# Seismic Liquefaction of Layered-Subsoil with Two Adjacent Foundations[1]

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**ABSTRACT :**

Seismic liquefaction of subsoil with two adjacent foundations on layered soil half space is presented in this paper. A cone model for two foundations on layered soil half space is presented to study dynamic interaction between two foundations (which are two scattering sources), In the study the total stress field of the subsoil is divided into the free-field and the scattering field. Seed's simplified method is adopted for the free-field analysis, while the cone-model is proposed to analyze dynamic scattering stress field, the shear stress field and the compressive stress field in layered stratum with two adjacent foundations are calculated by shear cone and compressive cone, respectively. In order to study dynamic interaction between two scattering sources, the stress wave fields of layered subsoil with two foundations are divided into six zones, the P wave and S wave are analyzed in each zone. This method is applied to evaluate the seismic deformation of subsoil under two adjacent foundations. Hopefully, the proposed method could be also available for other complicated earthquake engineering problems with multi-scattering sources .

**KEYWORDS:** seismic liquefaction, layered subsoil with two adjacent foundations, cone model.

## 1. Introduction

Up-to-date, studies on soil-structure interaction are in general restricted for a single foundation on elastic half space medium, As we known, quite often one building with its adjacent building(s) are founded on layered soil, due to interaction the scattering stress field of the subsoil with two adjacent foundations would different from that without adjacent buildings , Wave propagation in layered subsoil with two adjacent foundations is very complicated. Few papers were published on studying earthquake response of subsoil with two adjacent foundations (which are regarded as two multi-scattering sources). In this paper, the cone model is applied to evaluate the seismic stress field of subsoil with two adjacent foundations. Some numerical results are provided and some factors to influence the interaction of two foundations are studied. Seismic liquefaction of subsoil with two adjacent foundations is firstly studied , which is very important and necessary .

## 2Stresses in subsoil with two foundations on layered half space

2.1 Mechanism of cone model

In order to simplify the analysis of soil-structure interaction, cone model was suggested by Meek and Wolf (Meek, J. W. & Wolf , J. P. , 1992,1993, 2001). A half-space medium is replaced by a cone according to the material mechanics theory, cone model is very effective and widely adopted and developed by other researchers (Fu-lu Men & Jie Cui ,1996,Wenhua Chen 2001, 2002, 2003).it has been proved to be suitable for soil medium and for low frequency wave(like earthquake wave). In order to study added-stress in subsoil, all waves are assumed to propagates along cones, whether they are reflected waves or translational waves, or shear waves and compressive waves.

2.2 Stresses in subsoil with two foundations

If two adjacent foundations are not far, interactions of two foundations must be taken into account in the analysis of earthquake response. Two massless rigid strip foundations resting on layered soil (seen in

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Fig.1) are assumed. The stress field in the subsoil with adjacent buildings can be divided into six zones, i.e. Zones I, II, III, and VI. Stresses in the free field can be calculated by using the method proposed by Chen (1997), the added stress field of each zone can be calculated by using the cone model, and the total stress field equal to their summation is listed in Table 1.

After *N* times reflection and transmission, the shear stress at a point ($z$, $t$) in each zone can be calculated by the method suggested by Chen (2000).

Tab 1 total stress field of each zone

| zone | Total Stress field | |
|---|---|---|
| I | $\sigma_t = \gamma z$ | $\tau_t = 0.65\gamma\gamma_d z \frac{a_{max}}{g}$ |
| II | $\sigma_t = \left(\frac{z_0^\sigma}{z_0^\sigma + z}\right)^2 \frac{W}{A} + (\gamma z)$ | $\tau_t = (z,t) = \frac{z_0^\tau}{z_0^\tau + z} S_1(t - \frac{z}{c_{S1}}) + \left(0.65\gamma_d \gamma z \frac{a_{max}}{g}\right)$ |
| III | $\sigma_t(z,t) = \frac{z_{01}^\sigma}{z_{01}^\sigma + z} P_1(t - \frac{z}{c_{p1}}) + \left(\frac{z_{01}^\sigma}{z_{01}^\sigma + z}\right)^2 \frac{W}{A} + (\gamma z)$ | $\tau_t = (z,t) = \frac{z_0^\tau}{z_0^\tau + z} S_1(t - \frac{z}{c_{S1}}) + \left(0.65\gamma_d \gamma z \frac{a_{max}}{g}\right)$ |
| IV | $\sigma_t = \gamma z$ | $\tau_t(z,t) = \frac{z_0^\tau}{z_0^\tau + z} S_1(t - \frac{z}{c_{S1}}) + \frac{z_0^\tau}{z_0^\tau + z} S_2(t - \frac{z}{c_{S2}}) + \left(0.65\gamma_d \gamma z \frac{a_{max}}{g}\right)$ |
| V | $\sigma_t(z,t) = \frac{z_{01}^\sigma}{z_{01}^\sigma + z} P_1(t - \frac{z}{c_{p1}}) + \left(\frac{z_{01}^\sigma}{z_{01}^\sigma + z}\right)^2 \frac{W}{A} + \gamma z$ | $\tau_t(z,t) = \frac{z_0^\tau}{z_0^\tau + z} S_1(t - \frac{z}{c_{S1}}) + \frac{z_0^\tau}{z_0^\tau + z} S_2(t - \frac{z}{c_{S2}}) + \left(0.65\gamma_d \gamma z \frac{a_{max}}{g}\right)$ |
| VI | $\sigma_t(z,t) = \frac{z_{01}^\sigma}{z_{01}^\sigma + z} P_1(t - \frac{z}{c_{p1}}) + \frac{z_{02}^\sigma}{z_{02}^\sigma + z} P_2(t - \frac{z}{c_{p2}}) + \left(\frac{z_{01}^\sigma}{z_{01}^\sigma + z}\right)^2 \frac{W}{A} + \left(\frac{z_{02}^\sigma}{z_{02}^\sigma + z}\right)^2 \frac{W}{A} + (\gamma z)$ | |
| | $\tau_t(z,t) = \frac{z_0^\tau}{z_0^\tau + z} S_1(t - \frac{z}{c_{S1}}) + \frac{z_0^\tau}{z_0^\tau + z} S_2(t - \frac{z}{c_{S2}}) + 0.65\gamma_d \gamma z \frac{a_{max}}{g}$ | |

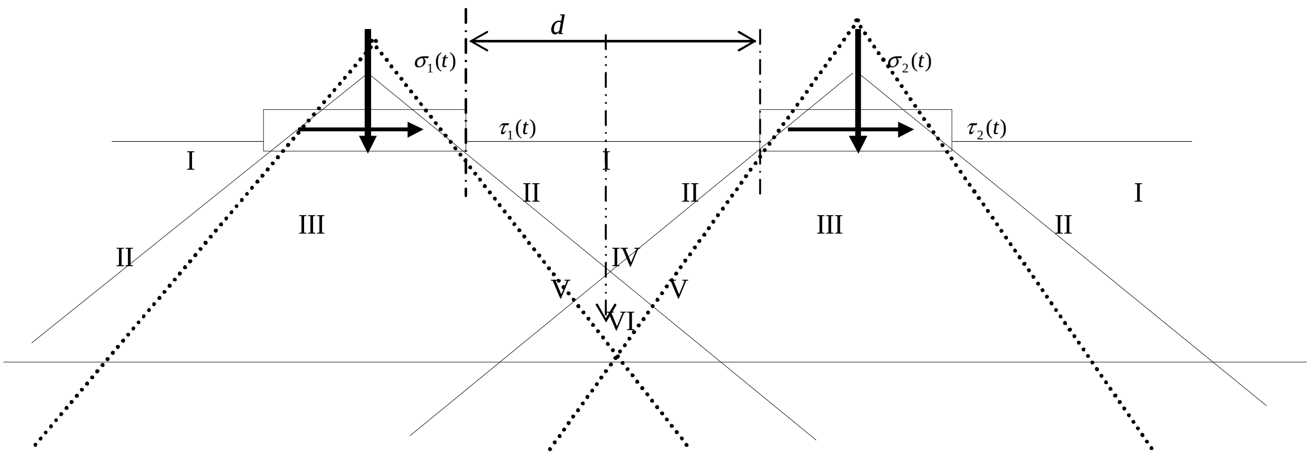


Fig 1 Zones of stress in subsoil of two foundations

## 3 Interaction of two foundations

Zones-division affected by two foundations is very useful in earthquake engineering. From Fig.3, Zones IV, V and VI sustain the effect of the scattered S wave from two foundations, in the same time. Zone VI is affected by scattered P wave from two foundations, but zone IV is affected only by the scattered P wave from one of the two foundations. Soil in this zone could be easily destroyed due to the significant amount of additional dynamic shear stress. The added stresses in the zones affected by only one of the foundations can be calculated by the method suggested by Chen (2003). The size of each zone is affected by the diffuse angle of the cone, which is the ratio of the rigidities of the subsoil and the foundation and the distance between the two foundations.

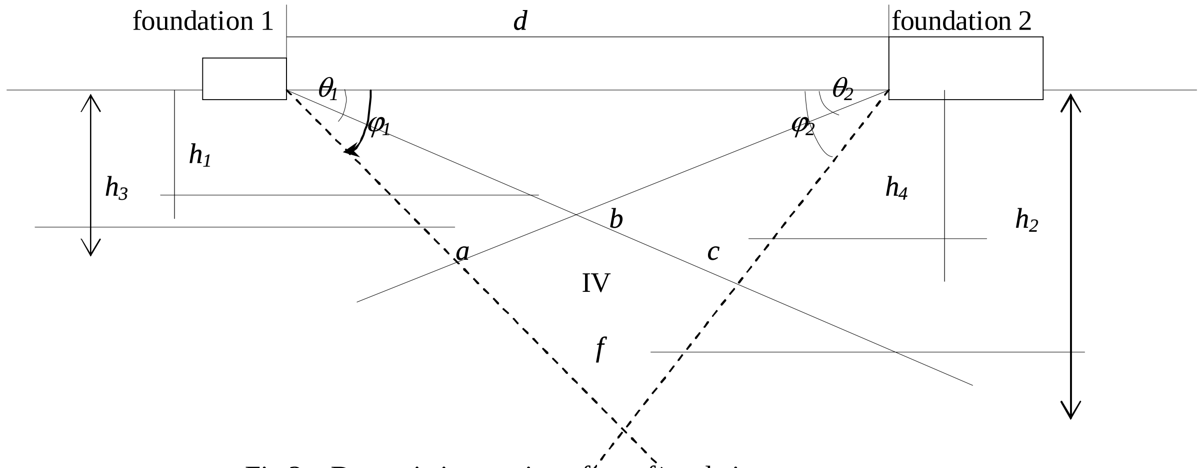


Fig 2 Dynamic interaction of two foundations

In the calculation section , it will be shown that Zone IV exhibits an extreme stress state. Here, attention is focused on how the buried depth, the size of the zone, and the added shear stress in this zone change depending on the distance of two adjacent foundations under horizontal excitation. A model of interaction between two foundations is shown in Fig.1. Denoting $h_1$ as the buried depth of zone, and d as the distance of the inside edge of the boundary of two foundations, the angle of the compressive cone and shear cone for two foundations are: 90°-$\theta_1$, 90°-$\varphi_1$, 90°-$\theta_2$ and 90°-$\varphi_2$, respectively,

$$h_1=\frac{tg\theta_1 tg\theta_2}{tg\theta_1+tg\theta_2}d \qquad h_2=\frac{tg\varphi_1 tg\varphi_2}{tg\varphi_1+tg\varphi_2}d \qquad h_3=\frac{tg\theta_1 tg\varphi_2}{tg\theta_1+tg\varphi_2}d \qquad h_4=\frac{tg\varphi_1 tg\theta_2}{tg\varphi_1+tg\theta_2}d \tag{1}$$

The average added shear stress of point a of IV zone is :

$$\tau_{ad}(z,t)=\frac{z_0^{\tau}}{z_0^{\tau}+z}\tau_1(t-\frac{z}{c_{s_1}})+\frac{z_0^{\tau}}{z_0^{\tau}+z}\tau_2(t-\frac{z}{c_{s_2}})=\frac{1}{1+\frac{h_1}{tg\theta_1 b_1}}\tau_1(t-\frac{z}{c_{s_1}})+\frac{1}{1+\frac{h_1}{tg\theta_1 b_2}}\tau_2(t-\frac{z}{c_{s_2}}) \tag{2}$$

if z=$h_1$ then

$$\tau_{ad}(h_1,t)=\frac{z_0^{\tau}}{z_0^{\tau}+h_1}\tau_1(t-\frac{h_1}{c_{s_1}})+\frac{z_0^{\tau}}{z_0^{\tau}+h_1}\tau_2(t-\frac{h_1}{c_{s_2}})=\frac{1}{1+\frac{h_1}{tg\theta_1 b_1}}\tau_1(t-\frac{h_1}{c_{s_1}})+\frac{1}{1+\frac{h_1}{tg\theta_1 b_2}}\tau_2(t-\frac{h_1}{c_{s_2}}) \tag{3}$$

input shear stress :

$$S_1(t)=\tau_1(t-\frac{z}{c_{s,1}})=A_1\sin\alpha_1$$

$$S_2(t)=\tau_2(t-\frac{z}{c_{s2}})=A_2\sin\alpha_2$$

$$\tau_{ad}(h_1,t)=\frac{1}{1+\frac{h_1}{tg\theta_1 b_1}}A_1\sin\alpha_1+\frac{1}{1+\frac{h_1}{tg\theta_1 b_2}}A_2\sin\alpha_2 \tag{4}$$

the equation (4) can also be written into:

$$\tau_{ad}(h_1,t)=\frac{1}{1+\frac{h_1}{tg\theta_1 b_1}}A_1(\lambda_1\sin\omega_1+\lambda_2\sin\omega_2+...+\lambda_i\sin\omega_i)+\frac{1}{1+\frac{h_1}{tg\theta_1 b_2}}A_2(\psi_1\sin\phi_1+\psi_2\sin\phi_2+...+\psi_i\sin\phi_i) \tag{5}$$

$$\lambda_1=1\ ;\ \psi_1=1\ ;\lambda_{i+1}=\lambda_i\frac{1}{1+\frac{h_1}{tg\theta_1 b_1}+\frac{H}{tg\theta_1}};\quad \psi_{i+1}=\psi_i\frac{1}{1+\frac{h_1}{tg\theta_2 b_2}+\frac{H}{tg\theta_2}} \tag{6}$$

H is thickness of soil layer.

If $\theta_1=\theta_2$, $b_1=b_2=b$, $A_1=A_2=0.2$, N=10, $\theta_1$=45°, $b$=10, $H$=20 then

$$h_1 = \frac{tg\theta_1 tg\theta_2}{tg\theta_1 + tg\theta_2} d = \frac{d}{2} tg\theta_1 \qquad h_2 = \frac{tg\varphi_1 tg\varphi_2}{tg\varphi_1 + tg\varphi_2} d = \frac{d}{2} tg\varphi_1 \tag{7}$$

$$\tau_{ad}(h_1, t) = \frac{0.15}{1 + \frac{d}{2b}} \left[ \sum_{i=1}^{10} \lambda_i \sin \omega_i + \sum_{i=1}^{10} \psi_i \sin \phi_i \right] \tag{8}$$

after N times cycles, the equation (8) can be written as:

$$\tau_{ad}(h_1, t) \approx \frac{1}{1 + 0.5d} \tag{9}$$

By the equation (8) and (9),the relationship of the add shear stress, depth and Area of zone IV with distance of two foundations($d$) are shown in Fig.3 ,Fig.4 and Fig.5.

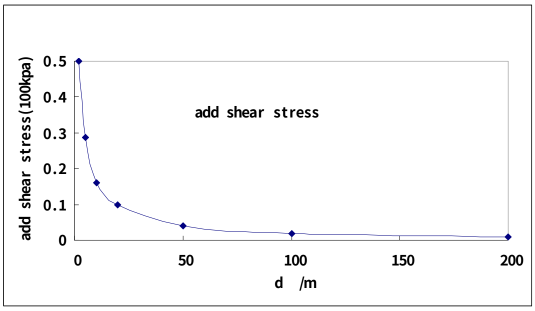


Fig.3 Curve of τ-d

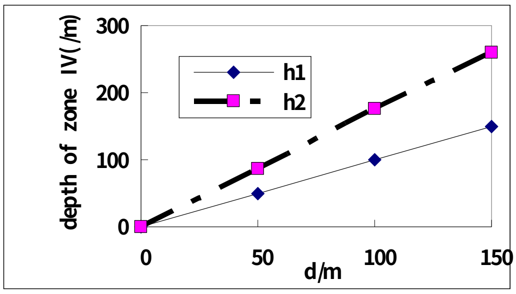


Fig.4 Depth of zone IV change with d

$\alpha_1 = \alpha_2$ $b_1 = b_2 = b$ $A_1 = A_2 = 0.15$ b=10m H =20m

$Q$ is the area of zone of IV, then,

$$Q = \tfrac{1}{2} dh_2 - \tfrac{1}{2} dh_3 - \tfrac{1}{2} dh_4 + \tfrac{1}{2} dh_1 = \frac{d^2}{2} \left( \frac{tg\varphi_1 tg\varphi_2}{tg\varphi_1 + tg\varphi_2} - \frac{tg\theta_1 tg\varphi_2}{tg\theta_1 + tg\varphi_2} - \frac{tg\varphi_1 tg\theta_2}{tg\varphi_1 + tg\theta_2} + \frac{tg\theta_1 tg\theta_2}{tg\theta_1 + tg\theta_2} \right) \tag{10}$$

If $\theta_1 = \theta_2$, $\varphi_1 = \varphi_2$ then $Q = \frac{d^2}{2} \left( \frac{tg\varphi_1 + tg\theta_1}{2} - 2 \frac{tg\theta_1 tg\varphi_2}{tg\theta_1 + tg\varphi_2} \right)$

$$\text{If} \quad K = \frac{tg\varphi_1 + tg\theta_1}{2} - 2\frac{tg\theta_1 tg\varphi_2}{tg\theta_1 + tg\varphi_2} \quad \text{then} \quad Q = \frac{K}{2} d^2 \tag{11}$$

For example $\theta_1 = \theta_2 = 45°$, $\varphi_1 = \varphi_2 = 60°$ then

$$Q = 0.099 d^2 \tag{12}$$

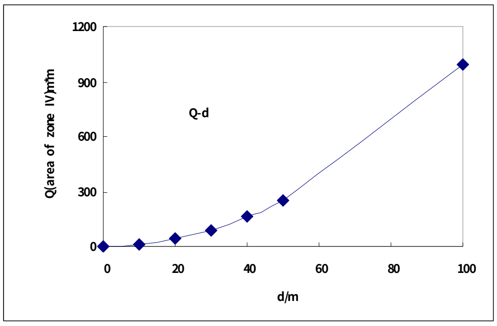


Fig.5 Curve of Q-d

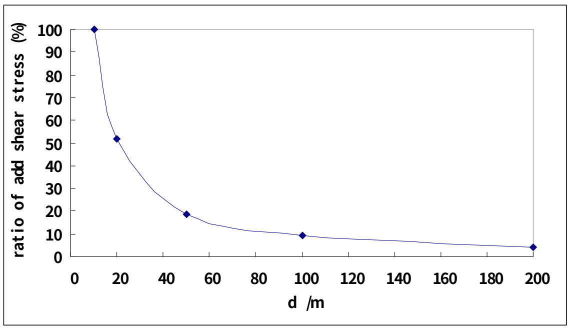


Fig.6 Change of $\tau_{add}$ with distance
*( compared with the $\tau_{add}$ of distance is 10 m)*

$\tau_{add}$ change with distance compared with $\tau_{add}$ that distance is 10 m.

$$r(d) = \frac{\tau_{add}(d)}{\tau_{add}(d)\big|_{d=10m}} \tag{13}$$

The above figures show that as the distance between the two foundations increases, the buried depth of the interaction in Zone IV deepens, the effected area enlarges, and the average added shear stress reduces quickly (shown in Fig.5). the distance between two foundations is 10 meters($d$ =10 m), the added shear stress is $\tau_{add}|_{d=10}$ ;the effected depth $h_1$ is 2.9 m; if $d$ = 20m, the add shear stress is about 60% of $\tau_{add}|_{d=10}$ ; If $d$ =50m, the add shear stress is about 20% of $\tau_{add}|_{d=10}$ .

## 4 Seismic liquefaction of subsoil with two foundations

$\tau/\sigma$ can be calculated as following:

in the zone I

$$\frac{\tau_t}{\sigma_t} = 0.65\gamma_d \frac{a_{\max}}{g} \tag{14}$$

in the zone II

$$\frac{\tau_t}{\sigma_t} = \left\{0.65\gamma_d z + \left(\frac{z_0^\tau}{z_0^\tau + z}\right)^2 \frac{W}{A}\right\} \frac{a_{\max}}{g} \Big/ \gamma\ z \tag{15}$$

in the zone III

$$\frac{\tau_t}{\sigma_t} = \left\{0.65\gamma_d z + \left(\frac{z_0^\tau}{z_0^\tau + z}\right)^2 \frac{W}{A}\right\} \frac{a_{\max}}{g} \Bigg/ \left(\gamma\ z + \left(\frac{z_0}{z_0 + z}\right)^2 \frac{W}{A}(1 \pm \frac{a_{\max}}{2g})\right) \tag{16}$$

in the zone IV

$$\frac{\tau_t}{\sigma_t} = 2\left\{0.65\gamma_d z + \left(\frac{z_0^\tau}{z_0^\tau + z}\right)^2 \frac{W}{A}\right\} \frac{a_{\max}}{g} \Big/ \gamma\ z \tag{17}$$

in the zone V

$$\frac{\tau_t}{\sigma_t} = 2\left\{0.65\gamma_d z + \left(\frac{z_0^\tau}{z_0^\tau + z}\right)^2 \frac{W}{A}\right\} \frac{a_{\max}}{g} \Bigg/ \left(\gamma\ z + \left(\frac{z_0}{z_0 + z}\right)^2 \frac{W}{A}(1 \pm \frac{a_{\max}}{2g})\right) \tag{18}$$

in the zone VI

$$\frac{\tau_t}{\sigma_t} = 2\left\{0.65\gamma_d z + \left(\frac{z_0^\tau}{z_0^\tau + z}\right)^2 \frac{W}{A}\right\} \frac{a_{\max}}{g} \Bigg/ \left(\gamma\ z + 2\left(\frac{z_0}{z_0 + z}\right)^2 \frac{W}{A}(1 \pm \frac{a_{\max}}{2g})\right) \tag{19}$$

$\tau/\sigma$ can be used to evaluate seismic liquefaction(Seed H. B , 1971) in different zone of subsoil with two foundations, $\tau/\sigma$ of zone of IV is the biggest, zone of IV is easy to liquefy , $\tau/\sigma$ of zone of III and zone of VI are the smallest, those zones are difficult to liquefy, The position and area of zones of will change with the distance of two foundations and depth of soil layer.

## 5 Calculation and discussion

Two identical strip foundations on layered soil are studied. The width of the foundations $b1$=$b2$=10m; the distance between the two are d=20m, $\theta$1= $\theta$2,=45° , $\varphi$1 =$\varphi$2 =60° ; the apex heights of the shear cone and the compressive cone are height of top of shear cone is 2.89$m$ , height of top of compressive cone is 4.19m, respectively; and the velocities of the S and P waves are: $c$s=200m/s, $c$p=1000m/s. The excitation forces acting on the surface foundation are: $S_1(t) = S_2(t) = 1.5\sin 2t$ (kpa), $\sigma_1(t) = \sigma_2(t) = 1.55\sin 2t$ (kpa)**,** the soil layer thickness is $H$=10m; foundation width is $B$=10m; the distance between the two foundations is $d$= 20m; and the soil unit weight is $r$ = 20KN/m$^3$. The calculated added stresses ratio and pore water pressure ratio in different zones at the same level(z=5m) are shown in Fig.7.

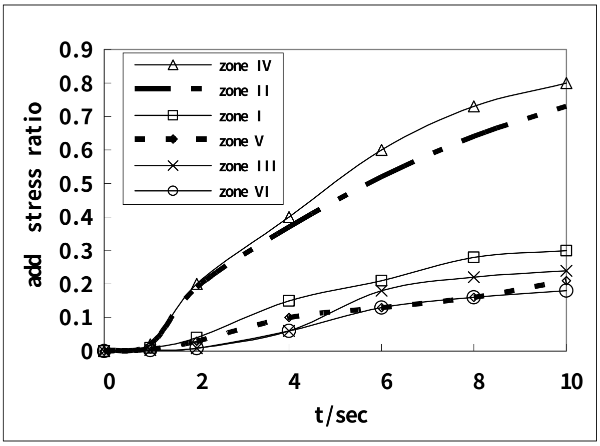


Fig.7 Added stress ratio in each zone(z=5m)

## 6 Conclusion

Seismic liquefaction of subsoil with two foundations is very different with single foundation, the cone model was extended to study the vibration of two surface foundations. By using a layered cone model, the stress field in the subsoil with two adjacent foundations was analyzed and divided into six zones, and the P and S waves in each zone were calculated . In the present analysis, special attention was given to the interaction region, i.e., zone IV, where extreme soil stress would occur under ground shaking. The numerical results show that for two adjacent surface foundations with an identical width of 10m, dynamic interaction must be taken into account if the distance between them is less than 50 m., For practical applications, the cone model can be further extended to the study of vibration of layered soil with multi-buildings..

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