TIME-DOMAIN MODAL ANALYSIS OF NON-PROPORTIONALLY DAMPED TWO-WAY ASYMMETRIC ELASTIC BUILDINGS

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ABSTRACT :

This study investigates the effectiveness of the modal analysis using three-degree-of freedom (3DOF) modal equations of motion to deal with the seismic analysis of two-way asymmetric elastic systems with supplemental damping. The 3DOF modal equations of motion possessing the non-proportional damping property enable the two modal translations and one modal rotation to be not proportional in an elastic state. The conventional approximation method is to use the single degree-of-freedom (SDOF) modal equations of motion, which is obtained by neglecting the off-diagonal elements of the transformed damping matrix. One one-story and one three-story non-proportionally damped two-way asymmetric buildings under the excitation of bi-directional seismic ground motions are analyzed. The analytical results are obtained by using the proposed method, the noted conventional approximation method and the direct integration of the equation of motion. It is seen that the proposed method can significantly improve the accuracy of the analytical results compared with those obtained by using the conventional approximation method. Moreover, the proposed method does not substantially increase the computational efforts.

KEYWORDS: asymmetric buildings, non-proportional damping, bi-directional ground motion, modal response history analysis

1. INTRODUCTION

The building with the center of stiffness (CR) not coincident with the center of mass (CM) along the two horizontal plane axes is defined as a two-way asymmetric building in this paper. The noted buildings with supplemental damping, e.g. viscous dampers in the braces, usually belong to the non-proportionally or non-classically damped structures whose damping matrix can not be diagonalized by the mode shapes of the undamped systems. Although the research on the dynamic responses of non-proportionally damped symmetric structures was conducted at a much earlier time [Itoh, 1973], the study of non-proportionally damped asymmetric structures was performed much later [Goel, 1998; Lin and Chopra, 2001]. Four key parameters of supplemental damping were identified [Lin and Chopra, 2001]. The noted parameters of supplemental damping are the damping ratio, the normalized eccentricity, the normalized radius of gyration and the relative amount oriented in the direction orthogonal to the direction of ground motion. According to the literature review [Goel, 2001], the analysis methods of non-proportionally damped systems were grouped into four categories and the corresponding shortcomings are briefly stated as follows. The first approach is to directly integrate the equation of motion of the original multi-degree-of–freedom (MDOF) structure. The stated approach is numerically inefficient for structural systems with a lot of degrees of freedom. Clough and Mojtahedi [1976] proposed to directly integrate the truncated set of the coupled modal equations of motion, which is more efficient than dealing with the whole set of equation of motion of the original structural system. The second approach is the mode superposition method using complex mode shapes [Igusa *et al*., 1984] which results in doubling the size of the eigenvalue problems and difficulties associated with the use of complex numbers in the dynamic response analysis. The third approach is the hybrid time-domain procedure [Ibrahimbegovic *et al*., 1990], which iteratively solves the coupled modal equations of motion in time domain. However, this method cannot be implemented on most commercially available structural analysis programs. The last approach, also the most common and simplest approach, is to simply neglect the off-diagonal elements of the transformed damping matrix which is appealing to the engineering practice because it enables the use of the traditional modal analysis methods.

Goel [2001] investigated the effects of neglecting the off-diagonal terms of the transformed damping matrix on the seismic responses of non-proportionally damped one-way asymmetric systems. The specific aim of that study was to identify the range of system parameters for which this simplification can be used without introducing significant errors in the seismic responses of the asymmetric systems. Goel [2001] concluded that the aforementioned approximation method is suitable for use over a wide range of parameters. The error parameter becomes excessive when the value of the normalized supplemental damping eccentricity $\bar{e}_{sd}$ is close to -0.5. This conclusion indicates that the stated approximation method should not be used for asymmetric-plane systems with a large normalized supplemental damping eccentricity.

The proposed method of this study is the modal analysis using the 3DOF modal equations of motion [Lin and Tsai, 2008a] instead of the single-degree-of-freedom (SDOF) modal equations of motion, which are obtained by neglecting the off-diagonal terms of the transformed damping matrix. The stated 3DOF modal equations of motion, which are sets of three coupled equations, are obtained by using the partition of matrices. The effectiveness of this proposed method is verified by two numerical examples.

2. THEORETICAL BACKGROUND

2.1. SDOF Modal Equations of Motions

The equation of motion for a typical N-story building where each floor is represented by a rigid diaphragm with three DOFs (two are translational DOFs and the other one is rotational DOF) is:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\boldsymbol{\iota}\ddot{u}_g \tag{2.1}$$

where the $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ corresponds to the mass, damping and stiffness matrices related to the deformation $\mathbf{u}$(t), $\boldsymbol{\iota}$ is the influence vector, and $\ddot{u}_g(t)$ is the ground acceleration. The damping matrix can be expressed as:

$$\mathbf{C} = \mathbf{C}_0 + \mathbf{C}_{sd} \tag{2.2}$$

where $\mathbf{C}_0$ is the inherent damping matrix and $\mathbf{C}_{sd}$ is the damping matrix due to supplemental dampers. The matrix $\mathbf{C}_0$ is defined as:

$$\mathbf{C}_0 = \alpha\mathbf{M} + \beta\mathbf{K} \tag{2.3}$$

where α and β are determined by the damping ratios of two specific modes. The transformed damping matrix is equal to

$$\mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi} = \begin{bmatrix}\boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \cdots & \boldsymbol{\varphi}_{3N}\end{bmatrix}^T \mathbf{C}\begin{bmatrix}\boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \cdots & \boldsymbol{\varphi}_{3N}\end{bmatrix} \tag{2.4}$$

where $\boldsymbol{\varphi}_i$ is the i-th mode shape of the undamped system. In general, the transformed damping matrix is not a diagonal matrix for non-proportionally damped structures, i.e. $\boldsymbol{\varphi}_n^T\mathbf{C}\boldsymbol{\varphi}_m \neq 0, \quad m \neq n$. The conventional approximation method is neglecting the off-diagonal terms of the transformed damping matrix. By using the stated conventional approximation method, Eqn. 2.1 is decomposed into $3N$ SDOF modal equations of motion:

$$\ddot{D}_n + 2\omega_n\xi_n\dot{D}_n + \omega_n^2 D_n = -\ddot{u}_g(t) \qquad n = 1 \sim 3N \tag{2.5}$$

in which D_n is the n-th modal coordinate. The corresponding damping ratio, ξ_n, and the square of the circular frequency, ω_n^2, are

$$\xi_n = \frac{\boldsymbol{\varphi}_n^T \mathbf{C} \boldsymbol{\varphi}_n}{2\omega_n \boldsymbol{\varphi}_n^T \mathbf{M} \boldsymbol{\varphi}_n} \qquad \omega_n^2 = \frac{\boldsymbol{\varphi}_n^T \mathbf{K} \boldsymbol{\varphi}_n}{\boldsymbol{\varphi}_n^T \mathbf{M} \boldsymbol{\varphi}_n} \tag{2.6}$$

By solving Eqn. 2.5 to obtain $\mathbf{u}_n(t)$, the displacement history of the non-proportionally damped system is approximated as:

$$\mathbf{u}(t) = \sum_{n=1}^{3N} \mathbf{u}_n(t) = \sum_{n=1}^{3N} \Gamma_n \boldsymbol{\varphi}_n D_n(t) \tag{2.7}$$

where Γ_n is the n-th modal participating factor defined as:

$$\Gamma_n = \frac{\boldsymbol{\varphi}_n^T \mathbf{M} \boldsymbol{\iota}}{\boldsymbol{\varphi}_n^T \mathbf{M} \boldsymbol{\varphi}_n} \tag{2.8}$$

2.2. *3DOF modal equations of motion*

The motivation of deriving the 3DOF modal equations of motion and the verification of the consistency of the stated equations with the SDOF modal equations of motion are shown in Lin and Tsai [2008a]. In order to keep the completeness of this paper, the derivation of the 3DOF modal equations of motion is briefly restated in this section. The coordinate system adopted in this study, which X- and Z-axis are the two horizontal axes and the Y-axis is vertically upward, is the same as that used in Lin and Tsai [2008a]. When the building is under bi-directional seismic ground motions, the right-hand side of Eqn. 2.1 is written as:

$$\begin{aligned} & -\mathbf{M}\boldsymbol{\iota}_x \ddot{u}_{gx}(t) - \mathbf{M}\boldsymbol{\iota}_z \ddot{u}_{gz}(t) \\ & = -\sum_{n=1}^{3N} \Gamma_{xn} \mathbf{M} \boldsymbol{\varphi}_n \ddot{u}_{gx}(t) - \sum_{n=1}^{3N} \Gamma_{zn} \mathbf{M} \boldsymbol{\varphi}_n \ddot{u}_{gz}(t) \\ & = -\sum_{n=1}^{3N} \left(\Gamma_{xn} \ddot{u}_{gx} + \Gamma_{zn} \ddot{u}_{gz}\right) \mathbf{M} \boldsymbol{\varphi}_n = -\sum_{n=1}^{3N} \left(\Gamma_{xn} \ddot{u}_{gx} + \Gamma_{zn} \ddot{u}_{gz}\right) \mathbf{s}_n \end{aligned} \tag{2.9}$$

where $\mathbf{s}_n$ is equal to $\mathbf{M}\boldsymbol{\varphi}_n$ and $\Gamma_{xn}\ddot{u}_{gx}+\Gamma_{zn}\ddot{u}_{gz}$ is the synthetic ground motion for the n-th mode. Γ_{xn} and Γ_{zn} are the n-th X-directional and Z-directional modal participation factors, respectively, defined as:

$$\Gamma_{xn} = \frac{\boldsymbol{\varphi}_n^T \mathbf{M} \boldsymbol{\iota}_x}{\boldsymbol{\varphi}_n^T \mathbf{M} \boldsymbol{\varphi}_n}, \quad \Gamma_{zn} = \frac{\boldsymbol{\varphi}_n^T \mathbf{M} \boldsymbol{\iota}_z}{\boldsymbol{\varphi}_n^T \mathbf{M} \boldsymbol{\varphi}_n}, \quad \boldsymbol{\iota}_x = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}_{3N\times 1}, \quad \boldsymbol{\iota}_z = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \end{bmatrix}_{3N\times 1} \tag{2.10}$$

It is assumed that only the n-th modal displacement, $\mathbf{u}_n$, of the non-proportionally damped system will be excited under the excitation of $-(\Gamma_{xn}\ddot{u}_{gx}+\Gamma_{zn}\ddot{u}_{gz})\mathbf{s}_n$, namely,

$$\mathbf{M}\ddot{\mathbf{u}}_n + \mathbf{C}\dot{\mathbf{u}}_n + \mathbf{K}\mathbf{u}_n = -\left(\Gamma_{xn}\ddot{u}_{gx} + \Gamma_{zn}\ddot{u}_{gz}\right)\mathbf{s}_n \quad n = 1 \sim 3N \tag{2.11}$$

The mass, damping and stiffness matrices shown in Eqn. 2.11 are partitioned as:

$$\mathbf{M} = \begin{bmatrix} \mathbf{m}_x & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{m}_z & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_0 \end{bmatrix}_{3N\times 3N} \quad \mathbf{C} = \begin{bmatrix} \mathbf{c}_{xx} & \mathbf{c}_{xz} & \mathbf{c}_{x\theta} \\ \mathbf{c}_{zx} & \mathbf{c}_{zz} & \mathbf{c}_{z\theta} \\ \mathbf{c}_{\theta x} & \mathbf{c}_{\theta z} & \mathbf{c}_{\theta\theta} \end{bmatrix}_{3N\times 3N} \quad \mathbf{K} = \begin{bmatrix} \mathbf{k}_{xx} & \mathbf{k}_{xz} & \mathbf{k}_{x\theta} \\ \mathbf{k}_{zx} & \mathbf{k}_{zz} & \mathbf{k}_{z\theta} \\ \mathbf{k}_{\theta x} & \mathbf{k}_{\theta z} & \mathbf{k}_{\theta\theta} \end{bmatrix}_{3N\times 3N} \tag{2.12}$$

where $\mathbf{m}_x$, $\mathbf{m}_z$ and $\mathbf{I}_0$ are the X-directional mass and the Z-directional mass and the mass moment of inertia of the building system, respectively. The subscript x, z and θ denote the sub-matrix relating to X-translational, Z-translational and Y-rotational degrees of freedom, respectively. The n-th modal displacement is also partitioned as:

$$\mathbf{u}_n=\begin{bmatrix}\mathbf{u}_{xn}\\ \mathbf{u}_{zn}\\ \mathbf{u}_{\theta n}\end{bmatrix}_{3N\times 1}=\begin{bmatrix}D_{xn}\boldsymbol{\varphi}_{xn}\\ D_{zn}\boldsymbol{\varphi}_{zn}\\ D_{\theta n}\boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3N\times 1}=\begin{bmatrix}\boldsymbol{\varphi}_{xn} & 0 & 0\\ 0 & \boldsymbol{\varphi}_{zn} & 0\\ 0 & 0 & \boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3N\times 3}\begin{bmatrix}D_{xn}\\ D_{zn}\\ D_{\theta n}\end{bmatrix}_{3\times 1} \tag{2.13}$$

where $\boldsymbol{\varphi}_{xn}$, $\boldsymbol{\varphi}_{zn}$ and $\boldsymbol{\varphi}_{\theta n}$ are the components of the n-th undamped mode shape associated with X- and Z-translational and Y-rotational DOFs, respectively, i.e. $\boldsymbol{\varphi}_n=\begin{bmatrix}\boldsymbol{\varphi}_{xn}^T & \boldsymbol{\varphi}_{zn}^T & \boldsymbol{\varphi}_{\theta n}^T\end{bmatrix}^T$. If D_{xn}, D_{zn} and $D_{\theta n}$ are equal to each other, Eqn. 2.13 would be the same as the conventional definition of $\mathbf{u}_n$. By pre-multiplying both sides of Eqn. 2.11 with $\begin{bmatrix}\boldsymbol{\varphi}_{xn} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\varphi}_{zn} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3N\times 3}^T$ and substituting Eqn. 2.13 into it, Eqn. 2.11 becomes:

$$\mathbf{M}_n\ddot{\mathbf{D}}_n+\mathbf{C}_n\dot{\mathbf{D}}_n+\mathbf{K}_n\mathbf{D}_n=-\mathbf{M}_n\mathbf{1}\left(\Gamma_{xn}\ddot{u}_{gx}+\Gamma_{zn}\ddot{u}_{gz}\right),\quad n=1\sim 3N \tag{2.14}$$

where

$$\mathbf{D}_n=\begin{bmatrix}D_{xn}\\ D_{zn}\\ D_{\theta n}\end{bmatrix}_{3\times 1}\qquad \mathbf{M}_n=\begin{bmatrix}\boldsymbol{\varphi}_{xn}^T\mathbf{m}_x\boldsymbol{\varphi}_{xn} & 0 & 0\\ 0 & \boldsymbol{\varphi}_{zn}^T\mathbf{m}_z\boldsymbol{\varphi}_{zn} & 0\\ 0 & 0 & \boldsymbol{\varphi}_{\theta n}^T\mathbf{I}_0\boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3\times 3}\qquad \mathbf{1}=\begin{bmatrix}1\\1\\1\end{bmatrix}_{3\times 1} \tag{2.15}$$

$$\mathbf{C}_n=\begin{bmatrix}\boldsymbol{\varphi}_{xn}^T\mathbf{c}_{xx}\boldsymbol{\varphi}_{xn} & \boldsymbol{\varphi}_{xn}^T\mathbf{c}_{xz}\boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{xn}^T\mathbf{c}_{x\theta}\boldsymbol{\varphi}_{\theta n}\\ \boldsymbol{\varphi}_{zn}^T\mathbf{c}_{zx}\boldsymbol{\varphi}_{xn} & \boldsymbol{\varphi}_{zn}^T\mathbf{c}_{zz}\boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{zn}^T\mathbf{c}_{z\theta}\boldsymbol{\varphi}_{\theta n}\\ \boldsymbol{\varphi}_{\theta n}^T\mathbf{c}_{\theta x}\boldsymbol{\varphi}_{xn} & \boldsymbol{\varphi}_{\theta n}^T\mathbf{c}_{\theta z}\boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{\theta n}^T\mathbf{c}_{\theta\theta}\boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3\times 3}\qquad \mathbf{K}_n=\begin{bmatrix}\boldsymbol{\varphi}_{xn}^T\mathbf{k}_{xx}\boldsymbol{\varphi}_{xn} & \boldsymbol{\varphi}_{xn}^T\mathbf{k}_{xz}\boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{xn}^T\mathbf{k}_{x\theta}\boldsymbol{\varphi}_{\theta n}\\ \boldsymbol{\varphi}_{zn}^T\mathbf{k}_{zx}\boldsymbol{\varphi}_{xn} & \boldsymbol{\varphi}_{zn}^T\mathbf{k}_{zz}\boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{zn}^T\mathbf{k}_{z\theta}\boldsymbol{\varphi}_{\theta n}\\ \boldsymbol{\varphi}_{\theta n}^T\mathbf{k}_{\theta x}\boldsymbol{\varphi}_{xn} & \boldsymbol{\varphi}_{\theta n}^T\mathbf{k}_{\theta z}\boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{\theta n}^T\mathbf{k}_{\theta\theta}\boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3\times 3}$$

Eqn. 2.14 is the so-called n-th 3DOF modal equation of motion. Each 3DOF modal equation of motion has a corresponding 3DOF modal stick [Lin and Tsai, 2008a]. D_{xn}, D_{zn} and $D_{\theta n}$ are denoted as the modal translations and the modal rotation of the n-th mode, respectively. The modal damping matrix, $\mathbf{C}_n$, given in Eqn. 2.15 is equal to:

$$\mathbf{C}_n=\begin{bmatrix}\boldsymbol{\varphi}_{xn} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\varphi}_{zn} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3N\times 3}^T\mathbf{C}\begin{bmatrix}\boldsymbol{\varphi}_{xn} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\varphi}_{zn} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3N\times 3} \tag{2.16}$$

If the original MDOF building is a proportionally damped system, i.e.

$$\mathbf{C}=\begin{bmatrix}\mathbf{c}_{xx} & \mathbf{c}_{xz} & \mathbf{c}_{x\theta}\\ \mathbf{c}_{zx} & \mathbf{c}_{zz} & \mathbf{c}_{z\theta}\\ \mathbf{c}_{\theta x} & \mathbf{c}_{\theta z} & \mathbf{c}_{\theta\theta}\end{bmatrix}_{3N\times 3N}=\alpha\mathbf{M}+\beta\mathbf{K} \tag{2.17}$$

, the modal damping matrix would be:

$$\mathbf{C}_n=\begin{bmatrix}\boldsymbol{\varphi}_{xn} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\varphi}_{zn} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3N\times 3N}^T(\alpha\mathbf{M}+\beta\mathbf{K})\begin{bmatrix}\boldsymbol{\varphi}_{xn} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\varphi}_{zn} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{\varphi}_{\theta n}\end{bmatrix}_{3N\times 3N}=\alpha\mathbf{M}_n+\beta\mathbf{K}_n \tag{2.18}$$

Therefore, if the original MDOF building is a non-proportionally damped system, i.e.

$$\mathbf{C} \neq \alpha\mathbf{M} + \beta\mathbf{K} \tag{2.19}$$

, the modal damping matrix would also be non-proportional, i.e.

$$\mathbf{C}_n \neq \alpha\mathbf{M}_n + \beta\mathbf{K}_n \tag{2.20}$$

It implies that a non-proportionally damped system will result in 3N non-proportionally damped 3DOF modal equations of motion, which are able to take the out-of-phase motions between the modal translations and the modal rotation into account. Thus, the 3DOF modal equations of motion are more appropriate to be used in the modal analysis of non-proportionally damped two-way asymmetric structures than the SDOF modal equations of motion. The n-th modal displacement history, $\mathbf{D}_n(t)$, is obtained by direct integration of the corresponding 3DOF modal equation of motion, Eqn. 2.14. The total displacement history of the non-proportionally damped two-way asymmetric building is calculated as:

$$\mathbf{u}(t) \approx \sum_{n=1}^{p} \mathbf{u}_n(t) = \sum_{n=1}^{p} \begin{bmatrix} \boldsymbol{\varphi}_{xn} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\varphi}_{zn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\varphi}_{\theta n} \end{bmatrix}_{3N\times 3} \mathbf{D}_n(t) = \sum_{n=1}^{p} \begin{bmatrix} \boldsymbol{\varphi}_{xn} D_{xn}(t) \\ \boldsymbol{\varphi}_{zn} D_{zn}(t) \\ \boldsymbol{\varphi}_{\theta n} D_{\theta n}(t) \end{bmatrix}_{3N\times 1} \tag{2.21}$$

where p is the number of modes to be used in the modal analysis, $p \leq 3N$. The 3DOF modal equations of motion possess the non-proportionally damped property at the expense of increasing two DOFs in the modal coordinate. The proposed 3DOF modal equations of motion still can be easily computed by commercially available structural analysis programs. On the other hand, the proposed method keeps the clarity and the simplicity of the modal analysis in calculating the seismic responses of structures.

3. ANALYTICAL EXAMPLE

3.1. Selected Structural System, Ground Motion and Basic Assumptions

The one-story and the three-story asymmetric building with viscous dampers shown in Fig. 1 are analyzed by three methods, which include the direct integration of the equation of motion, conventional modal analysis and the proposed method. The results obtained by using the direct integration of the equation of motion are the benchmark solutions in this study. All of the beams and columns of the noted prototype buildings are symmetric making the CR coincident with the geometric center of each floor. The CM is eccentrically located as shown in Fig. 1. The left and the upper sides of CR, shown in Fig. 1(c), are denoted as the stiff sides. The sides opposite to the stiff sides are denoted as flexible sides. It is seen in this research that the analytical errors resulted from the use of the conventional modal analysis are great when the center of supplemental damping (CSD) is on the stiff sides of each floor. Thus, the viscous dampers are purposely placed on the stiff sides in order to intensify the demonstration of the accuracy of the analytical results obtained by using the proposed method. According to the investigation of the errors in responses of the one-story one-way asymmetric buildings [Goel, 2001], the errors introduced by conventional approximate method are over 20% when the normalized supplemental damping eccentricity, $\bar{e}_{sd} = e_{sd}/a$, is equal to -0.5. The values of e_{sd} and a represent the distance from CM to CSD and the plane dimension of the building perpendicular to the seismic ground motion, respectively. The CM is eccentrically located making the values of normalized supplemental damping eccentricities in two horizontal directions both equal to -0.75. Therefore, choosing $\bar{e}_{sd} = -0.75$ is large enough to verify the effectiveness of the proposed method. The damping coefficients of dampers, C_x and C_z, along the X-axis and Z-axis are calculated as:

$$C_x = 2m_x\omega_x\xi_{sdx} \qquad C_z = 2m_z\omega_z\xi_{sdz} \tag{3.1}$$

where ξ_{sdx} and ξ_{sdz} are the supplemental damping ratios along the X- and Z-axis, respectively. ω_x and ω_z are the vibration circular frequencies of the first X- and Z-translational dominant modes, respectively. The supplemental damping ratios, ξ_{sdx} and ξ_{sdz}, used in these two prototype buildings are both equal to 30%. The properties of the one-storey buildings are shown in Tables 1 and 2. The units used in these tables are *kN*, *m* and *sec*. The n-th column vector of the matrix $\mathbf{\Phi}$ shown in Table 2 is the n-th undamped mode shape, φ_n, of the original MDOF building. The matrices $\mathbf{\Phi}$ shown in Table 2 have been normalized which make $\mathbf{\Phi}^T\mathbf{M\Phi}$ equal to identity matrices. The n-th diagonal elements of matrices $\mathbf{\Phi}^T\mathbf{C\Phi}$ and $\Lambda^{1/2}$ shown in Table 2 are the values of $2\omega_n\xi_n$ and ω_n, respectively. The noted values are used in the n-th SDOF modal equation of motion shown in Eqn. 2.5. The floors are simulated as rigid diaphragms. The Rayleigh damping is assumed as the inherent damping of the two prototype buildings. The damping ratios of the first and the third mode of the two prototype buildings are specified as 2%. The properties of the three-storey building and the corresponding vibration modes can be found in Lin and Tsai [2008b].
The ground acceleration records used in this study are the NS and EW components of 1940 El Centro earthquake. The noted NS and EW components of ground acceleration records are scaled down and applied along the Z- and X-axis, respectively. The peak ground accelerations (PGA) of NS/EW components are equal to 0.14g/0.086g and 0.1g/0.061g for the one-story and the three-story building, respectively. The two buildings both remain elastic under the excitation of the noted ground motions.

Table 1. The properties of the one-story building

M			**C**						**K**		
			$\mathbf{C}_0$			$\mathbf{C}_{sd}$					
9.45		*symm.*	10.355		*symm.*	161.48		*symm.*	8638.4		*symm.*
0	9.45		-0.002	8.053		0	114.32		-3.226	4599	
0	0	23.03	-3.704	2.947	43.688	-363.3	385.84	2119.7	-6501	5171.3	53437

Table 2. The eigenvectors, $\mathbf{\Phi}$, eigenvalues, Λ, and the transformed damping matrix of the one-story building

$\mathbf{\Phi}=\begin{bmatrix}\phi_{x1} & \phi_{x2} & \phi_{x3}\\ \phi_{z1} & \phi_{z2} & \phi_{z3}\\ \phi_{\theta1} & \phi_{\theta2} & \phi_{\theta3}\end{bmatrix}$			$\Lambda^{1/2}=\begin{bmatrix}\omega_1 & 0 & 0\\ 0 & \omega_2 & 0\\ 0 & 0 & \omega_3\end{bmatrix}$			$\mathbf{\Phi}^T\mathbf{C\Phi}$		
0.06163	0.30769	-0.08573	20.163	0	0	3.9103		*symm.*
-0.31137	0.07734	0.05375	0	28.48	0	-5.5397	13.956	
0.04562	0.04604	0.19804	0	0	50.035	-9.4736	1.7029	107.21

3.2. Seismic responses of one-story and three-story building

The analytical results obtained by using direct integration of the equation of motion, modal analysis with SDOF and 3DOF modal equations of motion are denoted as RHA, SMA and 3MA, respectively, in the following contents of this paper. The 3×3 $\mathbf{M}_n$, $\mathbf{C}_n$ and $\mathbf{K}_n$ matrices, defined in Eqs. 2.14 and 2.15, of the first three modes of the one-story building are shown in Table 3. The sum of the nine elements of matrix $\mathbf{M}_n$ is equal to one. The sum of the nine elements of matrix $\mathbf{C}_n$ and $\mathbf{K}_n$ are equal to the values of the n-th diagonal element of the matrix $\mathbf{\Phi}^T\mathbf{C\Phi}$ and Λ, respectively, shown in Table 2. The modal translations, D_{xn} and D_{zn}, and the modal rotation, $D_{\theta n}$, of the elastic one-story building calculated by using 3MA are no longer equal to each other as shown in Fig. 2. It is caused by the non-proportional damping effect. The total responses of this non-proportionally damped building obtained by using 3MA and SMA compared with those obtained by using RHA are shown in Figs. 3(a) and 3(b), respectively. Fig. 3(a) shows that the analytical results obtained by using 3MA are almost the same as those obtained by using RHA. Fig. 3(b) shows that the responses obtained by using SMA are obviously deviated from the benchmark solutions. The errors of the peak X- and Z- translational and Y-rotational responses obtained by using SMA are equal to 19.5%, 0.04% and 31.9%, respectively. The analytical results of the three-storey building are not shown here and can be found in Lin and Tsai [2008b].

From the verification of these two examples, it shows that the 3DOF modal equations of motion possessing the non-proportional damping characteristic are more appropriate for the modal analysis of non-proportionally damped two-way asymmetric buildings. Since the errors of the translational and the rotational responses simultaneously occurred at CM, it may amplify the errors of the translational responses at corners and deteriorate the applicability of the conventional approximation method to asymmetric buildings. The proposed 3MA approach provides a better alternative to deal with this kind of structures.

Table 3. The modal matrices, $\mathbf{M}_n$, $\mathbf{C}_n$ and $\mathbf{K}_n$, of the one-story building

*n*th mode	$\mathbf{M}_n$			$\mathbf{C}_n$			$\mathbf{K}_n$		
*1*st	0.036		*symm.*	0.653		*symm.*	32.806		*symm.*
	0	0.916		0.000	11.864		0.062	445.870	
	0	0	0.048	-1.032	-5.523	4.503	-18.278	-73.462	111.230
*2*nd	0.895		*symm.*	16.268		*symm.*	817.820		*symm.*
	0	0.057		0.000	0.732		-0.077	27.507	
	0	0	0.049	-5.200	1.384	4.586	-92.096	18.414	113.280
*3*rd	0.069		*symm.*	1.263		*symm.*	63.490		*symm.*
	0	0.027		0.000	0.354		0.015	13.287	
	0	0	0.903	6.232	4.139	84.849	110.370	55.048	2095.800

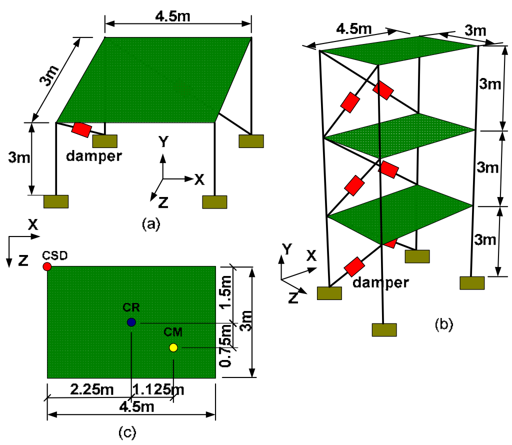

Figure 1. (a) one-story building
(b) three-story building
(c) typical floor plane of the two buildings.

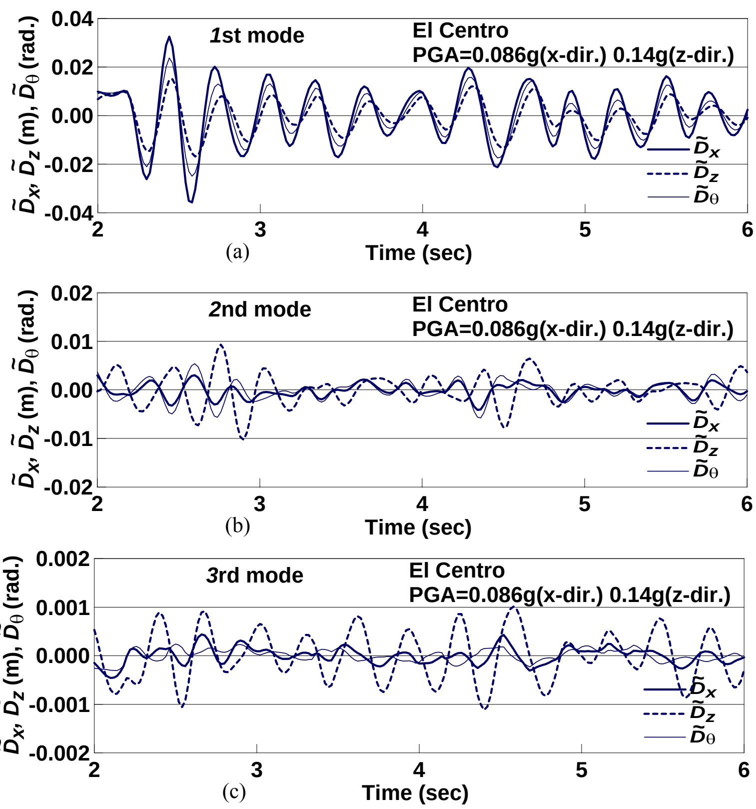

Figure 2. The (a) 1th (b) 2nd (c) 3rd modal responses of the one-story building obtained by using 3MA.

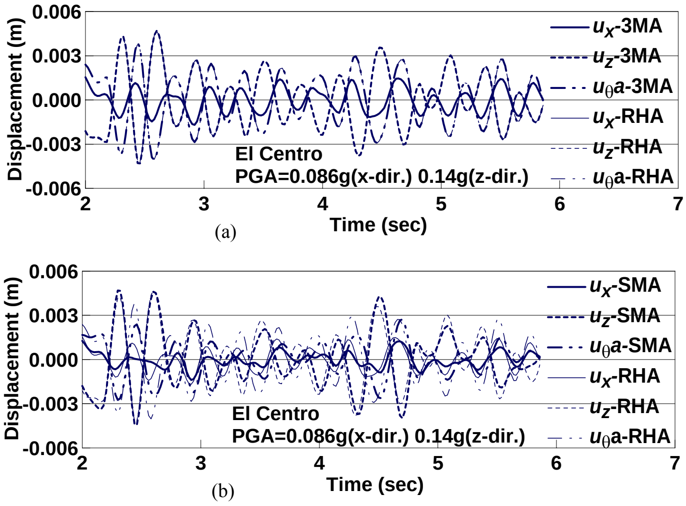

Figure 3. The total responses of the one-story building obtained by using (a) 3MA and RHA (b) SMA and RHA.

4. CONCLUSIONS

This study develops a method to analyze the seismic responses of non-proportionally damped two-way asymmetric buildings under bi-directional seismic ground motions. The proposed method is similar to the conventional modal analysis except using the 3DOF modal equations of motion instead of the SDOF modal equations of motion. The proportionalities of the damping matrices in the 3DOF modal equations of motion depend on that of the damping matrix of the original MDOF building. It makes the modal translations and modal rotation to be different from each other for a non-proportionally damped structure. The 3DOF modal equations of motion are closer to the realistic structural behavior than the SDOF modal equations of motion. The accuracy of the analytical results obtained by using the proposed method was verified by two numerical examples in this study. The proposed method inherits the advantages of the conventional modal analysis without the complexity of other developed methods. Hence, it is more appealing to the practical engineering.

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