STUDY ON BI-AXIAL EARTHQUAKE RESPONSES OF REINFORCED CONCRETE STRUCTURES WITH SLIPPING TYPE RESTORING FORCE CHARACTERISTICS

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ABSTRACT :

In this paper, slipping type bi-axial non-linear restoring force model proposed by authors in previous study was explained firstly. The proposed model, which a tensor was used to express slipping behaviors, was an extended model of a elastic-perfectly plastic type bi-axial model that was based on an analogy to the theory of plasticity. Then earthquake response analyses with one-mass-system were carried out. One of the features of earthquake responses of bi-axial slipping model was that two-directional shear force response was radial and tended to concentrate in orthogonal direction of yielded direction. Numerical results of two-dimensional analyses were compared with that of one-dimensional analyses of a maximum displacement response in all horizontal direction. A tendency can be seen that plastic ductility ratios of two-directional analyses were a little smaller than that of one-directional analyses, and total input energies of two-directional and one-directional analyses were hardly different.

KEYWORDS: reinforced concrete, bi-axial restoring force model, earthquake response analysis, slipping behavior, theory of plasticity, one-mass-system

1. INTRODUCTION

It is necessary to model restoring force characteristics of building structures properly when dynamic behaviors of the structures subjected to earthquake excitation are analyzed. R/C members have tendency to show slipping behaviors, which deformation increase and decrease with little varying lateral force in the quite low load condition, in case of one-directional cyclic load combined bending and shear. R/C beams with insufficient bond of longitudinal bars and R/C walls shows slipping behaviors. In the previous study, three-directional loading tests of R/C members without bond were carried out to understand slipping behaviors (Nishimura 2006). R/C members without bond of Longitudinal bar shows slipping behaviors in their one-directional force-deformation relationships. Then bi-axial non-linear restoring force model of slipping type was proposed (Nishimura 2006). The proposed model, which a tensor was used to express slipping behaviors, was an extended model of a elastic-perfectly plastic type bi-axial model that was based on an analogy to the theory of plasticity. It was confirmed that the proposed model could roughly describe the test results in the previous study.

In this paper, two-directional dynamic behaviors of slipping type were studied. Firstly, bi-axial non-linear restoring force model was explained. Then earthquake response analyses were carried out. In the response analyses, natural periods were ranged from 0.1 sec to 4.0 sec, and yield-shear force ratios, which were ratios to the weight of the mass, were ranged from 0.1 to 1.0. Three earthquake ground motions those were El Centro 1940, Kobe 1995, and Chi Chi 1999 were inputted. Numerical results of two-directional analyses were compared with results of one-directional analyses of maximum displacement response in all horizontal directions.

2. RESTORING FORCE MODEL

2.1. Outline of Restoring Force Model

The bi-axial non-linear restoring force model of slipping type, which a tensor was used to express slipping behaviors, was an extended model of a elastic-perfectly plastic type bi-axial model that was based on an analogy to the theory of plasticity. A concept of the model can be drawn as Figure 2.1 that a roller, which consist with a mass and an elastic member, moves in slipping area surrounded with rigid wall.

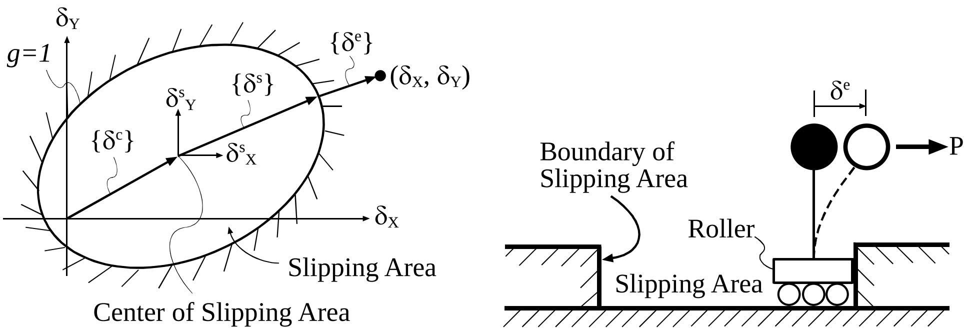

Figure 2.1 Slipping Model

As shown in Fig.1, total displacement vector $\{\delta\}$ is assumed as follows.

$$\{\delta\} = \{\delta^e\} + \{\delta^s\} + \{\delta^c\} \tag{2.1}$$

$\{\delta^e\}$, $\{\delta^s\}$, and $\{\delta^c\}$ are vectors of elastic displacement, slipping displacement, and center of slipping area, respectively. Those vectors consist of X and Y components. $\{\delta^s\}$ and increment of $\{\delta^c\}$ are assumed as follows.

$$\{\delta^s\} = [D] \cdot \{n\} \tag{2.2}$$

$$[dD] = \frac{k_s}{\left|\{d\delta^p\}\right|} \cdot \begin{bmatrix} d\delta_X^{p\,2} + k_o \cdot d\delta_Y^{p\,2} & (1-k_o) \cdot d\delta_X^p \cdot d\delta_Y^p \\ (1-k_o) \cdot d\delta_X^p \cdot d\delta_Y^p & k_o \cdot d\delta_X^{p\,2} + d\delta_Y^{p\,2} \end{bmatrix} \tag{2.3}$$

$$\{d\delta^c\} = (1-k_s) \cdot \{d\delta^p\} \tag{2.4}$$

$[D]$ is tensor for expressing the slipping area and $[dD]$ is increment of $[D]$. $\{n\}$ is a unit vector and $\{d\delta^p\}$ is increment of plastic displacement vector in elastic-plastic state of the model. As shown in Figure 2.2 and Figure 2.3, a coefficient k_s is degree of slipping behaviors, and a coefficient k_o is expanding ratio of slipping area in orthogonal direction in elastic-plastic state. Figure 2.4 shows expanding stage of slipping area.

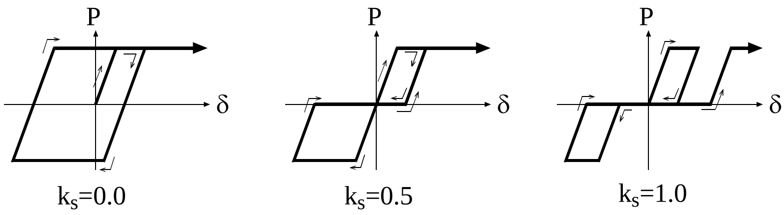

Figure 2.2 Coefficient k_s in Equation (2.2)

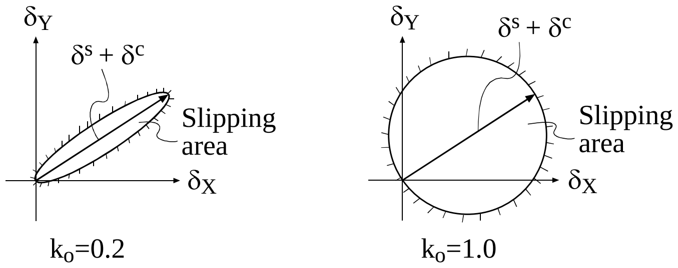

Figure 2.3 Coefficient k_o in Equation (2.3)

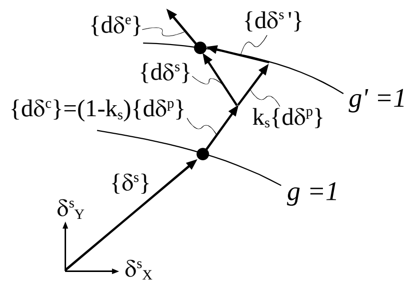

Figure 2.4 Expansion of slipping area

As shown in Figure 2.1, direction of force vector $\{P\}$ must be in the same direction as normal direction of slipping area, and $\{P\}$ is also given by elastic rigidity $[K^e]$. Therefore, the force vector can be given as follows, where slipping area is expressed by an equation $g(\delta^s)$=1 and $\{\nabla g\}$ is gradient of the function $g(\delta^s)$.

$$\{P\} = \kappa \cdot \{\nabla g\} \quad , \quad (\kappa \geq 0) \tag{2.5}$$

$$\{P\} = [K^e] \cdot \{\delta^e\} \tag{2.6}$$

The function $g(\delta^s)$ is given by Equation (2.2) and an equation $\{n\}^T\{n\}$=1 because $\{n\}$ is a unit vector.

$$g = \frac{\{\delta^s\}^T \cdot [T] \cdot \{\delta^s\}}{2} \quad \text{, where} \quad [T] = 2 \cdot [D]^{-1} \cdot [D]^{-1} \tag{2.7}$$

Figure 2.5 shows a yield surface that is assumed as an equation $f(P)$=1. The function $f(P)$ is given as follows, where components of a matrix $[U]$ are constants when its characteristics is assumed as perfectly plastic.

$$f = \{P\}^T \cdot [U] \cdot \{P\} \tag{2.8}$$

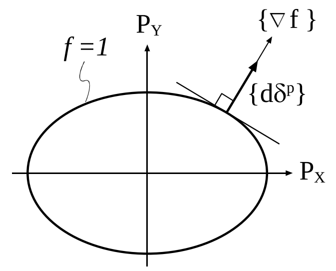

Figure 2.5 Yield surface

The flow rule of the theory of plasticity can be written as follows. $\{\nabla f\}$ is gradient of $f(P)$.

$$\{d\delta^p\} = d\lambda \cdot \{\nabla f\} \tag{2.9}$$

2.2. Incremental Force and Displacement Relationships

2.2.1 Elastic state

In elastic state, relationship between incremental force and displacement is written as follows because total of incremental displacement is equal to incremental elastic displacement.

$$\{dP\} = [K^e] \cdot \{d\delta\} \tag{2.10}$$

2.2.2 Slipping state

In slipping state, incremental force-displacement relationship is written as follows, where $[K^s]$ is a zero matrix, because total of incremental displacement is equal to incremental slipping displacement and there is no elastic displacement.

$$\{dP\} = [K^s] \cdot \{d\delta\} \tag{2.11}$$

2.2.3 Elastic-slipping state

In elastic-slipping state, total of incremental displacement is equal to the sum of $\{d\delta^e\}$ and $\{d\delta^s\}$, where $\{d\delta^e\}$ and $\{d\delta^s\}$ are increment of $\{\delta^e\}$ and $\{\delta^s\}$, respectively. $\{P\}$ and increment of $\{P\}$, $\{dP\}$, are given as follows by Equations (2.5) and (2.7).

$$\{P\} = \kappa \cdot [T] \cdot \{\delta^s\} \tag{2.12}$$

$$\{dP\} = d\kappa \cdot [T] \cdot \{\delta^s\} + \kappa \cdot [T] \cdot \{d\delta^s\} \tag{2.13}$$

The following equation is given by dg=0.

$$\{\nabla g\}^T \cdot \{d\delta^s\} = 0 \tag{2.14}$$

Relationship of $\{dP\}$ and $\{d\delta\}$ is given as follows by Equations (2.6), (2.13), and (2.14).

$$\{dP\} = [K^{es}] \cdot \{d\delta\} \ , \tag{2.15}$$

$$\text{where} \quad [K^{es}] = [K^e] \cdot [A]^{-1} \cdot \frac{\{\nabla g\} \cdot \{\nabla g\}^T}{\{\nabla g\}^T \cdot [A]^{-1} \cdot \{\nabla g\}} \cdot [A]^{-1} \cdot [K^e] + \kappa \cdot [T] \cdot [A]^{-1} \cdot [K^e]$$

$$\text{and} \quad [A] = [K^e] + \kappa \cdot [T]$$

2.2.4 Elastic-slipping-plastic state

In elastic-slipping-plastic state, total of incremental displacement is equal to the sum of $\{d\delta^e\}$, $\{d\delta^s\}$, and $\{d\delta^c\}$. The following equations are given by replacing $\{d\delta^s\}$ in Equations (2.13) and (2.14) with

($\{d\delta^s\} - k_s\{d\delta^p\}$).

$$\{dP\} = d\kappa \cdot [T] \cdot \{\delta^s\} + \kappa \cdot [T] \cdot \left(\{d\delta^s\} - k_s \cdot \{d\delta^p\} \right) \tag{2.16}$$

$$\{\nabla g\}^T \cdot \left(\{d\delta^s\} - k_s \cdot \{d\delta^p\} \right) = 0 \tag{2.17}$$

The following equation is given by df=0.

$$\{\nabla f\}^T \cdot \{dP\} = 0 \tag{2.18}$$

Incremental force and displacement relationship is given as follows by Equations (2.4), (2.6), (2.8), (2.9), (2.16), (2.17), and (2.18).

$$\{dP\} = \left[K^{esp}\right] \cdot \{d\delta\} \;, \tag{2.19}$$

$$\text{where} \quad \left[K^{esp}\right] = \left[K^{es}\right] - \frac{\left[K^{es}\right] \cdot \{\nabla f\} \cdot \{\nabla f\}^T \cdot \left[K^{es}\right]}{\{\nabla f\}^T \cdot \left[K^{es}\right] \cdot \{\nabla f\}}$$

3. EARTHQUAKE RESPONSE ANALYSIS

3.1. Numerical Program

Earthquake responses were examined with a one-mass-system. The constants of restoring force model are shown in Table 3.1. Parameters of the analyses were natural period, T, and yield-shear force ratio, α. The natural periods were ranged from 0.1 sec to 4.0 sec, and the yield-shear force ratios, which were ratios to the weight of the mass, were ranged from 0.1 to 1.0. Newmark's method [β=1/4] was used for the response analysis, and damping ratio was equal to 3% associating with elastic rigidity. Three earthquake ground motions those were El Centro 1940, Kobe 1995, and Chi Chi 1999 were inputted.

Table 3.1 Constants of restoring force model

T_{EW} / T_{NS}	α_{EW} / α_{NS}	k_S in Eq.(3)	k_O in Eq.(3)
1.0	1.0	0.5	0.2

3.2. Numerical Results

Two-directional responses were compared with one-directional responses that direction had response of maximum μ in all direction, which responses were calculated every an angle of 1 degree. Figures 3.1, 3.2, and 3.3 show numerical results. A plastic ductility ratio, μ, was assumed as μ=($|\{\delta\}_{MAX}| - \delta_Y$)/δ_Y, where δ_Y and $|\{\delta\}_{MAX}|$ are yield displacement and length of maximum displacement vector. An equivalent velocity was calculated from total input energy. The results shown in these Figures were chosen that μ were in the range 0.0 < μ < 10.0.

As shown in Figure 3.1, a tendency can be seen that μ of two-directional analyses were a little smaller than that of one-directional analyses. However direction of the maximum displacement vector of two-directional analysis didn't always agreed with that of one-directional analysis, as shown in Figure

3.2. As shown in Figure 3.3, V_E of two-directional analyses were a little larger than that of one-directional analyses, however it can be said that these were hardly different.

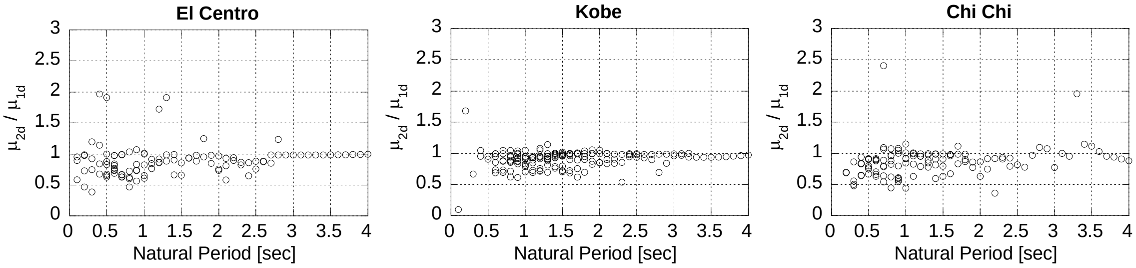

Figure 3.1 Plastic ductility ratio

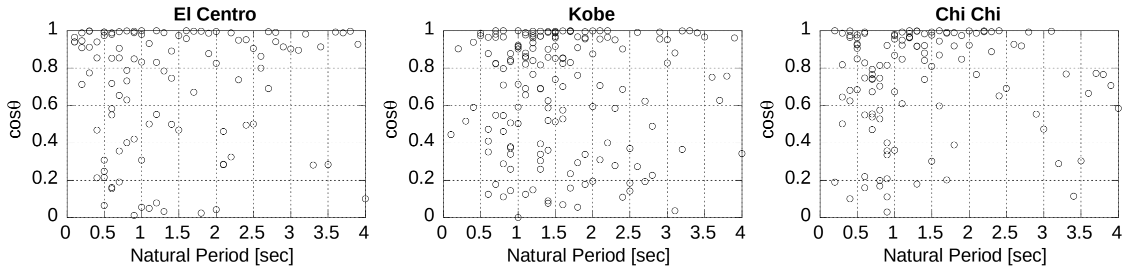

Figure 3.2 Cosine between maximum directions of 2D-analysis and 1D-analysis

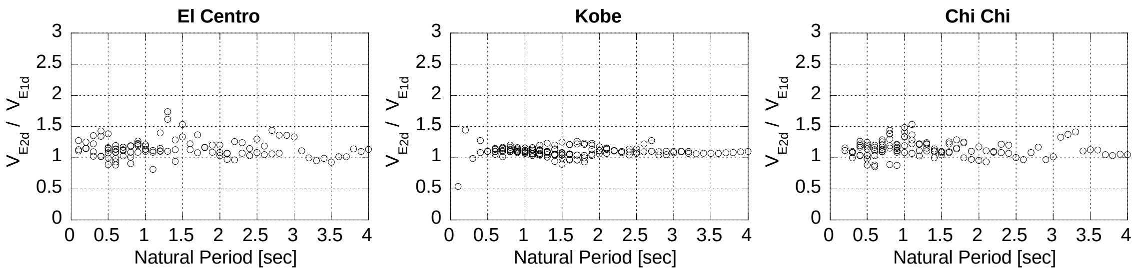

Figure 3.3 Equivalent velocity

Figure 3.4 shows the numerical result of response to El Centro earthquake when the natural period and the yield-shear force ratio were equal to 0.3 sec and 0.5, respectively. One of the features of this slipping model was that two-directional shear force response was radial and tented to concentrate in orthogonal direction of yielded direction.

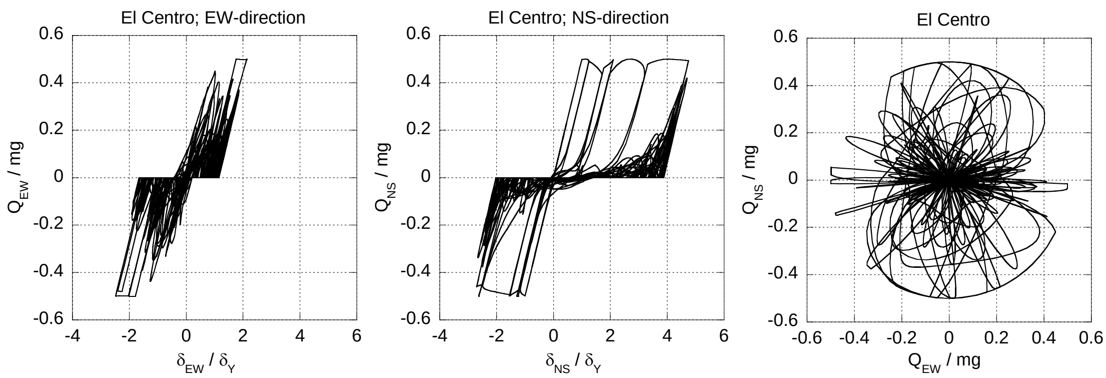

Figure 3.4 Response to earthquake of El Centro (T= 0.3 sec, α= 0.5)

4. CONCLUSION

In this paper, firstly, bi-axial non-linear restoring force model (Nishimura 2006) was explained, and then earthquake response analyses with one-mass-system were carried out. Numerical results of two-dimensional analyses, which plastic ductility ratios were ranged from 0.0 to 10.0, were compared with that of one-dimensional analyses of a maximum displacement response in all horizontal direction. As results, the following conclusions were found.

1) One of the features of earthquake responses of bi-axial slipping model was that two-directional shear force response was radial and tended to concentrate in orthogonal direction of yielded direction.

2) A tendency can be seen that plastic ductility ratios of two-directional analyses were a little smaller than that of one-directional analyses, and direction of the maximum displacement vector of two-directional analysis didn't always agreed with that of one-directional analysis.

3) Total input energies of two-directional and one-directional analyses were hardly different.

REFERENCES

Nishimura, K., Nguyen, H. H., and Takiguchi, T. (2006), "Study on Formulation of Slipping Type Multi- Axial Restoring Force Characteristics of R/C Members", Journal of Structural and Construction Engineering, Architectural Institute of Japan, No.609, pp145-154, (in Japanese).