecturer in Dynamics ,University of Bristol, UK*
[3] *University lecturer in Engineering Science, University of Oxford, UK*
[4] *Associate Director, Arup, UK*
Email: adri.kerciku@gmail.com, S.Bhattacharya@bristol.ac.uk,

ABSTRACT :

A pile becomes laterally unsupported when the soil liquefies during strong earthquakes. This makes it vulnerable to buckling instability. Buckling is a non-ductile method of failure which results in a rapid collapse and it should be avoided in the design process. This paper presents a simple method, based on an elastic analysis, which may be used to estimate the unsupported buckling length of piles in liquefied soil. The method would be applicable to simple structures such as bridges or jetties, which provide no moment or lateral restraint at the top end of the pile. Most research in the area of pile stability is based on the use of Winkler foundation (p-y method), which models the lateral restraining effect of the soil on the pile as a set of discrete one-dimensional springs distributed along the length of the pile. This paper investigates the stability of pile foundations in liquefied soils via a more accurate three dimensional (continuum) model. The program ABAQUS has been used to build and analyse a finite-element (FE), perfectly elastic, continuum, soil-pile model. Results from the FE, elastic-continuum model, for a pile embedded in [non-liquefied] soil, have been compared with documented equivalent Winkler foundation analytical studies, and experimental results. The FE, elastic-continuum model has then been used to analyse the buckling of pile foundations in liquefied soils for the parameters: depth of liquefaction, stiffness of the liquefied soil, and exposed length of the piles.

KEYWORDS: Pile, Foundation, Liquefaction, Buckling, Eigenvalue, Fixity.

1. INTRODUCTION

Piles are slender structural elements with lateral support offered from the surrounding soil. When axially-loaded piles lose the lateral support due to soil liquefaction, they behave like unsupported, axial and lateral load bearing structural elements. Bhattacharya et al. (2004) suggested that axially loaded piles may collapse as a result of buckling instability if the soil bracing effect is removed due to liquefaction. Reliable methods for estimating the buckling capacity of piles in liquefied soils have not been widely introduced to the industry and they are not included in the recommendations of design codes such as JRA (1996), NEHRP (2000) and Eurocode 8 (1998). Buckling is a non-ductile method of failure which results in a rapid collapse and it should be avoided in the design process.

The stability of pile foundations in seismically liquefied soil is a concept only recently introduced (Bhattacharya et al., 2004). Consequently any numerical, analytical and experimental studies on the stability of pile foundations in liquefied soil are a relatively new initiative. Most research in the topic of pile stability is based on the use of Winkler foundation, or p-y method, which offers a straightforward analytical tool, which models the lateral restraining effect of the soil on the pile as a set of discrete one-dimensional springs distributed along the length of the pile. But modelling the three-dimensional stiffness of the soil as a one-dimensional system may lead to inaccuracies (Davisson and Gill, 1963). In this study the buckling behaviour of pile foundations embedded in (liquefied and non-liquefied) soil is analysed using an elastic-continuum FE model.

2. THE SOIL-PILE MODEL

The programs ABAQUS CAE and ABAQUS/Standard (ABAQUS, 2002) have been used to build and analyse the soil-pile model for buckling behaviour. Second order elements have been used for higher accuracy. The pile in our soil-pile is a hollow-cylindrical thick-walled steel member, assumed to behave perfectly elastically, Young's modulus of $E=210\ GPa$ and Poisson's ratio of $\nu=0.3$. The soil has also been assumed to behave linearly and conditions of undrained soil analysis have been applied with a Poisson's ratio of ν=0.499. No slip is assumed at the pile/soil interface. For computational ease, and due to its symmetry, only half of the soil-pile model has been built and the adequate boundary conditions have been applied as shown in Figure 1. The embedment length, L_s has been kept constant, whilst the exposed (unembedded) length, L_u has been varied in the analysis. For higher accuracy the soil model has been meshed at higher element densities near the pile, as shown in Figure 2.

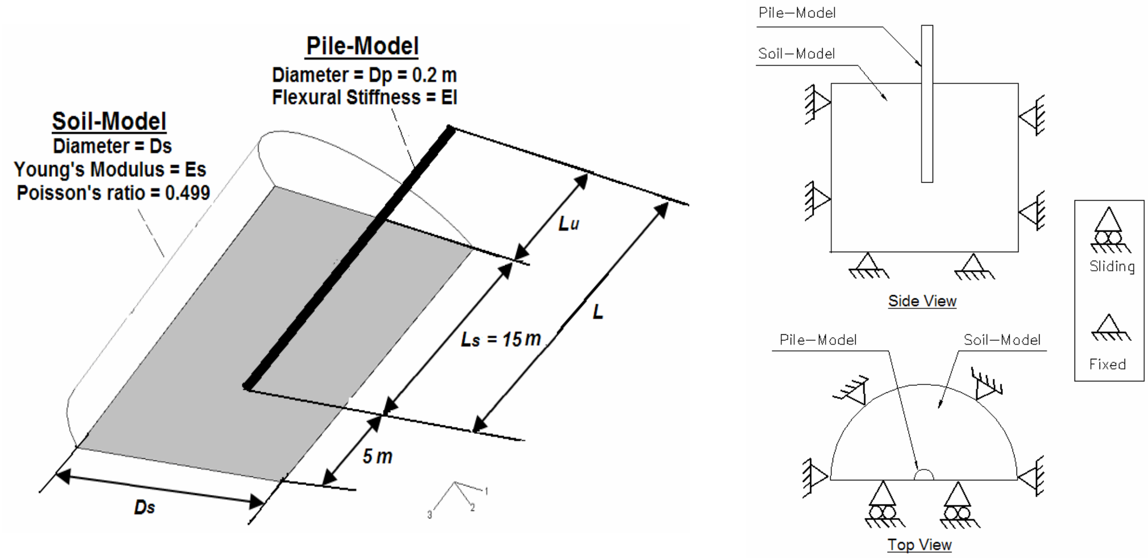

Figure 1: Model dimensions and nomenclature (left). Boundary conditions applied to the model (right)

3.1. Soil Model Diameter

To determine the sensitivity of the system to the soil model size, the buckling load has been calculated for different soil diameters D_S.

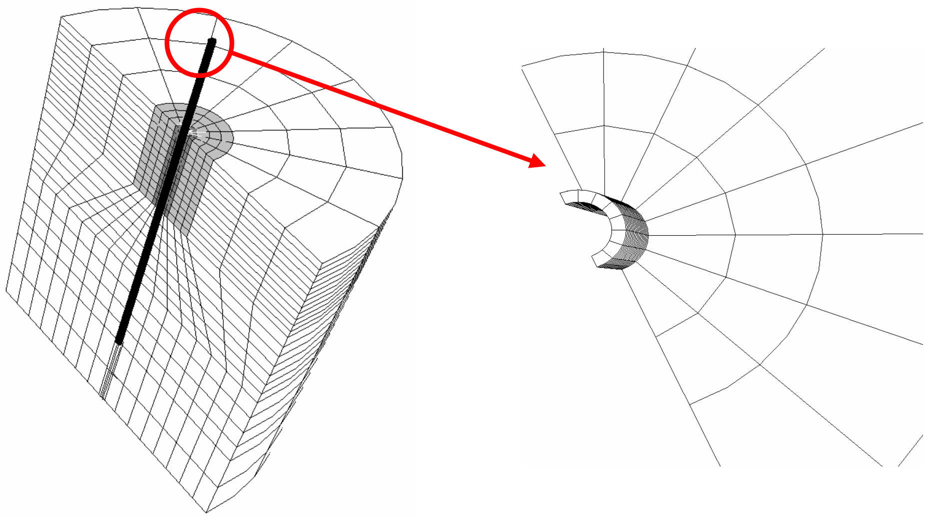

Figure 2: Mesh density used for the soil model. The denser partition is shown in grey

The results of this analysis for a hexahedral elements mesh is presented in Figure 3 where P_C is the critical buckling load computed by ABAQUS and the *soil model diameter to pile diameter ratio* (D_s/D_p). P_C converges as the soil diameter increases. It can be seen that the change in buckling load converges as the soil-model becomes larger. Therefore the pile-model becomes less sensitive to the size of the soil-model for larger soil-model diameters, D_s.

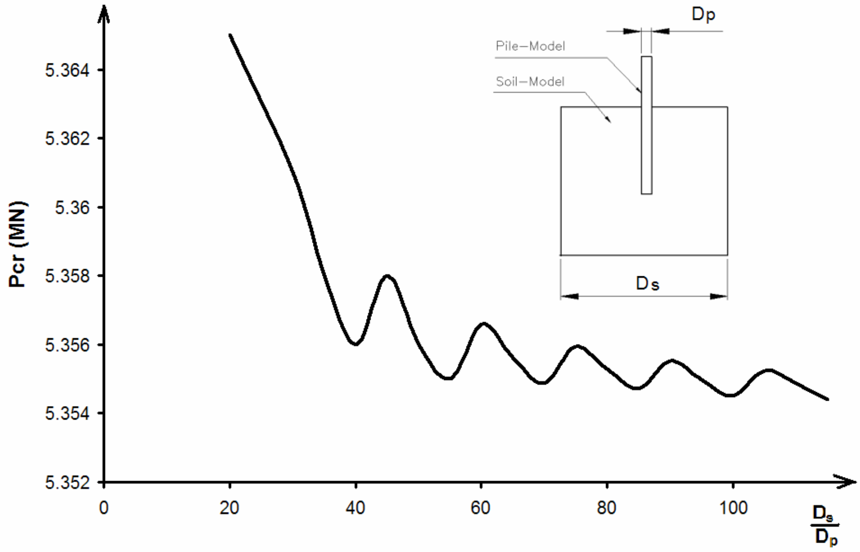

Figure 3:Results for the soil model size optimisation analysis for a mesh of linear Hexahedral elements for the buckling load P_c with increase in soil model diameter

From Figure 3, it can be derived that the change in buckling load becomes insignificant (less then 0.02%) at D_S/D_P values larger than *80*. Therefore, it would be reasonable to use a circular soil-model of diameter at least *80* times the diameter of pile.

3. ABAQUS SOIL-PILE MODEL VALIDATION FOR STABILITY ANALYSIS

Studies on the topic of buckling of piles focus predominantly on the topic of partially exposed piles (in non-liquefied soils). In this section the buckling instability of pile foundations partially embedded in [non-liquefied] soil is studied using ABAQUS FE modelling. The computed buckling loads are then compared with analytical and numerical studies detailed in Fleming et al. (1992) and Heelis et al. (2004). The purpose of this is to build confidence in the relevance and accuracy of the FE continuum stability analysis for piles in liquefied soil.
The study of buckling of exposed pile foundations dates back to Hetenyi (1946), who used the Winkler foundation model and set up a series of differential equations to compute the critical buckling loads of exposed columns/piles, assuming constant and linear subgrade moduli. This analytical solution was developed further by Davisson and Robinson (1965), who suggested that for engineering calculations, of buckling and bending of piles/columns partially embedded in soil, it may be assumed that they are fixed at a point below the ground. This depth, the depth of fixity, L'_S, is described in Figure 4. Davisson and Robinson (1965) used the Winkler foundation analytical model to provide a relationship between the soil stiffness, embedment ratio and depth of fixity. Defining the embedment ratio as:

$$\delta = \frac{L_s}{L} \tag{3.1}$$

where L is the total length of the pile (See Figure 1).

Further, Heelis el al. (2004) updated the differential equations of Davisson and Robinson (1965) by taking into account the shaft friction. The authors compared their results to experimental data and the analysis of Davisson and Robinson (1965), and Fleming et al. (1992), showing that the new algorithm produced more accurate results.

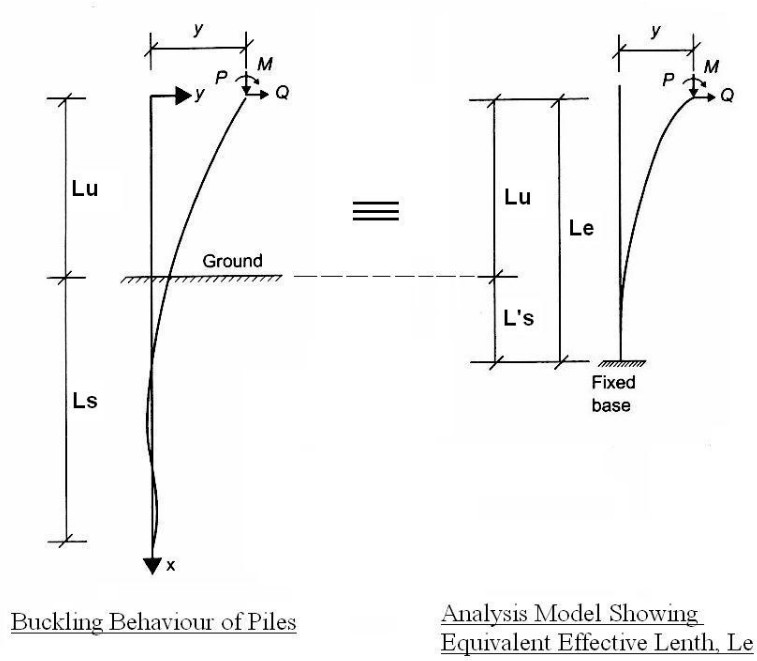

Figure 4: Depth of fixity according to Davisson and Robinson (1965)

The model described in Section 2 has been meshed with denser elements nearer the top soil-pile interface in order to get more accurate stress-strain contours at the region of largest soil-pile displacement. The soil model has been partitioned into three annuli to mesh the model with a radial decreasing density (varying from 2 - 20 elements/m), as shown in Figure 2. Hexahedral, continuum, second order finite elements were used to model the soil and a 25mm-thick hollow section. The FE meshing was set up to allow acceptable element sizes – length-to-width ratio for elements were chosen of greater than 0.1 to avoid errors and convergence problems. The hollow pile walls were meshed with of 18 continuum elements per annulus. The critical buckling load (deduced from the first/largest eigenvalue), P_C, was computed for a varying soil Young's modulus E_S and a varying embedment ratio. The results are presented in Figure 5.

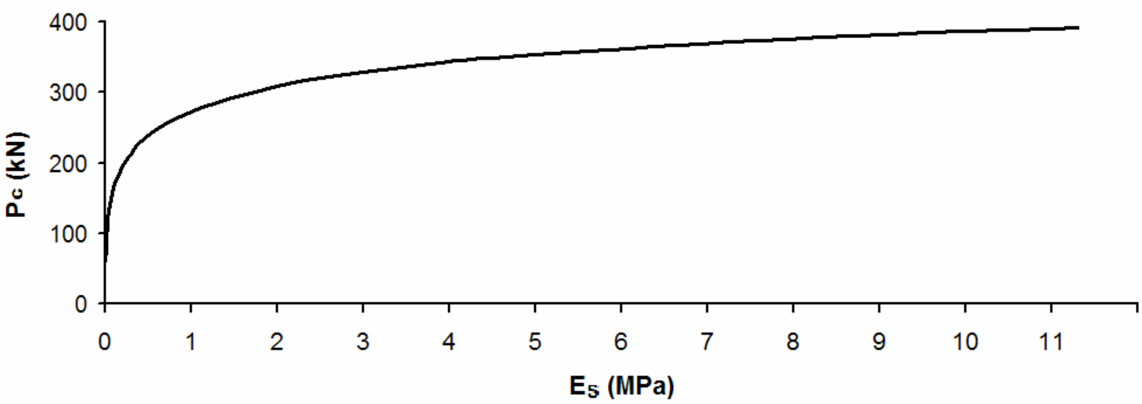

Figure 5: Variation of computed buckling load P_C with increasing soil stiffness E_S from *0.005* to *11 MPa*, and $L_S=15m$, $L=20m$, exposed length $L_U=5$, $\delta=0.75$, $EI=6\ MNm^2$, $\nu_{steel}=0.3$

To compare these results with the ones presented in the literature it is necessary to convert this data into the respective non-dimensional variables. Fleming et al. (1992) presents the outcome of his study in the plot P_C/P_E

against λ. P_C/P_E is the non-dimensional buckling capacity of the embedded pile, where P_C is the buckling load of soil-pile system, and P_E is the Euler buckling load of the pile assuming both ends are pinned:

$$P_E = \frac{\pi^2 EI}{L^2} \tag{3.2}$$

λ is a non-dimensional value describing the soil stiffness:

$$\lambda = \sqrt{\frac{kL^4}{EI}} \tag{3.3}$$

Terzaghi (1955) defined the relationship between the k (the subgrade modulus) and the soil's Young's modulus E_s through a theoretical analysis based on theory of elasticity for undrained soil conditions:

$$k = \frac{E_s}{1.35} \tag{3.4}$$

Therefore, λ may be written in terms of the soil's Young's Modulus as:

$$\lambda = \sqrt{\frac{E_S L^4}{1.35 EI}} \tag{3.5}$$

The experimental FE results of P_C vs. E_S in Figure 5 have been converted to the non-dimensional values above and compared to the results of the analytical (Winkler Foundation model) studies of Fleming et al. (1992) and Heelis et al. (2004). Figure 6 shows that our results fit in well with the results of Fleming et al. (1992) and Heelis et al. (2004).

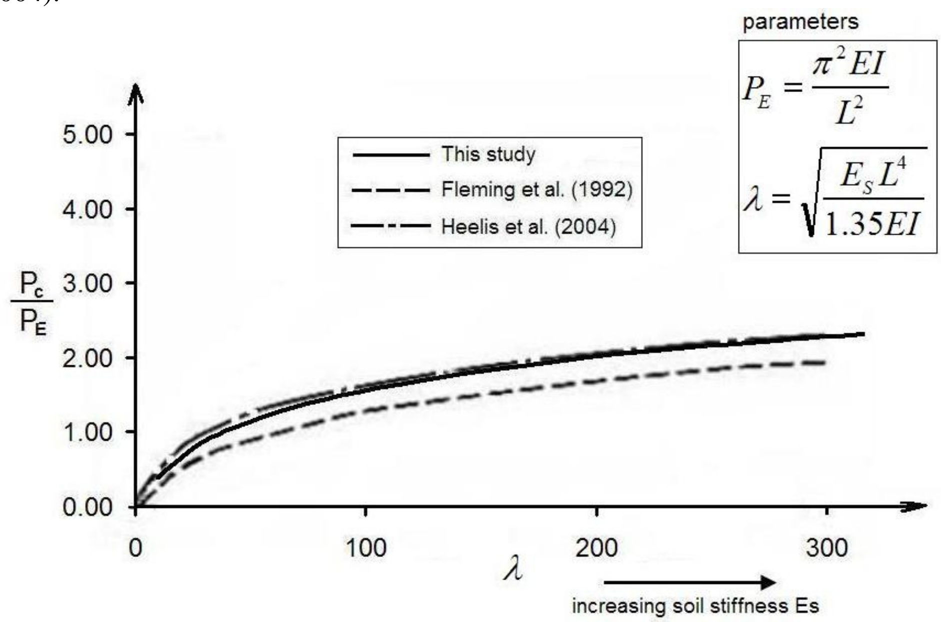

Figure 6: Buckling loads from Fleming et al. (1992) and Heelis et al. (2004) compared to the ABAQUS analysis results presented in this study for a fixed-free pile in homogeneous soil: for *δ=0.75*

Heelis et al. (2004) incorporate a more advanced analytical soil-pile model including shaft friction - assumptions of uniform skin friction with depth and no slip on the soil-pile interface have been made. These are directly compatible with the friction conditions of the soil-pile model in this paper. This would explain why our curve matches the one from Heelis et al. (2004) better. Further, our results match the Winkler foundation model form Heelis et al. (2004) better at high soil stiffness (λ>200). This can be explained by the fact that a stiffer soil will lead to smaller pile and soil displacements, and therefore, the subsequent effect of the continuum stress-strain behaviour of the soil is less emphasised, and the one-dimensional Winkler foundation model represents more accurately the real behaviour of the soil.

4. STABILITY OF PILE FOUNDATIONS PARTIALLY EMBEDDED IN LIQUEFIED SOIL

In the previous section we demonstrated that the results of the elastic-continuum ABAQUS FE model compares well with other documented Winkler foundation approaches. The same FE model was used to investigate the buckling behaviour of pile foundations embedded in liquefied soil, with the varying parameters: depth of liquefaction, liquefied soil stiffness, and pile embedment ratio. For this parametric study the ABAQUS soil-model has been partitioned into *1*m-deep segments, and their Young's modulus has in turn been reduced to model the soil stiffness degradation during liquefaction. The loss of shear strength of liquefied soil has been determined according to Yasuda et al. (1998 and 1999), according to the defined "*Liquefaction stiffness degradation ratio*", φ:

$$\varphi = \frac{E_s'}{E_s} \times 100\% \tag{4.1}$$

At full liquefaction Yasuda et al. (1999) reported that the shear modulus of the soil decreased from *10%* down to *0.1%*, depending on the relative density, of its non-liquefied value. These results are comparable with the findings of Ishihara (1997) who suggested that φ has a magnitude between *10%* and *1%*.
The results of this analysis have been plotted as the non-dimensional variables S'_R and P_R, as indicated in Figure 7.

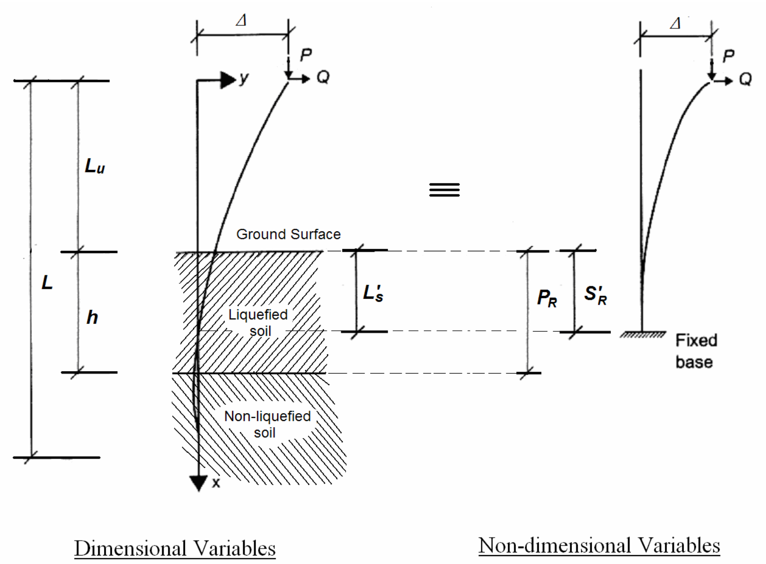

Figure 7: Visual definition of the variable L, Lu, h, L'_S, and non-dimensional variables P_R and S'_R. Adapted from Davison and Robinson (1965)

The non-dimensional constants are defined as:

$$S_R' = \frac{L_S'}{R'} \quad \rightarrow \text{Non-dimensional variable for depth of fixity} \tag{4.2}$$

$$P_R = \frac{h}{R'} \quad \rightarrow \text{Non-dimensional variable for liquefaction depth} \tag{4.3}$$

where h is the depth of liquefaction. The apostrophe on the constants indicates the liquefied soil properties. $R`$ is defined as:

$$R' = \sqrt[4]{\frac{EI}{k'}} = \sqrt[4]{\frac{1.35EI}{E_S'}} \tag{4.4}$$

The depth of fixity can be defined as:

$$L_S' = \sqrt{\frac{\pi^2 EI}{4P_c}} - L_U \tag{4.5}$$

The output of the FE analyses has been presented in the non-dimensional plots in Figure 8. The analysis represents the buckling load results of the soil-pile model for a varying liquefaction depth, and constant depth of fixity and constant soil stiffness degradation. The lines in red plot the results for a constant embedment ratio of 0.75, and constant values of soil stiffness degradation ratio of 10% and 1%. The lines in black represent the results for a constant soil stiffness degradation ratio of 1% and three constant embedment ratios of 1, 0.75, 0.5.

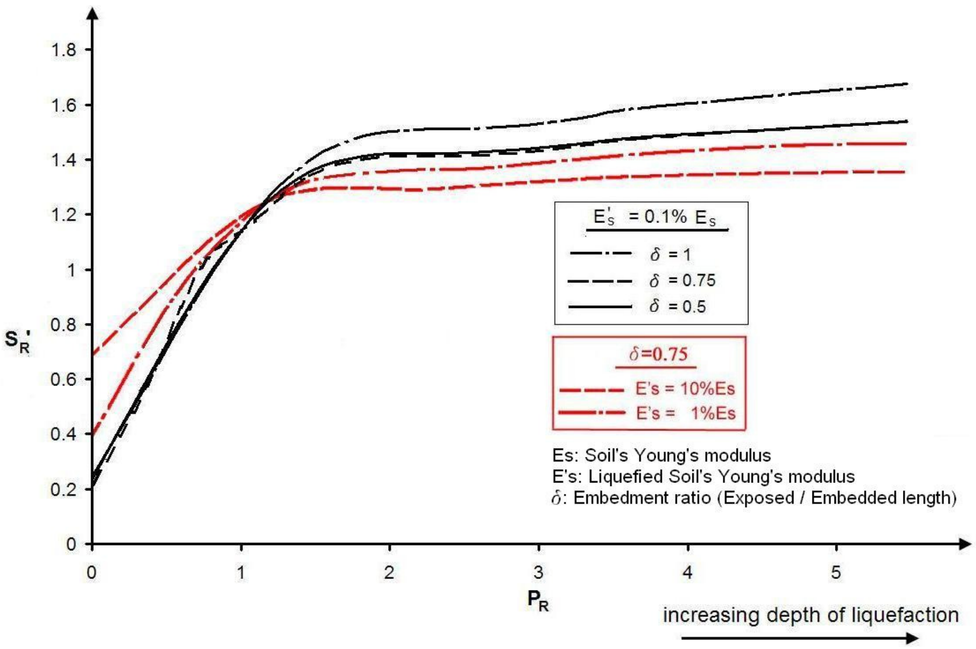

Figure 8: Non-dimensional plot of the liquefaction depth from the parametric study. The change in depth of fixity with depth of liquefaction for constant values of embedment ratio, and soil stiffness degradation.

5. CONCLUSIONS AND DISCUSSION

The analysis above demonstrates that the buckling behaviour of piles embedded in liquefied soil is governed by the depth of liquefaction, and the degree of soil stiffness degradation. The embedment ratio was found to not affect majorly behaviour of piles in liquefied soils. From Figure 8 the curves for embedment ratios of 0.5 and 0.75, overlap - this corresponds to the conclusion reached by Davison and Robinson (1965) that the depth

of fixity is insensitive to embedment ratio after a certain exposed length for constant soil stiffness.
The main factor governing the behaviour of piles in liquefied soil is the depth of liquefaction. For both analyses the non-dimensional depth fixity S'_R increases at an approximately constant rate with the non-dimensional depth of liquefaction, P_R. At a non-dimensional liquefaction depth value of approximately 1.5, the depth fixity converges to a value of S'_R of approximately 1.7. The stiffness degradation factor, φ being the main influence of the gradient and value of convergence. The curve of maximum stiffness degradation ($\varphi=0.1\%$) may be used to conservatively estimate the depth of fixity for any pile embedded in liquefied soil, and the embedment ratio $\delta=1$, would give a worst case scenario for a pile supporting a typical bridge or jetty.
A conservative recommendation may be to set $S'_R = 1.7$ and therefore use in calculations a buckling effective length of:

$$L = \gamma\left(L_U + 1.7\sqrt[4]{\frac{1.35EI}{0.001E_S}}\right) \tag{5.1}$$

where γ is an arbitrary safety factor applied by the designer. Equation 5.1 may be applied to steel, circular-hollow piles which have no rotational or lateral restriction to the top of the pile and are partially embedded in soil strata where the topmost layers are prone to liquefaction.
The elastic-continuum FE study in this paper is based on elastic soil-pile behaviour, linear soil stiffness with depth, and no slip conditions at the pile/soil interface. In reality the interaction between the soil and pile is much more complex, especially under liquefied conditions. Further, liquefaction often happens in the deep soil layers producing different boundary conditions for buckling mode for the pile. The study of the buckling behaviour of pile foundations in liquefied soils is a new but critical subject which requires careful attention by researchers and designers.

8. REFFERENCES

ABAQUS (2002) ABAQUS/CAE, "Standard user's manual, Version 6.3". Hibbitt Kaelsson and Sorensen; Providence, RI
Bhattacharya, S., Madabhushi, S. P. G. and Bolton, M. D. (2004*) "An Alternative Mechanism of Pile Failure in Liquefiable Deposits during Earthquakes,"* Geotechnique 54, No.3, p203-213.
Davisson M. T. and Gill H. L. (1963) *Laterally loaded piles in a layers soil system.* Proceedings of the American Society of civil engineers, Vol. 89, SM3 p63-94.
Davisson M. T. and Robinson K. E. (1965). *Bending and Buckling of partially embedded piles.* Proceedings of the 6th International conference in Soil Mechanics and Foundation Engineering, Vol 2, p243-246.
Eurocode 8: Part 5 [1998]. *Design provisions for earthquake resistance of structures- foundations, retaining structures and geotechnical aspects*, European Committee for standardization, Brussels.
Heelis M. E. Palovic M. N. and West R. P. (2004). The analytical prediction of the buckling loads of fully and partially embedded piles. Geotechnique 54, No.6, p363-373.
Hetenyi M. (1946). *Beams on Elastic Foundation*, Ann Arbor, University of Michigan Press.
Fleming, W.G.K., Weltman, A.J., Randolph, M.F. and Elson, W.K (1992). "Piling Engineering". Second edition, John Wiley & Sons.
Ishihara K. (l997). *Geotechnical aspects of the 1995 Kobe earthquake.* Proceedings of the 14th International Conference on Soil mechanics and Foundation Engineering, Terzaghi Oration, Hamburg, p2047-2073
Japanese Road Association (1996), *Specification for Highway Bridges, Part V, Seismic Design*. Tokyo, Japan
NEHRP (2000): National National Earthquake Hazards Reduction Program (NEHRP, 2000): *Commentary for Federal Emergency Management Agency on seismic regulations for new buildings (FEMA, USA 369),* USA.
Terzaghi K. (1955). Evaluation of coefficients of subgrade reaction. Geotechnique 4, p186-215.
Yasuda S., Terauchi T., Morimoto H., Erken A. and Yoshida N. (1998) *Post liquefaction behaviour of several sands*. 11th European Conference on Earthquake Engineering 1998 Balkema, Rotterdam.
Yasuda S., Yoshida N., Kiku H., Adachi K., and Gose S. (1999) *A simplified method to evaluate liquefaction-induced deformation.* Earthquake Geotechnical Engineering., Balkema, Roterdam